Objectives

- Solve equations using the Addition Property of Equality.
- Solve equations using the Multiplication Property of Equality.

Definition 1. *A number is a solution of an equation when the variable in the equation can be replaced with the number and the resulting equation is a true statement.*

For example, 2 is a solution to $2 \cdot x + 3 = 7$ since $2 \cdot 2 + 3 = 7$.

Definition 2. *To solve an equation means to find all of its solutions.*

Definition 3. *Two equations are called equivalent when they have the same solutions.*

The Addition Property of Equality

If $a = b$, then $a + c = b + c$.

Adding a number to each side of an equation produces an equivalent equation.

Example Solve: $x + 3 = 1$

$$\begin{aligned} x + 3 &= 1 \\ x + 3 + (-3) &= 1 + (-3) && \text{add } -3 \text{ to both sides} \\ x &= -2 && \text{solution equation} \end{aligned}$$

The Multiplication Property of Equality

If $a = b$, then $c \cdot a = c \cdot b$.

Multiplying each side of an equation by a nonzero real number c produces an equivalent equation.

Example Solve: $-\frac{1}{5}x = 6$

$$\begin{aligned} -\frac{1}{5}x &= 6 \\ (-5) \cdot \left(-\frac{1}{5}x \right) &= (-5) \cdot 6 && \text{multiply both sides by } -5 \\ 1 \cdot x &= -30 \\ x &= -30 && \text{solution equation} \end{aligned}$$

Exercises

1. Determine whether the given number is a solution of the given equation.
   a) Is 0 a solution to $x + 3 = 3 + 5x$?
   b) Is 12 a solution to $x + 3 = 14$?
   c) Is 3 a solution to $17 = 2 + 5x$?

2. Solve using the Addition Property of Equality
   a) $x + 23 = 17$ b) $-3 = 5 + x$ c) $17 = x - 3$ d) $x - 7 = -13$

3. Solve using the Multiplication Property of Equality
   a) $\frac{4}{5}x = 16$ b) $-\frac{2}{3}x = 12$ c) $-7 = \frac{x}{6}$ d) $4x = -24$
   e) $\frac{3x}{5} = 9$ f) $8 = 16x$ g) $-x = 4$ h) $-7x = 56$

Answers: 1a) yes, b) no, c) yes, 2a) -6, b) -8, c) 20, d) -6, 3a) 20, b) -18, c) -42, d) -6, e) 15, f) $\frac{1}{2}$, g) -4, h) -8