Test 4 **Name:** ____________________

No Calculators or Computing Devices allowed! Use Algebraic Notation AND Show All of Your Work.

---

1. (6 points) Find the inverse of $C = \begin{bmatrix} 1 & -2 & -4 \\ 2 & -3 & -6 \\ -3 & 6 & 15 \end{bmatrix}$ if it exists.

1. ____________

2. (a) (2 points) Write a matrix equation equivalent to the following system.

$$\begin{cases} 3x + 2y = 14 \\ x - 2y = 2 \end{cases}$$

(a) ____________

(b) (4 points) Find the inverse of the coefficient matrix, and use it to solve the system.

(b) ____________

3. (5 points) Solve $\left\{\begin{matrix} 2x + y = 1 \\ 3x + 4y = 14 \end{matrix}\right\}$ using Cramer's Rule.

3. ____________

4. Let $A = \begin{bmatrix} 1 & -5 \\ -3 & 7 \end{bmatrix}$, $B = \begin{bmatrix} -2 & -6 \\ 2 & 7 \\ 1 & 0 \end{bmatrix}$, $C = \begin{bmatrix} 1 & 3 & 1 \\ -2 & 7 & 2 \\ 0 & 2 & 4 \end{bmatrix}$

Carry out the indicated operation, or explain, using complete sentences, why it cannot be performed.

(a) (2 points) $A + B$

(b) (2 points) AB

(c) (2 points) $BA - 3A$

(d) (2 points) B^{-1}

(e) (2 points) $\det(B)$

5. (6 points) Find the partial fraction decomposition of $\dfrac{7x-2}{x^2-4}$.

5. ____________

6. Only one of the following two matrices has an inverse.

$$A = \begin{bmatrix} 2 & 3 & -1 \\ 0 & 2 & 4 \\ -2 & 5 & 6 \end{bmatrix}, \qquad B = \begin{bmatrix} 1 & 3 & 7 \\ 2 & 0 & 8 \\ 0 & 2 & 2 \end{bmatrix}$$

(a) (5 points) Find the determinant of each matrix. (a) ____________

(b) (1 point) Use the determinants from part (a) to identify which matrix has an inverse.

(b) ____________

7. Let $A = \begin{bmatrix} 2 & -5 \\ -6 & 2 \\ 2 & -8 \end{bmatrix}$, $B = \begin{bmatrix} -1 & 3 \\ 3 & -4 \\ 1 & 0 \end{bmatrix}$, $C = \begin{bmatrix} 3 & 0 & 1 \\ -2 & 4 & 6 \\ 2 & 2 & 5 \end{bmatrix}$

Carry out the indicated operation, or explain, using complete sentences, why it cannot be performed.

(a) (4 points) CA

(b) (4 points) $2B - 3A$

8. (6 points) Find the complete solution of the system, or show that no solution exists.

$$\begin{cases} x - y + 5z = \text{-}2 \\ 2x + y + 4z = 2 \\ 2x + 4y - 2z = 8 \end{cases}$$

8. ____________

9. (6 points) Sketch the graph (and label the vertices, or boundary intersections) of the solution set of ordered pairs of the system.

$$\begin{cases} x \geq 0 \\ y \geq 0 \\ x \leq 5 \\ x + y \leq 7 \end{cases}$$

10. (6 points) Use Gaussian elimination to find the complete solution of the system, or show that no solution exists.

$$\begin{cases} x - y + 2z = 0 \\ 2x - 4y + 5z = -5 \\ 2y - 3z = 5 \end{cases}$$

10. ____________

11. (6 points) Write the given system as an augmented matrix. Use Elementary Row Operations to derive equivalent matrices and find the complete solution of the system, or show that no solution exists.

$$\begin{cases} x - 3y + 2z = 12 \\ 2x - 5y + 5z = 14 \\ x - 2y + 3z = 20 \end{cases}$$

11. ______________

12. (6 points) Sketch the graph (and label the vertices, or boundary intersections) of the solution set of ordered pairs of the system.

$$\begin{cases} x - y < 2 \\ x > 2 \\ y \leq 3 \end{cases}$$

13. (6 points) Find the partial fraction decomposition of $\dfrac{3x-4}{x^3+4x^2}$.

13. ___________

14. (6 points) Find the partial fraction decomposition of $\dfrac{2x-3}{x^3+3x}$.

14. ____________