2.2 Functions **Name:** ____________________

1. A function f is defined by the formula $f(x) = x^2 + 4$.
   (a) Express in words how f acts on the input x to produce output $f(x)$.
   (b) Evaluate $f(3)$, $f(-2)$ and $f(\sqrt{5})$.
   (c) Find the domain and range of f

2. Suppose $f(x) = 3x^2 + x - 5$. Evaluate the function at the values indicated below.
   (a) $f(-2)$ (a) __________
   (b) $f(0)$ (b) __________
   (c) $f(4)$ (c) __________
   (d) $f(\frac{1}{2})$ (d) __________

3. Suppose $C(x) = \left\{ \begin{array}{ll} 39 & \text{if } 0 \le x \le 400 \\ 39 + 0.20(x - 400) & \text{if } x > 400 \end{array} \right\}$. Evaluate the piecewise defined function at the values indicated below.
   (a) $C(100)$ (a) __________
   (b) $C(400)$ (b) __________
   (c) $C(480)$ (c) __________

4. Suppose $g(x) = \left\{ \begin{array}{ll} 2x & \text{if } x < 0 \\ x + 3 & \text{if } 0 \leq x \leq 2 \\ (x-1)^2 & \text{if } x > 2 \end{array} \right\}$. Evaluate the piecewise defined function at the values indicated below.

(a) $g(-5)$ (a) ___________

(b) $g(0)$ (b) ___________

(c) $g(1)$ (c) ___________

(d) $g(2)$ (d) ___________

(e) $g(5)$ (e) ___________

5. If an astronaut weighs 130 pounds on the surface of the earth, then her weight when when she is h miles above the earth is given by $w(h) = 130\left(\dfrac{3960}{3960 + h}\right)^2$. What is her weight when she is 100 mi above earth?

Definition 1. *If n is any positive integer, then the* *principal n^{th} root* *of a is defined as follows:*

$$\sqrt[n]{a} = b \quad \textit{means} \quad b^n = a$$

If n is even, we must have $a \geq 0$ and $b \geq 0$.

Definition 2. *The domain of a function is the set of all real-valued inputs which produce real-valued outputs .*

Find the domain of each function given below. Express the domain set using interval notation.

6. $C(x) = \dfrac{1}{x-5}$

7. $h(x) = \sqrt{x}$

8. $f(x) = \dfrac{2}{\sqrt{x}}$

9. $f(x) = \dfrac{2}{\sqrt[3]{2x-4}}$

10. $g(x) = \sqrt[3]{x}$

11. $f(x) = \dfrac{2}{\sqrt[4]{x}}$

12. $h(x) = \sqrt{9 - x^2}$

13. $f(x) = \dfrac{2}{\sqrt{2x-4}}$

14. Find all real values of x such that $f(x) = 0$ if $f(x) = -3x - 24$.

14. ___________

15. Find all real values of x such that $f(x) = 0$ if $f(x) = x^2 - 5x + 6$.

15. ___________

16. Determine if the equation $x^2 + y = 9$ represents y as a function of x.

16. ___________

17. Determine if the equation $y^2 + x = 9$ represents y as a function of x.

17. ___________

18. Evaluate and simplify the expression $\dfrac{f(x+h)-f(x)}{h}$ for $f(x) = 2x^2 - 1$. Assume x, h and $x+h$ are real numbers in the domain of f and that $h \neq 0$. Hint: You know you are finished when you can replace h with zero and not get a division by zero.

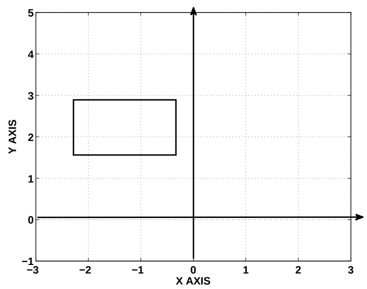
19. Evaluate and simplify the expression $\dfrac{f(x+h)-f(x)}{h}$ for $f(x) = x^2 - 3x + 2$.

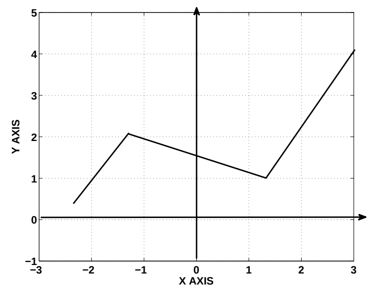
20. Evaluate and simplify the expression $\dfrac{f(x+h)-f(x)}{h}$ for $f(x) = \dfrac{3x}{1-x}$.

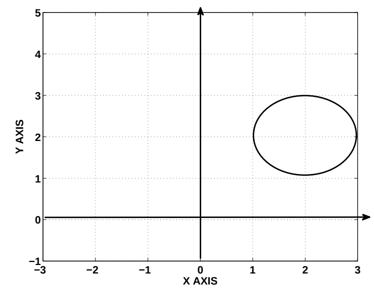
2.3 Analyzing Graphs of Functions **Name:** ____________________

1. Does the equation $x^2 + y^2 = 16$ define y as a function of x?

2. Use the Vertical Line Test to determine if the given graphs are the graphs of a functions.







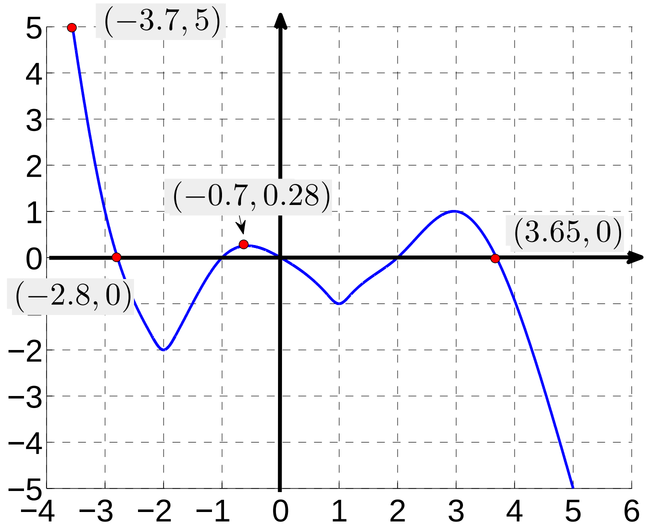
3. Find the zeros of each function.

(a) $f(x) = 3x^2 + x - 10$ (a) ____________

(b) $g(x) = \sqrt{9 - x^2}$ (b) ____________

(c) $h(t) = \dfrac{2t - 4}{t + 6}$ (c) ____________

4. (12 points) The graph of a function f is given in the figure (right). Assume the entire graph of the function is shown.



(a) Find all local and absolute maximum and minimum values of the function and the value of x at which each occurs.

(b) State the x intervals for which $f(x) > 0$.

(c) State the x intervals for which $f(x) < 0$.

(d) Find the intervals on which the function is ***increasing***.

(e) Find the intervals on which the function is ***decreasing***.

(f) Find $f(4)$. (f) ____________

(g) Find $f(5)$. (g) ____________

5. Find the average rates of change 0f $f(x) = x^3 - 3x$ from

(a) $x_1 = -2$ to $x_2 = -1$ (a) ____________

(b) $x_1 = 0$ to $x_2 = 2$ (b) ____________

6. The distance s (in feet) a moving car is from a stoplight is given by the function $s(t) = 20t^{3/2}$

7. Determine if the given function is even, odd or neither. If the graph of the function has symmetry, state which kind it has.

(a) $f(x) = -x + x^3$ (a) ____________

(b) $f(x) = 1 - x^2$ (b) ____________

(c) $f(x) = x^2 + 2x$ (c) ____________

2.4 Common Functions

Name: ______________________

1. Graph $C(x) = \left\{ \begin{array}{ll} 3 - x, & \text{if } x \leq -2 \\ 1 - x^2, & \text{if } x > -2 \end{array} \right\}$

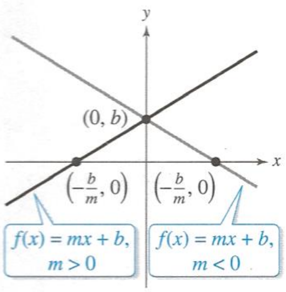
Graph $h(x) = \left\{ \begin{array}{ll} 2, & \text{if } x < -1 \\ \sqrt{1 - x^2}, & \text{if } -1 \leq x \leq 1 \\ 1 + x, & \text{if } x > 1 \end{array} \right\}$

3. Graph $f(x) = \left| x - 1 \right| = \left\{ \begin{array}{ll} -(x - 1), & \text{if } x < 1 \\ x - 1, & \text{if } x \geq 1 \end{array} \right\}$

GRAPHS OF PARENT FUNCTIONS

Linear Function

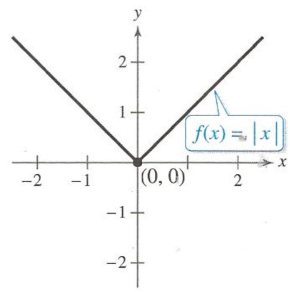
$f(x) = mx + b$



Domain: $(-\infty, \infty)$
Range: $(-\infty, \infty)$
x-intercept: $(-b/m, 0)$
y-intercept: $(0, b)$
Increasing when $m > 0$
Decreasing when $m < 0$

Absolute Value Function

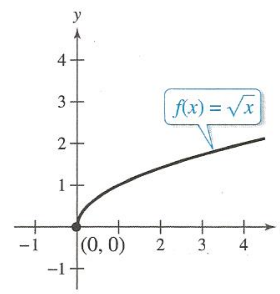
$$f(x) = |x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$$



Domain: $(-\infty, \infty)$
Range: $[0, \infty)$
Intercept: $(0, 0)$
Decreasing on $(-\infty, 0)$
Increasing on $(0, \infty)$
Even function
y-axis symmetry

Square Root Function

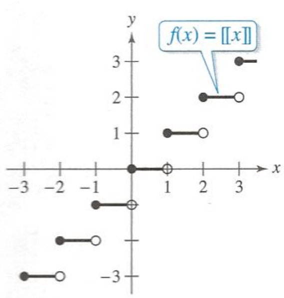
$f(x) = \sqrt{x}$



Domain: $[0, \infty)$
Range: $[0, \infty)$
Intercept: $(0, 0)$
Increasing on $(0, \infty)$

Greatest Integer Function

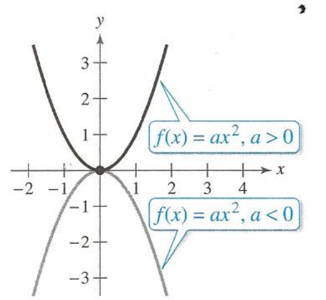
$f(x) = [\![x]\!]$



Domain: $(-\infty, \infty)$
Range: the set of integers
x-intercepts: in the interval $[0, 1)$
y-intercept: $(0, 0)$
Constant between each pair of consecutive integers
Jumps vertically one unit at each integer value

Quadratic (Squaring) Function

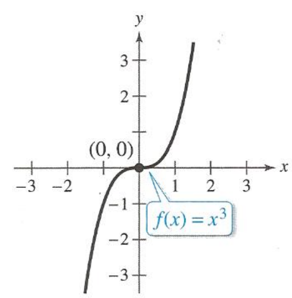
$f(x) = ax^2$



Domain: $(-\infty, \infty)$
Range $(a > 0)$: $[0, \infty)$
Range $(a < 0)$: $(-\infty, 0]$
Intercept: $(0, 0)$
Decreasing on $(-\infty, 0)$ for $a > 0$
Increasing on $(0, \infty)$ for $a > 0$
Increasing on $(-\infty, 0)$ for $a < 0$
Decreasing on $(0, \infty)$ for $a < 0$
Even function
y-axis symmetry
Relative minimum $(a > 0)$, relative maximum $(a < 0)$, or vertex: $(0, 0)$

Cubic Function

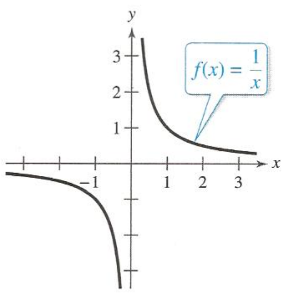
$f(x) = x^3$



Domain: $(-\infty, \infty)$
Range: $(-\infty, \infty)$
Intercept: $(0, 0)$
Increasing on $(-\infty, \infty)$
Odd function
Origin symmetry

Rational (Reciprocal) Function

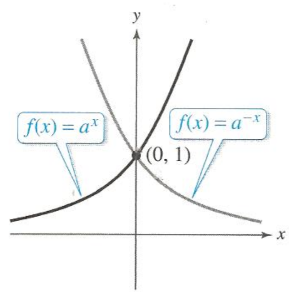
$f(x) = \dfrac{1}{x}$



Domain: $(-\infty, 0) \cup (0, \infty)$
Range: $(-\infty, 0) \cup (0, \infty)$
No intercepts
Decreasing on $(-\infty, 0)$ and $(0, \infty)$
Odd function
Origin symmetry
Vertical asymptote: y-axis
Horizontal asymptote: x-axis

Exponential Function

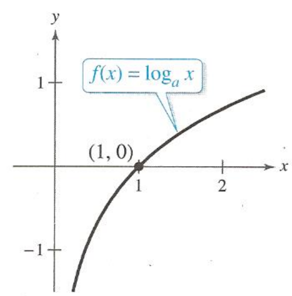
$f(x) = a^x,\ a > 1$



Domain: $(-\infty, \infty)$
Range: $(0, \infty)$
Intercept: $(0, 1)$
Increasing on $(-\infty, \infty)$
 for $f(x) = a^x$
Decreasing on $(-\infty, \infty)$
 for $f(x) = a^{-x}$
Horizontal asymptote: x-axis
Continuous

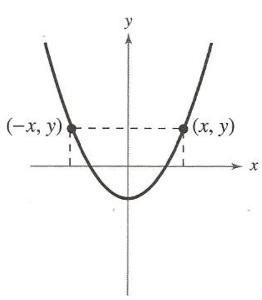
Logarithmic Function

$f(x) = \log_a x,\ a > 1$

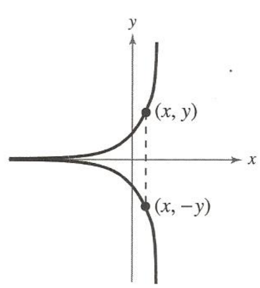


Domain: $(0, \infty)$
Range: $(-\infty, \infty)$
Intercept: $(1, 0)$
Increasing on $(0, \infty)$
Vertical asymptote: y-axis
Continuous
Reflection of graph of $f(x) = a^x$
 in the line $y = x$

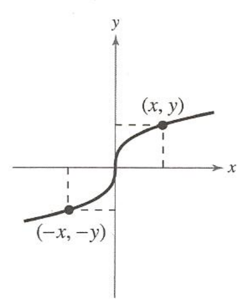
SYMMETRY



y-Axis Symmetry



x-Axis Symmetry



Origin Symmetry

2.5 Transformations **Name: ____________________**

Write an equation for the function described by the given characteristics.

1. The shape of $f(x) = |x|$, but shifted five units right and 3 units down.

2. The shape of $f(x) = x^3$, but shifted five units right and 3 units down.

3. The shape of $f(x) = x^2$, but shifted five units left and 3 units up.

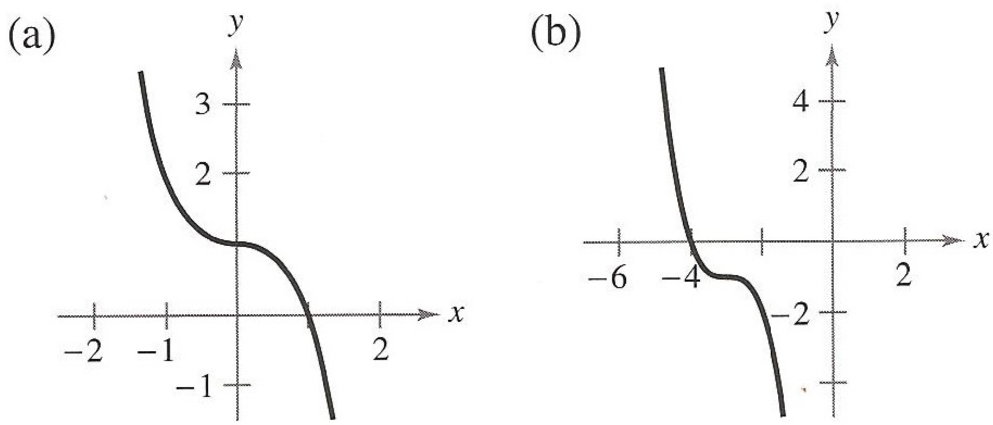
4. The shape of $f(x) = \sqrt{x}$, but shifted five units right, reflected in the x axis and 12 units down.

5. The shape of $f(x) = \sqrt[3]{x}$, but shifted four units left, reflected in the x axis and 3 units up.

6. The shape of $f(x) = \sqrt{x}$ shifted six units down and reflected in both the x axis and y axis.

7. $f(x) = \sqrt{9 - x^2}$; stretch vertically by a factor of 5, shift downward 10 units and shift 4 units right.

8. Use the graph of $f(x) = x^3$ to write an equation for each function.



Directions: Sketch the graph of the function, not by plotting points, but by starting with the graph of a standard function and applying transformations. Label at least 3 points on your final graph.

9. $C(x) = 2|x + 2| - 3$

10. $h(x) = 1 - \sqrt{x - 2}$

11. $R(x) = -2(x + 3)^2 - 1$

12. $P(x) = 2 - \sqrt{25 - x^2}$

2.6 Combining Functions **Name:** ______________________

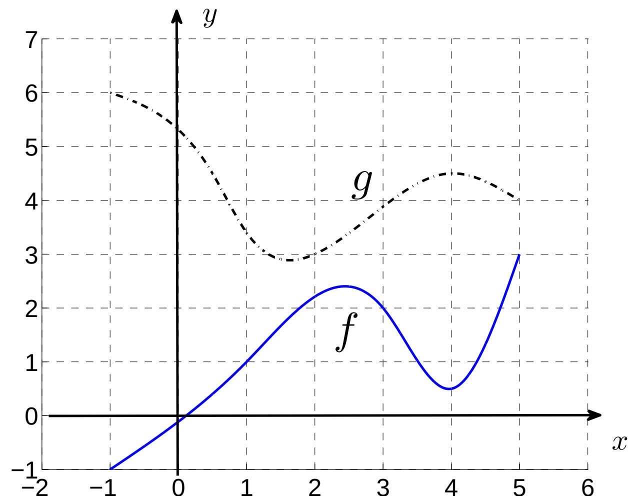
1. **Use the graph to find the indicated functional values.**

   (a) $(f+g)(3)$

   (b) $(f-g)(-1)$

   (c) $\dfrac{f}{g}(5)$

   (d) Find the domain and range of f



Find $f+g$, $f-g$, fg, and f/g and their domains.

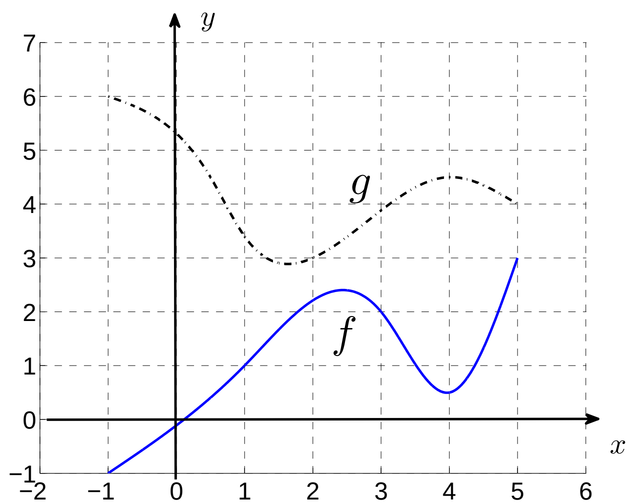
2. $f(x) = x^2 - 14x$ and $g(x) = x + 14$.

Find $f + g$ and f/g and their domains.

3. $f(x) = \sqrt{7 - x}$ and $g(x) = \sqrt{7 + x}$

4. $f(x) = \dfrac{7}{x - 14}$ and $g(x) = \dfrac{15}{x + 8}$

5. **Use the graph to find the indicated functional values.**



   (a) $f(g(3))$

   (b) $f(g(-1))$

   (c) $g(f(3))$

   (d) $g(g(3))$

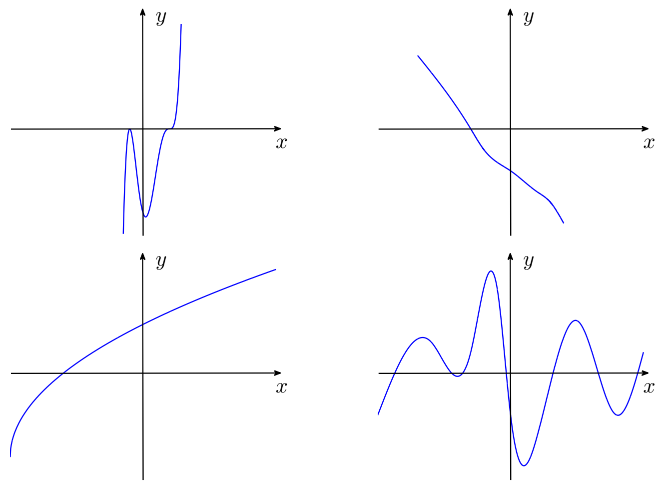
Find $f \circ g$ and $g \circ f$ and their domains.

6. $f(x) = x^2 + 7$ and $g(x) = x - 7$.

7. $f(x) = \dfrac{1}{x}$ and $g(x) = 4x - 9$.

2.7 Inverse Functions **Name:** ____________________

1. Determine whether each function f is one-to-one.



Determine whether f is one-to-one.

2. $f(x) = 2 - x - x^2$

4. $f(x) = 2(x-3)^4$

3. $f(x) = 4 + (x-3)^5$

5. $f(x) = x^{2/3}$

Assume that f is one-to-one

7. If $f(3) = -56$, find $f^{-1}(-56)$ and $(f(3))^{-1}$

8. If $f^{-1}(2) = 21$, find $f(21)$ and $(f^{-1}(2))^{-1}$

Find the inverse function of f.

10. $f(x) = \dfrac{3x}{x-2}$

11. $f(x) = (2 - x^3)^5$

12. $f(x) = x^3 + 5$

3.1 Quadratic Functions **Name:** ______________________

1. Find the vertex of $g(x) = 3(x-5)^2 + 7$. Does f open up or down?

1. ___________

2. What is the range of g?

2. ___________

3. Find the vertex of $g(x) = -2(x+8)^2 - 4$. Does f open up or down?

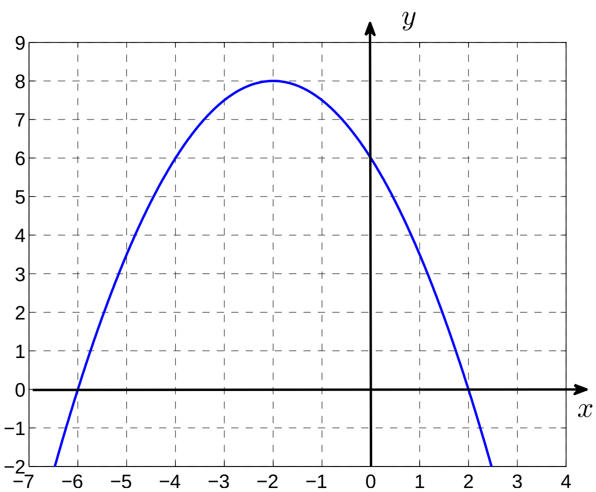
3. ___________

4. What is the range of g?

4. ___________

The graph of a quadratic function f is given. Find the coordinates of the vertex. Find the maximum or minimum of f. Find the intervals on which the function is increasing and on which the function is decreasing. Find the domain and range of f.

5. $f(x) = -\frac{1}{2}x^2 - 2x + 6$



Express the quadratic function in standard (vertex) form.

6. $g(x) = x^2 + 8x - 7$

7. $g(x) = x^2 + 10x - 4$

8. $g(x) = x^2 + 3x - 6$

9. $g(x) = 2x^2 + 3x - 7$

A quadratic function is given. Express the quadratic function in standard form. Find its vertex and its $x-$ and y- intercept(x). Sketch its graph. Then identify the vertex, axis of symmetry, domain and range.

10. $C(x) = -x^2 - 4x + 4$

11. $h(x) = 2x^2 + 4x - 5$

3.3 Dividing Polynomials

Name: ____________________

1. Divide $\dfrac{6x^3 - 9x^2 + 12x}{3x}$

1. __________

2. Divide $\dfrac{27x^4y^7 - 81x^5y^3}{-9x^3y^2}$

2. __________

3. Simplify $\dfrac{x^3 + 2x^2 - x - 2}{x - 1}$. Hint: Factor the numerator first (using the grouping technique) then divide out any common factors.

3. __________

4. Use long division to divide $\dfrac{x^3 + 2x^2 - x - 2}{x - 1}$.

4. __________

Find the quotient and remainder using long division.

5. $\dfrac{4x^2 - 3x - 7}{2x - 1}$

7. $\dfrac{2x^4 - x^3 + 9x^2}{x^2 + 4}$

6. $\dfrac{9x^2 - x + 5}{3x^2 - 7x}$

8. $\dfrac{x^3 + x^2 - 10x + 8}{x - 3}$

Find the quotient and remainder using synthetic division.

9. $\dfrac{x^3 + x^2 - 10x + 8}{x - 3}$

11. $\dfrac{x^3 - 7x + 3}{x + 4}$

10. $\dfrac{9x^2 - x + 5}{x + 5}$

12. $\dfrac{2x^3 + 5x^2 - 10x}{x - 3}$

Use synthetic division and the Remainder Theorem to evaluate $f(c)$.

13. $f(x) = x^3 - 8x^2 + 9x - 2;$
$c = -1$

14. $f(x) = 2x^5 - 13x^3 + 8x^2;$
$c = -4$

Use the Factor Theorem to show that $x - c$ is a factor of $f(x)$ for the given value of c.

15. $f(x) = x^3 + 2x^2 - 3x - 10;$
$c = 2$

16. $f(x) = x^4 + 3x^3 - 16x^2 - 27x + 63;$
$c = -3$

17. Use the Factor Theorem to find a polynomial function of degree 4 with zeros at $x = -2, 0, 2, 4$.

17. ___________

3.4 Rational Zeros Theorem **Name: ______________________**

Find all rational zeros of the polynomial, and write the polynomial in factored form.

1. $f(x) = x^3 + x^2 - 14x - 24$

2. $f(x) = 2x^4 - x^3 - 19x^2 + 9x + 9$

3. $f(x) = 6x^3 + 11x^2 - 3x - 2$

4. $f(x) = 6x^4 - 7x^3 - 12x^2 + 3x + 2$

3.2 Graphing Polynomials   Name: ____________________

Use the Leading Coefficient Test to determine the end behavior of each polynomial.

1. $f(x) = x^3 - x^2 + 14x - 8$

2. $f(x) = 2x^4 - x^3 - 19x^2 + 9x + 9$

3. $f(x) = -6x^3 + 11x^2 - 3x - 2$

4. $f(x) = -6x^4 - 7x^3 - 12x^2 + 3x + 2$

Solve the following polynomial inequalities.

5. $f(x) < 0$ for $f(x) = (x - 2)^2(x - 5)$

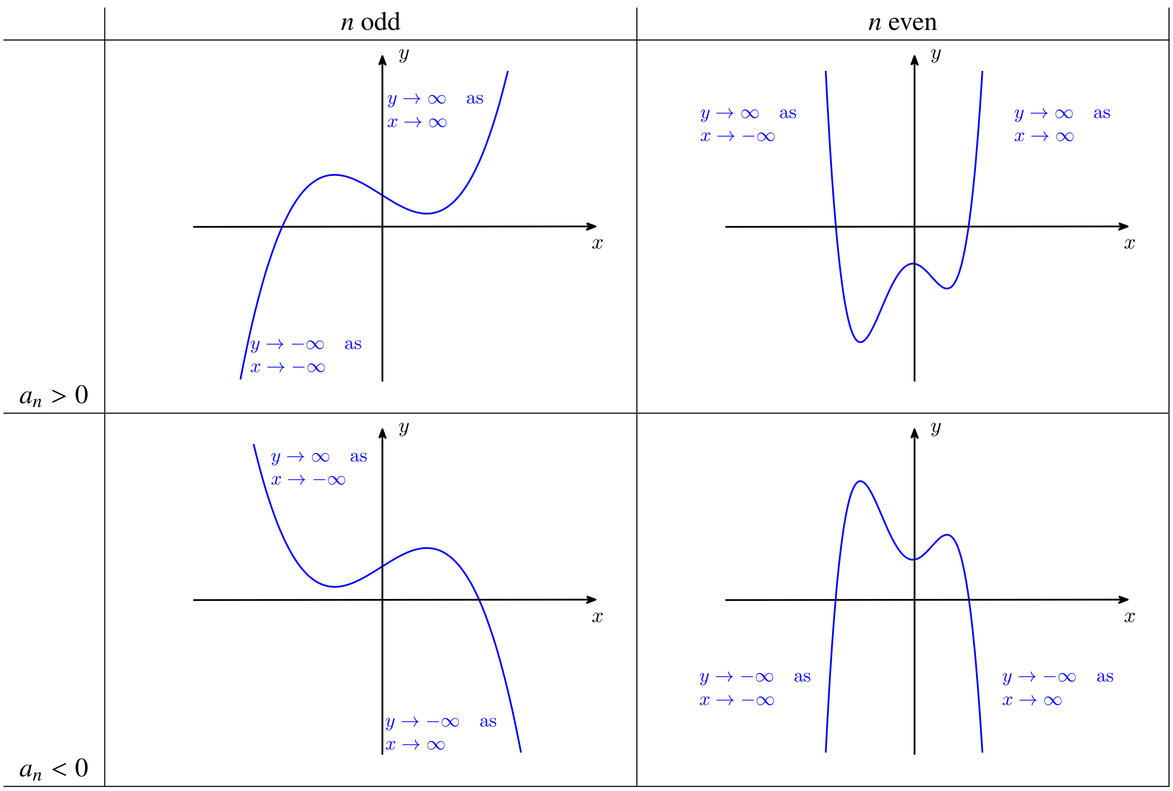
6. $f(x) > 0$ for $f(x) = x(x+1)^2(x-1)^3$

7. $-x^3 + 2x^2 + 4x \geq 8$

8. $x^5 - x^4 + 3x^3 - 3x^2 - 4x + 4 > 0$

GRAPHING POLYNOMIAL FUNCTIONS

1. Determine if the graph has any symmetry. Locate the y intercept.
2. Factor the polynomial and find the zeros.
3. Determine the x intervals for which $f(x) > 0$ (is above the y axis) and $f(x) < 0$ (is below the y axis).
4. Plot the zeros on the real number line. Label each zero as being either odd or even.
5. Make a table of values. Mark the end behavior of the graph.



6. Begin graphing starting at the left 'end behavior point' that you marked your graph with. Proceed to the first zero on the left:
   - if the zero is odd, pass through the x axis,
   - if the zero is even 'bounce off' the x axis.
7. Continue to the next zero and repeat the process.
8. When you have finished this procedure with the last zero on the right, the graph should connect with the right end behavior arrow that you marked in step 4.
9. As a 'check point' you can determine the sign (+ or -) of the y intercept (when $x = 0$) and see if the result is consistent with your graph.

9. Plot $f(x) = x^5 - x^4 + 3x^3 - 3x^2 - 4x + 4$

10. Plot $f(x) = x^3 + x^2 - 14x - 24$

11. Plot $f(x) = 2x^4 - x^3 - 19x^2 + 9x + 9$

Modeling and Variation **Name:** ____________________

Find a mathematical model for the verbal statement.

1. V varies directly as the square of r. 1. __________

2. y varies directly as the fifth root of x. 2. __________

3. h varies inversely as the square of r. 3. __________

4. y varies directly as the cube of x and inversely as the square of s.

   4. __________

5. V varies jointly as the square root of x and the cube y. 5. __________

6. The gravitational attraction F between two objects of masses m_1 and m_2 is jointly proportional to the masses and inversely proportional to the square of the distance *r* between the objects.

   6. __________

Write a sentence using the variation terminology of this section to describe the formula.

7. $r = d/t$ 7. __________

8. $V = \frac{4}{3}\pi r^3$ 8. __________

9. $V = \frac{1}{3}\pi r^3 h$ 9. __________

Find a mathematical model that represents the statement. Then determine the value of the constant of proportionality, *k*.

10. y varies inversely as x. ($y = 3$ when $x = 25$.) 10. __________

11. P varies directly as x and inversely as the square of y. It is known from experimental results that ($P = \frac{28}{3}$ when $x = 42$ and $y = 9$.)

    11. __________

4.1 – 4.2 Rational Functions **Name:** ________________________

Definition 3. *A* *Rational Function* *is a function that has the form:*

$$f(x) = \frac{p(x)}{q(x)}$$

where $p(x)$ *and* $q(x)$ *are polynomials. The domain of* f *is the set of all real numbers* x*, such that* $q(x) \neq 0$.

Examples:

$$f(x) = \frac{1}{x} \qquad f(x) = \frac{2-x}{3x+1} \qquad f(x) = \frac{4x-3}{x^2+3x-4}$$

Is $f(x)$ a rational function?

1. $f(x) = \dfrac{x^3 - x - 3}{x^{1/2} + 3}$ 1. ____________

Write $\text{dom}\big(f(x)\big)$ using interval notation.

2. $f(x) = \dfrac{x-2}{x^2+7x}$ 2. ____________

3. $f(x) = \dfrac{1-x^2}{x+3}$ 3. ____________

4. $f(x) = \dfrac{x^2-4}{x-2}$ 4. ____________

Finding Intercepts

- Set the numerator polynomial $p(x) = 0$ and solve for x to find the x intercept(s).

- Evaluate $f(0)$ to find the y intercept.

Find the intercepts of $f(x)$.

5. $f(x) = \dfrac{x - 2}{x^2 - 7x + 6}$ 5. ____________

6. $f(x) = \dfrac{1 - x^2}{x + 3}$ 6. ____________

Solving rational inequalities. Find all values of x for which $f(x) > 0$.

7. $f(x) = \dfrac{x^2 + 2x - 8}{x^3 + 2x^2}$ 7. ____________

8. $f(x) = \dfrac{(x+3)}{(x-5)^2}$

8. ___________

Identify the vertical asymptotes of $f(x)$, and write limit statements describing the behavior of the graph of f about its vertical asymptotes.

9. $f(x) = \dfrac{(x+3)}{(x-5)^2}$

9. ___________

10. $f(x) = \dfrac{x}{x^2 - 81}$

10. ___________

11. $f(x) = -\dfrac{2}{x+8}$ 11. ____________

Theorem 1. *Let* $f(x) = \dfrac{ax^n + ...}{bx^d + ...}$, *where* ax^n *and* bx^d *are the leading terms of the polynomial in the numerator and denominator. Note that n is the degree of the numerator polynomial and d is the degree of the denominator polynomial.*

1. *if* $n < d$, *then the HA occurs at* $y = 0$.
2. *if* $n = d$, *then the HA occurs at* $y = \dfrac{a}{b}$, *the ratio of the leading coefficients.*
3. *if* $n > d$, *then the graph of* $f(x)$ *has a slant asymptote; found by using long division to divide the polynomial in the numerator by the polynomial in the denominator. y equals the quotient is the equation representing the slant asymptote.*

End Behavior *As* $x \to \infty$ *and as* $x \to -\infty$, $f(x) \to c$, *where* $y = c$ *is the HA. If the graph has a slant asymptote, then the end behavior of f is the same as the end behavior of the the polynomial quotient after doing the long division.*

Identify the horizontal asymptote of $f(x)$. Then write a description of the end behavior of the graph of f.

12. $f(x) = \dfrac{4}{4x+3}$ 12. ____________

13. $f(x) = \dfrac{x+4}{2x-5}$ 13. ____________

14. $f(x) = \dfrac{x^2-3x+1}{x^2+6x-9}$ 14. ____________

15. $f(x) = \dfrac{x^2}{1-x^3}$ 15. ____________

16. $f(x) = \dfrac{x^3-2x+1}{x^2+1}$ 16. ____________

Graphing Rational Functions Algorithm

1. Write the numerator and denominator in factored form. Determine the locations of the x and y intercepts.

2. Determine the equation(s) that represent the Vertical Asymptote(s): set the denominator equal to zero and solve for x.

3. Determine the equation that represents the Horizontal or Slant Asymptote (HA or SA).

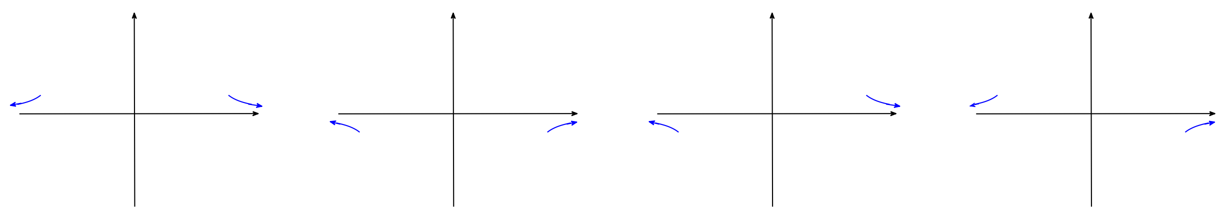
4. Write a statement describing the end behavior of the graph:

$$y \to HA \quad \text{as} \quad x \to \pm\infty \qquad \text{or} \qquad y \to SA \quad \text{as} \quad x \to \pm\infty$$

5. Determine the x intervals for which $f(x) < 0$ and $f(x) > 0$. Make a sign chart and use either the test point method or the multiplicity method to mark the sign (+ or -) of $f(x)$ for x in each interval.

6. Mark the "end behavior" of the graph, depending on whether $n < d$, $n = d$, or $n > d$; where n is the degree of the polynomial in the numerator and d is the degree of the polynomial in the denominator.

   Case 1: ($n < d$) The HA is $y = 0$. The end behavior of the graph will look like one of the following 4 pictures: Your inequality sign chart will indicate which one of the four you have;



   but you may have to compute a couple of y values be sure.

   Case 2: ($n = d$) The HA occurs at $y = \dfrac{a}{b}$. The end behavior will look like one of the four above graphs, but the HA is shifted up or down to $y = \dfrac{a}{b}$.

   Case 3: ($n > d$) The graph doesnt have a HA, it has a SA at $y = mx + b$, where $mx + b$ is the linear part of the resulting quotient, after the polynomial in the numerator has been divided by the polynomial in the denominator. The end behavior for the graph of the rational function will be the same as the end behavior of the SA. Draw the graph of $y = mx + b$ with a dotted line.

7. Identify and plot the zeros (also called roots) of the rational function.
   - Label these odd (O) or even (E) as in polynomial graphing
   - also plot the y intercept

8. Sketch the vertical asymptotes as vertical dotted lines: Label these VAs as being odd (O) or even (E) at the top of the dotted line depending on their multiplicities.

9. Determine the behavior of the graph (y value) around the VAs.

$$y \to \pm\infty \quad \text{as} \quad x \to a^- \quad \text{and} \quad y \to \pm\infty \quad \text{as} \quad x \to a^+$$

Whether or not y goes to plus or minus infinity depends on the sign of f(x) in that interval.

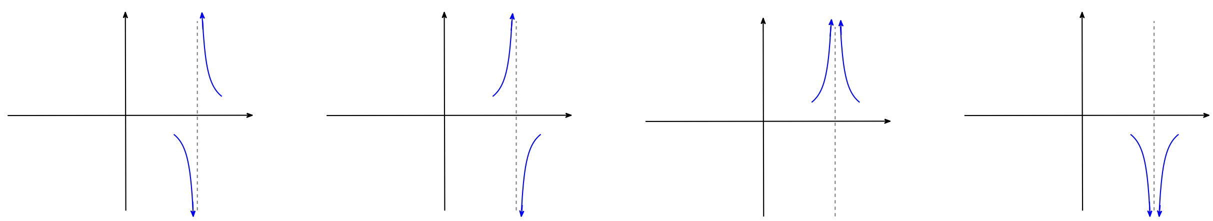
10. Begin graphing from the left end point and proceed to the first critical value
    - a critical value is a zero or a vertical asymptote
    - you may need to make a small table of values for better plotting accuracy.
11. At each zero proceed as in polynomial graphing, ie. If the zero is odd the graph goes thru the x axis; if the zero has even multiplicity, then it touches the axis and bounces off.
12. At each vertical asymptote, as you approach from the left go "north" to $+\infty$ or "south" to $-\infty$.

    At each **odd** vertical asymptote, as you step across the dotted line

    Switch signs from $+\infty$ or to $-\infty$ or from $-\infty$ or to $+\infty$

    At each **even** vertical asymptote, as you step across the dotted line

    Keep the **same signs** from $+\infty$ or to $+\infty$ or from $-\infty$ or to $-\infty$



13. Continue to the next critical value (zero or asymptote) repeat the process of step 8 or 9; ignore the vertical scale as you pass from one zero to the next; item When you have finished this procedure with the last zero on the right, the graph should connect with the right "endpoint" that you marked in step 4
14. As a "check point" you can determine the sign (+ or −) of the y intercept (when $x = 0$) and see if the result is consistent with your graph

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17. Graph $f(x) = \dfrac{1}{x}$

18. Graph $g(x) = \dfrac{1}{x+3}$

19. Graph $h(x) = -\dfrac{2}{x+3}$

20. Graph $w(x) = 1 - \dfrac{2}{x+3}$

4.3 Conics **Name:** ____________________

Find the focus and directrix of the parabola. Then sketch the parabola.

1. $y = -6x^2$ 1. __________

2. $y^2 = 3x$ 2. __________

Find the standard form of the equation of the parabola with the given characteristic(s) and vertex at the origin.

3. Focus: $(x, y) = \left(0, \frac{1}{2}\right)$ 3. __________

4. Directrix: $x = 4$ 4. __________

Find the standard form of the equation of the ellipse with the given characteristic(s) and center at the origin.

5. Vertices: $(x, y) = \left(0, \pm 8\right)$; Foci: $(x, y) = \left(0, \pm 4\right)$; 5. __________

6. Foci: $(x, y) = \left(\pm 2, 0\right)$; major axis of length 10 6. __________

Find the vertices of the ellipse then sketch the ellipse.

7. $\dfrac{x^2}{121} + \dfrac{y^2}{144} = 1$

8. $4x^2 + 9y^2 = 36$

Find the standard form of the equation of the hyperbola with the given characteristic(s) and center at the origin.

9. Vertices: $(x, y) = \left(\pm 4, 0\right)$; Foci: $(x, y) = \left(\pm 5, 0\right)$; 9. ___________

10. Vertices: $(x, y) = \left(\pm 4, 0\right)$; asymptotes: $y = \pm 3x$ 10. ___________

Find the vertices of the hyperbola then sketch the hyperbola using the asymptotes as an aid.

11. $\dfrac{y^2}{1} - \dfrac{x^2}{25} = 1$

12. $4x^2 - 9y^2 = 36$

4.4 Translated Conics **Name:** ____________________

Write the equation of a circle in standard form, and then find its center and radius.

1. $x^2 + y^2 - 10x - 6y + 25 = 0$ 1. __________

2. $2x^2 + 2y^2 - 2x - 2y - 7 = 0$ 2. __________

Find the standard form of the equation of the ellipse with the given characteristics.

3. Foci: $(x, y) = \left(0, 0\right),\ \left(0, 8\right);$ major axis of length 16 3. __________

Find the center, foci, and vertices of the ellipse then sketch the ellipse.

4. $16x^2 + 25y^2 - 32x + 50y + 16 = 0$

Find the vertex, focus and directrix of the parabola. Then sketch the parabola.

5. $(x + 2) + (y - 4)^2 = 0$ 5. ___________

Find the standard form of the equation of the parabola with the given characteristic(s).

6. Vertex: $(x, y) = \left(-1, 2\right)$; Focus: $(x, y) = \left(-1, 0\right)$ 6. ___________

Find the standard form of the equation of the hyperbola with the given characteristic(s).

7. Vertices: $\left(1, 2\right)$, $\left(5, 2\right)$; Foci: $\left(0, 2\right)$, $\left(6, 2\right)$ 7. ___________

Find the center, foci, and vertices of the hyperbola then sketch the hyperbola using the asymptotes as an aid.

8. $16y^2 - x^2 + 2x + 64y + 62 = 0$

5.1 Exponential Functions & their Graphs

Definition 4. *An exponential function with base number a is a function of the form*

$$f(x) = a^x,$$

where a and x are real numbers and

- *x is the independent VARIABLE of the function; and*
- *a is a number FIXED CONSTANT such that $a > 0$ and $a \neq 1$.*

Use translations theory to sketch the graphs of each function. What is the domain, range and horizontal asymptote for each? Write an end behavior statement for each.

1. $f(x) = 2^x$
2. $f(x) = 2^x + 3$
3. $f(x) = 1 - 2^x$
4. $f(x) = 2^{x+3}$
5. $f(x) = 2^{-x}$
6. $f(x) = 1 - 2^{-x+4}$

7. This is a ***Matching question*** associated with the theory on graphical translations of functions. Suppose $f(x) = 7^{-x}$. Relative to the graph of $f(x)$ the graphs of the following functions have been changed in what way?

____	$g(x) = 7^{-x} + 5$	a.) shifted 5 units left
____	$g(x) = 7^{(-x+5)}$	b.) reflected about the x axis
____	$g(x) = -2 \cdot 7^{-x}$	c.) shifted 5 units down
____	$g(x) = 7^{(-x-5)}$	d.) shifted 5 units right
____	$g(x) = 7^{-x} - 5$	e.) shifted 5 units vertically up

Use the One-to-One Property to solve each equation for x.

8. $2^{x-3} = 16$

9. $5^{x-2} = \dfrac{1}{125}$

Compound Interest Formula
If a principal P (dollars) is invested for t years at an annual rate r, and it is compounded n times per year, then the amount A, or ending balance, is given by

$$A = P\left(1 + \frac{r}{n}\right)^{n \cdot t}$$

10. Suppose you invest \$2500 at an interest rate of 3.5% per year. Find the amounts in the account after 3 years if interest is compounded annually, semi-annually, quarterly, monthly, and daily.

Compounding	n	Amount after 3 years
Annual	1	
Semiannual		
Quarterly		
Monthly		
Daily		
Continuously		

Use the One-to-One Property to solve each equation for x.

11. $e^{2x-3} = 1$

12. $e^{x^2-8} = \dfrac{1}{e^{2x}}$

5.2 Logarithmic Functions & their Graphs

Definition 5. *Let $a > 0$ and $a \neq 1$. Then $\log_a(x)$ is the exponent we raise the number a to get x.*

---

1. Write, in words, the meaning of $\log_5(25)$.

2. What number does $\log_5(25)$ represent? 2. ____________

3. Write, in words, the meaning of $\log_7(1)$.

4. What number does $\log_7(1)$ represent? 4. ____________

5. Write, in words, the meaning of $\log_{\frac{1}{2}}(16)$.

6. What number does $\log_{\frac{1}{2}}(16)$ represent? 6. ____________

Definition 6. *Let a be a positive number with $a \neq 1$. The logarithm function with base a, denoted* ***log***$_a$*, is defined by*

$$\left[\ \log_a(x) = y\ \right] \iff \left[\ a^y = x\ \right]$$

So $\log_a(x)$ is the exponent we raise a to get x.

Write each in logarithmic form.

7. $2^4 = 16$ 7. ____________

8. $10^4 = 10,000$ 8. ____________

9. $0.001 = 10^3$ 9. ____________

10. $\left(\frac{1}{3}\right)^{-2} = 9$ 10. ____________

11. $10^{x+5} = 10$ 11. ____________

Write each in exponential form.

12. $\log_3\left(\frac{1}{81}\right) = -4$ 12. ____________

13. $\log_7 49 = 2$ 13. ____________

14. $\log_5 125 = 3$ 14. ____________

15. $\log_4 x = \frac{1}{2}$ 15. ____________

16. $\log_x 9 = 2$ 16. ____________

Properties of Logarithms

Property	Reason
1. $\log_a(1) = 0$	We must raise a to the power 0 to get 1.
2. $\log_a(a) = 1$	We must raise a to the power 1 to get a.
3. $\log_a(a^x) = x$	We must raise a to the power x to get a^x.
4. $a^{\log_a(x)} = x$	$\log_a x$ is the power to which a must be raised to get x.

Evaluate Each Expression.

17. $\log_3 1$ 17. ____________

18. $\log_4 64$ 18. ____________

19. $\log_8 8^{17}$ 19. ____________

20. $\log_{10} \sqrt{10}$ 20. ____________

21. $3^{\log_3 8}$ 21. ____________

22. $10^{\log 5}$ 22. ____________

Use translations theory to sketch the graphs of each function. What is the domain, range and vertical asymptote for each? Write an end behavior statement for each.

23. $f(x) = \log_2(x)$

24. $f(x) = \log_2(x) + 3$

25. $f(x) = 1 - \log_2(x)$

26. $f(x) = \log_2(x + 3)$

27. $f(x) = \log_2(-x)$

28. This is a ***Matching question*** associated with the theory on graphical translations of functions. Suppose $f(x) = \log_2 x$. Relative to the graph of $f(x)$ the graphs of the following functions have been changed in what way?

_____	$g(x) = \log_2 x + 5$	a.) shifted 5 units left
_____	$g(x) = \log_2(x + 5)$	b.) reflected about the x axis
_____	$g(x) = -2 \cdot \log_2(x)$	c.) shifted 5 units down
_____	$g(x) = \log_2(x - 5)$	d.) shifted 5 units right
_____	$g(x) = \log_2(x) - 5$	e.) shifted 5 units vertically up

Definition 7. *The logarithm with base 10 is called the **common logarithm** and is described by omitting the base:*

$$\log(x) = \log_{10}(x)$$

Definition 8. *The logarithm with base e is called the **natural logarithm** and is denoted by **ln**:*

$$\ln(x) = \log_e(x)$$

Properties of Natural Logarithms

Property	Reason
1. $\ln(1) = 0$	We must raise e to the power 0 to get 1.
2. $\ln(e) = 1$	We must raise e to the power 1 to get e.
3. $\ln(e^x) = x$	We must raise e to the power x to get e^x.
4. $e^{\ln x} = x$	$\ln x$ is the power to which e must be raised to get x.

Evaluate Each Expression.

29. $\ln e^{-3}$ 29. ____________

30. $\ln\left(\dfrac{1}{e^4}\right)$ 30. ____________

5.3 Laws of Logs **Name:** ____________________

Laws of Logarithms

Let a be a positive number, with $a \neq 1$. Let A, B and C be any real numbers with $A > 0$ and $B > 0$.

1. $\log_a(AB) = \log_a A + \log_a B$
2. $\log_a\left(\frac{A}{B}\right) = \log_a A - \log_a B$
3. $\log_a(A^C) = C \cdot \log_a A$

1. **Evaluate each expression.**

(a) $\log_4 2 + \log_4 32$

(b) $\log_2 80 + \log_2 5$

(c) $-\frac{1}{3}\log 8$

2. **Use the Laws of Logarithms to expand each expression.**

(a) $\log_2(6x) + \log_5(x^3y^6)$

(b) $\ln\left(\frac{ab}{\sqrt[3]{c}}\right)$

3. Combine $4\log x + \frac{1}{2}\log(x+2)$ into a single logarithm.

4. Combine $3\ln x + \frac{1}{2}\ln y - 5\ln(x^2+2)$ into a single logarithm.

Change of Base Formula

$$\log_d n = \frac{\log_a n}{\log_a d} = \frac{\ln n}{\ln d} = \frac{\log n}{\log d}$$

Use the Change of Base Formula to rewrite the logarithm as a ratio of (a) common logarithms and (b) natural logarithms. Use a calculator to evaluate the logarithm and round it to the thousandth place.

5. $\log_3 6$ 5. ____________

6. $\log_7 48$ 6. ____________

7. $\log_{15} 97$ 7. ____________

Find the exact value of the logarithmic expression without using a calculator.

8. $\log_5\left(\frac{1}{125}\right)$ 8. ____________

9. $\log_6\left(\sqrt{6}\right)$ 9. ____________

10. $\log_3\left(81^{-3}\right)$ 10. ____________

11. $\ln \sqrt[4]{e^3}$ 11. ____________

12. $2\ln e^6 - \ln e^5$ 12. ____________

5.4 Exp. & Log Eqns **Name: ____________________**

Solve for x. Approximate your answers to 3 decimal places.

1. $4^{2x-1} = 7$

1. __________

2. $8e^{3x} = 17$

2. __________

3. $e^{3-2x} = 4$

3. __________

Solve for x.

4. $e^{2x} - 4e^{x} = 5$

4. ___________

5. $3xe^{x} + x^{2}e^{x} = 0$

5. ___________

6. $\log_2(x + 4) = 3$

6. ___________

7. $\ln x = 3$

7. ___________

8. $\log_3(1 - 2x) = 3$

8. ___________

9. $4 + 3\log(2x) = 16$

9. ___________

10. $\log(x + 2) + \log(x - 1) = 1$

10. ___________