Properties of Real Numbers

Let a, b, and c represent real numbers.

Property	*Verbal Description*
Closure Property of Addition	The sum of two real numbers is a real number.
$a + b$ is a real number.	Example: $1 + 5 = 6$, and 6 is a real number.
Closure Property of Multiplication	The product of two real numbers is a real number.
ab is a real number.	Example: $7 \cdot 3 = 21$, and 21 is a real number.
Commutative Property of Addition	Two real numbers can be added in either order.
$a + b = b + a$	Example: $2 + 6 = 6 + 2$
Commutative Property of Multiplication	Two real numbers can be multiplied in either order.
$a \cdot b = b \cdot a$	Example: $3 \cdot (-5) = -5 \cdot 3$
Associative Property of Addition	When three real numbers are added, it makes no difference which two are added first.
$(a + b) + c = a + (b + c)$	Example: $(1 + 7) + 4 = 1 + (7 + 4)$
Associative Property of Multiplication	When three real numbers are multiplied, it makes no difference which two are multiplied first.
$(ab)c = a(bc)$	Example: $(4 \cdot 3) \cdot 9 = 4 \cdot (3 \cdot 9)$
Distributive Properties	Multiplication distributes over addition.
$a(b + c) = ab + ac$ $(a + b)c = ac + bc$	Examples: $2(3 + 4) = 2 \cdot 3 + 2 \cdot 4$ $(3 + 4)2 = 3 \cdot 2 + 4 \cdot 2$
Additive Identity Property	The sum of zero and a real number equals the number itself.
$a + 0 = 0 + a = a$	Example: $4 + 0 = 0 + 4 = 4$
Multiplicative Identity Property	The product of 1 and a real number equals the number itself.
$a \cdot 1 = 1 \cdot a = a$	Example: $5 \cdot 1 = 1 \cdot 5 = 5$
Additive Inverse Property	The sum of a real number and its opposite is zero.
$a + (-a) = 0$	Example: $5 + (-5) = 0$
Multiplicative Inverse Property	The product of a nonzero real number and its reciprocal is 1.
$a \cdot \frac{1}{a} = 1, \quad a \neq 0$	Example: $7 \cdot \frac{1}{7} = 1$

The operations of subtraction and division are not listed above because they fail to possess many of the properties described in the list. For instance, subtraction and division are not commutative. To see this, consider $4 - 3 \neq 3 - 4$ and $15 \div 5 \neq 5 \div 15$. Similarly, the examples $8 - (6 - 2) \neq (8 - 6) - 2$ and $20 \div (4 \div 2) \neq (20 \div 4) \div 2$ illustrate the fact that subtraction and division are not associative.

Additional Properties of Real Numbers

Let a, b, and c be real numbers.

Properties of Equality	*Verbal Description*
Addition Property of Equality If $a = b$, then $a + c = b + c$.	Adding a real number to each side of a true equation produces another true equation.
Multiplication Property of Equality If $a = b$, then $ac = bc$, $c \neq 0$.	Multiplying each side of a true equation by a nonzero real number produces another true equation.
Cancellation Property of Addition If $a + c = b + c$, then $a = b$.	Subtracting a real number from each side of a true equation produces another true equation.
Cancellation Property of Multiplication If $ac = bc$ and $c \neq 0$, then $a = b$.	Dividing each side of a true equation by a nonzero real number produces another true equation.
Reflexive Property of Equality $a = a$	A real number always equals itself.
Symmetric Property of Equality If $a = b$, then $b = a$.	If a real number equals a second real number, then the second real number equals the first.
Transitive Property of Equality If $a = b$ and $b = c$, then $a = c$.	If a real number equals a second real number and the second real number equals a third real number, then the first real number equals the third real number.

Properties of Zero	*Verbal Description*
Multiplication Property of Zero $0 \cdot a = 0$	The product of zero and any real number is zero.
Division Property of Zero $\frac{0}{a} = 0, a \neq 0$	If zero is divided by any *nonzero* real number, the result is zero.
Division by Zero Is Undefined $\frac{a}{0}$ is undefined.	We do not define division by zero.

Properties of Negation	*Verbal Description*
Multiplication by -1 $(-1)a = -a$ $(-1)(-a) = a$	The opposite of a real number a can be obtained by multiplying the real number by -1.
Placement of Negative Signs $-(ab) = (-a)(b) = (a)(-b)$	The opposite of the product of two numbers is equal to the product of one of the numbers and the opposite of the other.
Product of Two Opposites $(-a)(-b) = ab$	The product of the opposites of two real numbers is equal to the product of the two real numbers.