Key

70 total pts

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College Algebra - Test 1 **Name:** Key

1. (6 points) Suppose $g(x) = \begin{cases} -3x & \text{if } x < 0 \\ \sqrt{16 - x^2} & \text{if } 0 \leq x < 4 \\ (x-4)^2 & \text{if } x \geq 4 \end{cases}$.

Evaluate the piecewise defined function at the values indicated below.

(a) $g(-1) = -3(-1) = 3$ (a) 3

(b) $g(-3) = -3(-3) = 9$ (b) 9

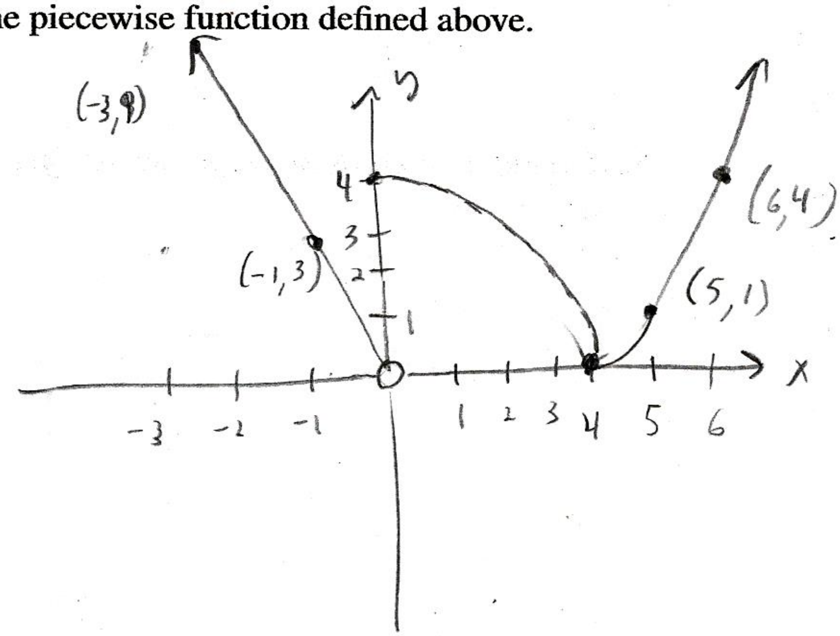
(c) $g(0) = 4$ (c) 4

(d) $g(4) = 0$ (d) 0

(e) $g(6) = (6-4)^2 = 2^2 = 4$ (e) 4

(f) $g(8) = (8-4)^2 = 4^2 = 16$ (f) 16

2. (4 points) Sketch the graph of the piecewise function defined above.



Key

3. (5 points) Write the domain of $f(x) = \dfrac{1}{4-x}$ using interval notation.

$4 - x = 0$ when $x = 4$

3. $(-\infty, 4) \cup (4, \infty)$

4

4. (5 points) Write the domain of $f(x) = \sqrt{2x+3}$ using interval notation.

4. $[-3/2, \infty)$

$2x + 3 \geq 0$

$2x \geq -3$

$-3/2$

$x \geq -3/2$

5. (5 points) Find f/g and its domain. $f(x) = \sqrt{25 - x^2}$ and $g(x) = \sqrt{2 + x}$

5. $f/g = \dfrac{\sqrt{25-x^2}}{\sqrt{2+x}}$

$$\boxed{\begin{array}{l} (f/g)(x) = \dfrac{\sqrt{25-x^2}}{\sqrt{2+x}} \\ \text{dom}(f/g) = (-2, 5] \end{array}}$$

x, dom(f)

-5 $\quad$ 5

$2 + x \geq 0$

x, dom(g)

$x \geq -2$

but $x = -2$ gives division by zero

-2

x intersection

-2 $\quad$ 5

6. (5 points) Find the average rate of change of $f(x) = 2x^2 - 3x$ from $x_1 = 2$ to $x_2 = 3$

$$\frac{f(x_2) - f(x_1)}{x_2 - x_1} = \frac{9 - 2}{3 - 2} = \frac{7}{1}$$

6. 7

$f(3) = 2 \cdot 9 - 3(3)$

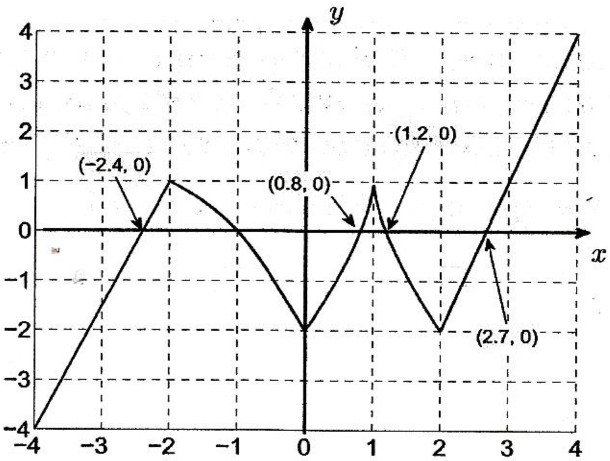
$= 18 - 9 = 9$

$f(2) = 2 \cdot 4 - 3(2)$

$= 8 - 6 = 2$

Key

7. (12 points) The graph of a function f is given. **Assume the entire graph of f is shown in the figure.**



(a) Find all ***local*** and absolute maximum and minimum values of the function and the value of x at which each occurs.

abs max $(4,4)$ abs min $(-4,-4)$

local max $(-2,1)$, $(1,1)$

local min $(0,-2)$, $(2,-2)$

(b) State the x intervals for which $f(x) > 0$.

$(-2.4,-1) \cup (0.8,1.2) \cup (2.7,4]$

(c) State the x intervals for which $f(x) < 0$.

$[-4,-2.4) \cup (-1,0.8) \cup (1.2,2.7)$

(d) Find the x intervals on which the function is ***increasing***.

$[-4,-2) \cup (0,1) \cup (2,4]$

(e) Find the x intervals on which the function is ***decreasing***.

$(-2,0) \cup (1,2)$

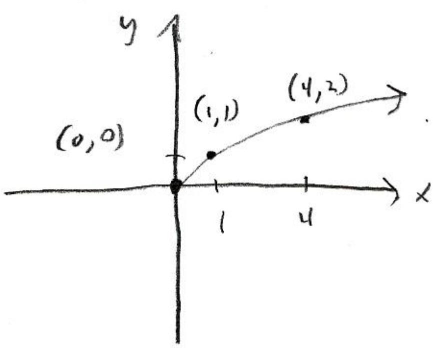
(f) Find $f(4)$. (f) 4

(g) Find $f(-1)$. (g) 0

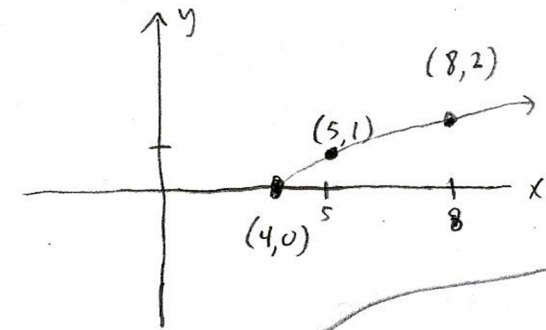
Directions: Sketch the graph of the function, not by plotting points, but by starting with the graph of a standard function and applying transformations. Label at least 3 points on your final graph.

8. (5 points) $h(x) = -3\sqrt{x-4} + 1$

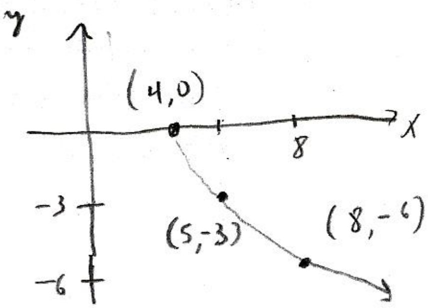
$f(x) = \sqrt{x}$



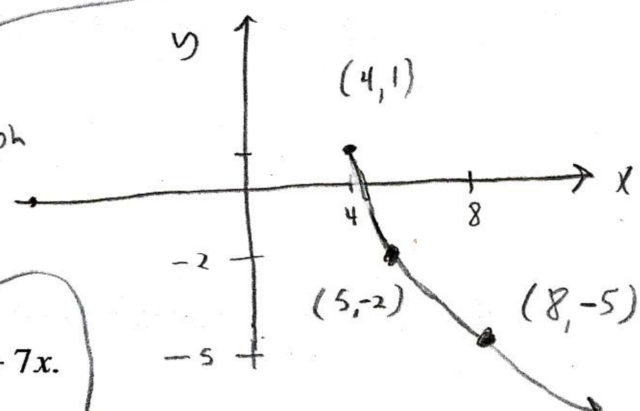
$y = \sqrt{x-4}$



$y = -3\sqrt{x-4}$



Final graph



Find $f \circ g$ and its domain.

9. (5 points) $f(x) = \dfrac{2}{1-x}$ and $g(x) = 2 + 7x$.

$$f(g(x)) = f(2+7x) = \frac{2}{1-(2+7x)} = \frac{2}{1-2-7x} = \frac{2}{-1-7x}$$

and $-1-7x=0$ when $-1 = 7x$ or when $x = -1/7$

$$\boxed{f \circ g = \frac{2}{-1-7x} \qquad \text{dom}(f \circ g) = \{x \mid x \neq -1/7\} \text{ or } (-\infty, -1/7) \cup (-1/7, \infty)}$$

10. (5 points) Find the inverse function of $f(x) = \dfrac{2x}{x+3}$

$$\boxed{f^{-1}(x) = \frac{-3x}{x-2}}$$

10. ______

$y = \frac{2x}{x+3}$

$x = \frac{2y}{y+3}$

$x(y+3) = 2y$

$xy + 3x = 2y$

$-2y \quad -2y$

$xy - 2y + 3x = 0$

$-3x \quad -3x$

$xy - 2y = -3x$

$xy - 2y = -3x$

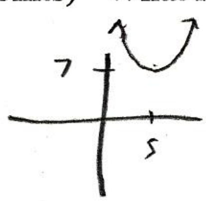
$(x-2)y = -3x$

$y = \frac{-3x}{x-2}$

11. (3 points) Find the vertex of $g(x) = -3(x+4)^2 - 7$. Does f open up or down?

11. down

12. (3 points) What is the range of $g(x) = 3(x-5)^2 + 7$?



12. $[7, \infty)$

Express the quadratic function in standard (vertex) form.

13. (5 points) $g(x) = 2x^2 + 4x - 7$

$$\boxed{g(x) = 2(x+1)^2 - 9}$$

13. ______

$= 2(x^2 + 2x) - 7$

$= 2(x^2 + 2x + \_\_) - 7 - \_\_$

$= 2(x^2 + 2x + \underline{1}) - 7 - \underline{2}$

$= 2(x+1)^2 - 9$