Chapter 10 Review Exercises

1. $\sqrt{81} = 9$ because $9^2 = 81$

2. $-\sqrt{\frac{1}{100}} = -\frac{1}{10}$ because $\left(-\frac{1}{10}\right)^2 = \frac{1}{100}$

3. $\sqrt[3]{-27} = -3$ because $(-3)^3 = -27$

4. $\sqrt[4]{-16}$
not a real number
The index is even and the radicand is negative.

5. $\sqrt[5]{-32} = -2$ because $(-2)^5 = -32$

6. $f(15) = \sqrt{2(15)-5} = \sqrt{30-5}$
$= \sqrt{25} = 5$
$f(4) = \sqrt{2(4)-5} = \sqrt{8-5} = \sqrt{3} \approx 1.73$
$f\left(\frac{5}{2}\right) = \sqrt{2\left(\frac{5}{2}\right)-5} = \sqrt{5-5}$
$= \sqrt{0} = 0$
$f(1) = \sqrt{2(1)-5} = \sqrt{2-5} = \sqrt{-3}$
not a real number

7. $g(4) = \sqrt[3]{4(4)-8} = \sqrt[3]{16-8} = \sqrt[3]{8} = 2$
$g(0) = \sqrt[3]{4(0)-8} = \sqrt[3]{-8} = -2$
$g(-14) = \sqrt[3]{4(-14)-8} = \sqrt[3]{-56-8}$
$= \sqrt[3]{-64} = -4$

8. To find the domain, set the radicand greater than or equal to zero and solve the resulting inequality.
$x - 2 \geq 0$
$x \geq 2$
The domain of f is $[2, \infty)$.

9. To find the domain, set the radicand greater than or equal to zero and solve the resulting inequality.
$100 - 4x \geq 0$
$-4x \geq -100$
$\frac{-4x}{-4} \leq \frac{-100}{-4}$
$x \leq 25$
The domain of g is $(-\infty, 25]$.

10. $\sqrt{25x^2} = 5|x|$

11. $\sqrt{(x+14)^2} = |x+14|$

12. $\sqrt{x^2 - 8x + 16} = \sqrt{(x-4)^2} = |x-4|$

13. $\sqrt[3]{64x^3} = 4x$

14. $\sqrt[4]{16x^4} = 2|x|$

15. $\sqrt[5]{-32(x+7)^5} = -2(x+7)$

16. $(5xy)^{\frac{1}{3}} = \sqrt[3]{5xy}$

17. $16^{\frac{3}{2}} = \left(\sqrt{16}\right)^3 = (4)^3 = 64$

18. $32^{\frac{4}{5}} = \left(\sqrt[5]{32}\right)^4 = (2)^4 = 16$

19. $\sqrt{7x} = (7x)^{\frac{1}{2}}$

20. $\left(\sqrt[3]{19xy}\right)^5 = (19xy)^{\frac{5}{3}}$

21. $8^{-\frac{2}{3}} = \frac{1}{8^{\frac{2}{3}}} = \frac{1}{\left(\sqrt[3]{8}\right)^2} = \frac{1}{(2)^2} = \frac{1}{4}$

22. $3x(ab)^{-\frac{4}{5}} = \frac{3x}{(ab)^{\frac{4}{5}}}$
$= \frac{3x}{\left(\sqrt[5]{ab}\right)^4}$
$= \frac{3x}{(ab)^{\frac{4}{5}}}$
$= \frac{3x}{a^{\frac{4}{5}}b^{\frac{4}{5}}}$

23. $x^{\frac{1}{3}} \cdot x^{\frac{1}{4}} = x^{\frac{1}{3}+\frac{1}{4}} = x^{\frac{4}{12}+\frac{3}{12}} = x^{\frac{7}{12}}$

24. $\dfrac{5^{\frac{1}{2}}}{5^{\frac{1}{3}}} = 5^{\frac{1}{2}-\frac{1}{3}} = 5^{\frac{3}{6}-\frac{2}{6}} = 5^{\frac{1}{6}}$

25. $\left(8x^6y^3\right)^{\frac{1}{3}} = 8^{\frac{1}{3}}x^{6\cdot\frac{1}{3}}y^{3\cdot\frac{1}{3}} = 2x^2y$

26. $\left(x^{-\frac{2}{3}}y^{\frac{1}{4}}\right)^{\frac{1}{2}} = x^{-\frac{2}{3}\cdot\frac{1}{2}}y^{\frac{1}{4}\cdot\frac{1}{2}}$

$= x^{-\frac{1}{3}}y^{\frac{1}{8}} = \dfrac{y^{\frac{1}{8}}}{x^{\frac{1}{3}}}$

27. $\sqrt[3]{x^9y^{12}} = \left(x^9y^{12}\right)^{\frac{1}{3}}$

$= x^{9\cdot\frac{1}{3}}y^{12\cdot\frac{1}{3}} = x^3y^4$

28. $\sqrt[9]{x^3y^9} = \left(x^3y^9\right)^{\frac{1}{9}} = x^{3\cdot\frac{1}{9}}y^{9\cdot\frac{1}{9}}$

$= x^{\frac{1}{3}}y = y\sqrt[3]{x}$

29. $\sqrt{x}\cdot\sqrt[3]{x} = x^{\frac{1}{2}}x^{\frac{1}{3}} = x^{\frac{1}{2}+\frac{1}{3}} = x^{\frac{3}{6}+\frac{2}{6}}$

$= x^{\frac{5}{6}} = \sqrt[6]{x^5}$

30. $\dfrac{\sqrt[3]{x^2}}{\sqrt[4]{x^2}} = \dfrac{x^{\frac{2}{3}}}{x^{\frac{2}{4}}} = x^{\frac{2}{3}-\frac{1}{2}}$

$= x^{\frac{4}{6}-\frac{3}{6}} = x^{\frac{1}{6}} = \sqrt[6]{x}$

31. $\sqrt[5]{\sqrt[3]{x}} = \sqrt[5]{x^{\frac{1}{3}}} = \left(x^{\frac{1}{3}}\right)^{\frac{1}{5}} = x^{\frac{1}{3}\cdot\frac{1}{5}}$

$= x^{\frac{1}{15}} = \sqrt[15]{x}$

32. Since 2012 was 27 years after 1985, find $f(27)$.

$f(27) = 350(27)^{\frac{2}{3}} = 350\left(\sqrt[3]{27}\right)^2$

$= 350(3)^2 = 350(9) = 3150$

Expenditures were \$3150 million or \$3,150,000,000 in the year 2012.

33. $\sqrt{3x}\cdot\sqrt{7y} = \sqrt{21xy}$

34. $\sqrt[5]{7x^2}\cdot\sqrt[5]{11x} = \sqrt[5]{77x^3}$

35. $\sqrt[6]{x-5}\cdot\sqrt[6]{(x-5)^4} = \sqrt[6]{(x-5)^5}$

36. $f(x) = \sqrt{7x^2-14x+7}$

$= \sqrt{7\left(x^2-2x+1\right)}$

$= \sqrt{7(x-1)^2} = \sqrt{7}\,|x-1|$

37. $\sqrt{20x^3} = \sqrt{4\cdot5\cdot x^2\cdot x} = \sqrt{4x^2\cdot5x}$

$= 2x\sqrt{5x}$

38. $\sqrt[3]{54x^8y^6} = \sqrt[3]{27\cdot2\cdot x^6\cdot x^2y^6}$

$= \sqrt[3]{27x^6y^6\cdot2x^2}$

$= 3x^2y^2\sqrt[3]{2x^2}$

39. $\sqrt[4]{32x^3y^{11}z^5} = \sqrt[4]{16\cdot2\cdot x^3y^8\cdot y^3\cdot z^5\cdot z}$

$= \sqrt[4]{16y^8z^4\cdot2x^3y^3z}$

$= 2y^2z\sqrt[4]{2x^3y^3z}$

40. $\sqrt{6x^3}\cdot\sqrt{4x^2} = \sqrt{24x^5} = \sqrt{4\cdot6\cdot x^4\cdot x}$

$= \sqrt{4x^4\cdot6x} = 2x^2\sqrt{6x}$

41. $\sqrt[3]{4x^2y}\cdot\sqrt[3]{4xy^4} = \sqrt[3]{16x^3y^5}$

$= \sqrt[3]{8\cdot2\cdot x^3\cdot y^3\cdot y^2}$

$= \sqrt[3]{8x^3y^3\cdot2y^2}$

$= 2xy\sqrt[3]{2y^2}$

42. $\sqrt[5]{2x^4y^3z^4}\cdot\sqrt[5]{8xy^6z^7}$

$= \sqrt[5]{16x^5y^9z^{11}}$

$= \sqrt[5]{16\cdot x^5\cdot y^5\cdot y^4\cdot z^{10}\cdot z}$

$= \sqrt[5]{x^5y^5z^{10}\cdot16y^4z}$

$= xyz^2\sqrt[5]{16y^4z}$

43. $\sqrt{x+1}\cdot\sqrt{x-1} = \sqrt{(x+1)(x-1)}$

$= \sqrt{x^2-1}$

44. $6\sqrt[3]{3}+2\sqrt[3]{3} = (6+2)\sqrt[3]{3} = 8\sqrt[3]{3}$

45. $5\sqrt{18}-3\sqrt{8}=5\sqrt{9\cdot 2}-3\sqrt{4\cdot 2}$
$=5\cdot 3\sqrt{2}-3\cdot 2\sqrt{2}$
$=15\sqrt{2}-6\sqrt{2}$
$=(15-6)\sqrt{2}=9\sqrt{2}$

46. $\sqrt[3]{27x^4}+\sqrt[3]{xy^6}$
$=\sqrt[3]{27x^3x}+\sqrt[3]{xy^6}$
$=3x\sqrt[3]{x}+y^2\sqrt[3]{x}$
$=\left(3x+y^2\right)\sqrt[3]{x}$

47. $2\sqrt[3]{6}-5\sqrt[3]{48}=2\sqrt[3]{6}-5\sqrt[3]{8\cdot 6}$
$=2\sqrt[3]{6}-5\cdot 2\sqrt[3]{6}$
$=2\sqrt[3]{6}-10\sqrt[3]{6}$
$=(2-10)\sqrt[3]{6}=-8\sqrt[3]{6}$

48. $\sqrt[3]{\frac{16}{125}}=\sqrt[3]{\frac{8\cdot 2}{125}}=\frac{2}{5}\sqrt[3]{2}$

49. $\sqrt{\frac{x^3}{100y^4}}=\sqrt{\frac{x^2\cdot x}{100y^4}}$
$=\frac{x}{10y^2}\sqrt{x}$ or $\frac{x\sqrt{x}}{10y^2}$

50. $\sqrt[4]{\frac{3y^5}{16x^{20}}}=\sqrt[4]{\frac{y^4\cdot 3y}{16x^{20}}}$
$=\frac{y}{2x^5}\sqrt[4]{3y}$ or $\frac{y\sqrt[4]{3y}}{2x^5}$

51. $\frac{\sqrt{48}}{\sqrt{2}}=\sqrt{\frac{48}{2}}=\sqrt{24}=\sqrt{4\cdot 6}=2\sqrt{6}$

52. $\frac{\sqrt[3]{32}}{\sqrt[3]{2}}=\sqrt[3]{\frac{32}{2}}=\sqrt[3]{16}=\sqrt[3]{8\cdot 2}=2\sqrt[3]{2}$

53. $\frac{\sqrt[4]{64x^7}}{\sqrt[4]{2x^2}}=\sqrt[4]{\frac{64x^7}{2x^2}}=\sqrt[4]{32x^5}$
$=\sqrt[4]{16\cdot 2\cdot x^4\cdot x}$
$=\sqrt[4]{16x^4\cdot 2x}=2x\sqrt[4]{2x}$

54. $\frac{\sqrt{200x^3y^2}}{\sqrt{2x^{-2}y}}=\sqrt{\frac{200x^3y^2}{2x^{-2}y}}=\sqrt{100x^5y}$
$=\sqrt{100x^4xy}=10x^2\sqrt{xy}$