

Definitions and Concepts

Examples

Section 10.7 Complex Numbers (continued)

To divide complex numbers, multiply the numerator and the denominator by the conjugate of the denominator in order to obtain a real number in the denominator. This real number becomes the denominator of a and b in the quotient $a + bi$.

$$\begin{aligned}\frac{5 + 2i}{4 - i} &= \frac{5 + 2i}{4 - i} \cdot \frac{4 + i}{4 + i} = \frac{20 + 5i + 8i + 2i^2}{16 - i^2} \\ &= \frac{20 + 13i + 2(-1)}{16 - (-1)} \\ &= \frac{20 + 13i - 2}{16 + 1} \\ &= \frac{18 + 13i}{17} = \frac{18}{17} + \frac{13}{17}i\end{aligned}$$

To simplify powers of i , rewrite the expression in terms of i^2 . Then replace i^2 with -1 and simplify.

Simplify: i^{27} .

$$i^{27} = i^{26} \cdot i = (i^2)^{13}i = (-1)^{13}i = (-1)i = -i$$

CHAPTER 10 REVIEW EXERCISES

10.1 In Exercises 1–5, find the indicated root, or state that the expression is not a real number.

- $\sqrt{81}$
- $-\sqrt{\frac{1}{100}}$
- $\sqrt[3]{-27}$
- $\sqrt[4]{-16}$
- $\sqrt[5]{-32}$

In Exercises 6–7, find the indicated function values for each function. If necessary, round to two decimal places. If the function value is not a real number and does not exist, so state.

6. $f(x) = \sqrt{2x - 5}$; $f(15), f(4), f\left(\frac{5}{2}\right), f(1)$

7. $g(x) = \sqrt[3]{4x - 8}$; $g(4), g(0), g(-14)$

In Exercises 8–9, find the domain of each square root function.

- $f(x) = \sqrt{x - 2}$
- $g(x) = \sqrt{100 - 4x}$

In Exercises 10–15, simplify each expression. Assume that each variable can represent any real number, so include absolute value bars where necessary.

- $\sqrt{25x^2}$
- $\sqrt{x^2 - 8x + 16}$
- $\sqrt[4]{16x^4}$
- $\sqrt{(x + 14)^2}$
- $\sqrt[3]{64x^3}$
- $\sqrt[5]{-32(x + 7)^5}$

10.2 In Exercises 16–18, use radical notation to rewrite each expression. Simplify, if possible.

- $(5xy)^{\frac{1}{3}}$
- $16^{\frac{2}{3}}$
- $32^{\frac{4}{5}}$

In Exercises 19–20, rewrite each expression with rational exponents.

19. $\sqrt{7x}$

20. $(\sqrt[3]{19xy})^5$

In Exercises 21–22, rewrite each expression with a positive rational exponent. Simplify, if possible.

21. $8^{-\frac{2}{3}}$

22. $3x(ab)^{\frac{4}{5}}$

In Exercises 23–26, use properties of rational exponents to simplify each expression.

23. $x^{\frac{1}{3}} \cdot x^{\frac{1}{4}}$

24. $\frac{5^{\frac{1}{2}}}{5^{\frac{1}{3}}}$

25. $(8x^6y^3)^{\frac{1}{3}}$

26. $(x^{-\frac{2}{3}}y^{\frac{1}{4}})^{\frac{1}{2}}$

In Exercises 27–31, use rational exponents to simplify each expression. If rational exponents appear after simplifying, write the answer in radical notation.

27. $\sqrt[3]{x^9y^{12}}$

28. $\sqrt[9]{x^3y^9}$

29. $\sqrt{x} \cdot \sqrt[3]{x}$

30. $\frac{\sqrt[3]{x^2}}{\sqrt[4]{x^2}}$

31. $\sqrt[5]{\sqrt[3]{x}}$

32. The function $f(x) = 350x^{\frac{2}{3}}$ models the expenditures, $f(x)$, in millions of dollars, for the U.S. National Park Service x years after 1985. According to this model, what were expenditures in 2012?

10.3 In Exercises 33–35, use the product rule to multiply.

33. $\sqrt{3x} \cdot \sqrt{7y}$

34. $\sqrt[5]{7x^2} \cdot \sqrt[5]{11x}$

35. $\sqrt[6]{x - 5} \cdot \sqrt[6]{(x - 5)^4}$

36. If $f(x) = \sqrt{7x^2 - 14x + 7}$, express the function, f , in simplified form. Assume that x can be any real number.

In Exercises 37–39, simplify by factoring. Assume that all variables in a radicand represent positive real numbers.

$$37. \sqrt{20x^3} \qquad 38. \sqrt[3]{54x^8y^6}$$

$$39. \sqrt[4]{32x^3y^{11}z^5}$$

In Exercises 40–43, multiply and simplify, if possible. Assume that all variables in a radicand represent positive real numbers.

$$40. \sqrt{6x^3} \cdot \sqrt{4x^2}$$

$$41. \sqrt[3]{4x^2y} \cdot \sqrt[3]{4xy^4}$$

$$42. \sqrt[5]{2x^4y^3z^4} \cdot \sqrt[5]{8xy^6z^7}$$

$$43. \sqrt{x+1} \cdot \sqrt{x-1}$$

10.4 Assume that all variables represent positive real numbers. In Exercises 44–47, add or subtract as indicated.

$$44. 6\sqrt[3]{3} + 2\sqrt[3]{3} \qquad 45. 5\sqrt{18} - 3\sqrt{8}$$

$$46. \sqrt[3]{27x^4} + \sqrt[3]{xy^6}$$

$$47. 2\sqrt[3]{6} - 5\sqrt[3]{48}$$

In Exercises 48–50, simplify using the quotient rule.

$$48. \sqrt[3]{\frac{16}{125}} \qquad 49. \sqrt{\frac{x^3}{100y^4}}$$

$$50. \sqrt[4]{\frac{3y^5}{16x^{20}}}$$

In Exercises 51–54, divide and, if possible, simplify.

$$51. \frac{\sqrt{48}}{\sqrt{2}} \qquad 52. \frac{\sqrt[3]{32}}{\sqrt[3]{2}}$$

$$53. \frac{\sqrt[4]{64x^7}}{\sqrt[4]{2x^2}} \qquad 54. \frac{\sqrt{200x^3y^2}}{\sqrt{2x^{-2}y}}$$

10.5 Assume that all variables represent positive real numbers.

In Exercises 55–62, multiply as indicated. If possible, simplify any radical expressions that appear in the product.

$$55. \sqrt{3}(2\sqrt{6} + 4\sqrt{15})$$

$$56. \sqrt[3]{5}(\sqrt[3]{50} - \sqrt[3]{2})$$

$$57. (\sqrt{7} - 3\sqrt{5})(\sqrt{7} + 6\sqrt{5})$$

$$58. (\sqrt{x} - \sqrt{11})(\sqrt{y} - \sqrt{11})$$

$$59. (\sqrt{5} + \sqrt{8})^2$$

$$60. (2\sqrt{3} - \sqrt{10})^2$$

$$61. (\sqrt{7} + \sqrt{13})(\sqrt{7} - \sqrt{13})$$

$$62. (7 - 3\sqrt{5})(7 + 3\sqrt{5})$$

In Exercises 63–75, rationalize each denominator. Simplify, if possible.

$$63. \frac{4}{\sqrt{6}}$$

$$64. \sqrt{\frac{2}{7}}$$

$$65. \frac{12}{\sqrt[3]{9}}$$

$$66. \sqrt{\frac{2x}{5y}}$$

$$67. \frac{14}{\sqrt[3]{2x^2}}$$

$$68. \sqrt[4]{\frac{7}{3x}}$$

$$69. \frac{5}{\sqrt[5]{32x^4y}}$$

$$70. \frac{6}{\sqrt{3}-1}$$

$$71. \frac{\sqrt{7}}{\sqrt{5} + \sqrt{3}}$$

$$72. \frac{10}{2\sqrt{5} - 3\sqrt{2}}$$

$$73. \frac{\sqrt{x} + 5}{\sqrt{x} - 3}$$

$$74. \frac{\sqrt{7} + \sqrt{3}}{\sqrt{7} - \sqrt{3}}$$

$$75. \frac{2\sqrt{3} + \sqrt{6}}{2\sqrt{6} + \sqrt{3}}$$

In Exercises 76–79, rationalize each numerator. Simplify, if possible.

$$76. \sqrt{\frac{2}{7}}$$

$$77. \frac{\sqrt[3]{3x}}{\sqrt[3]{y}}$$

$$78. \frac{\sqrt{7}}{\sqrt{5} + \sqrt{3}}$$

$$79. \frac{\sqrt{7} + \sqrt{3}}{\sqrt{7} - \sqrt{3}}$$

10.6 In Exercises 80–84, solve each radical equation.

$$80. \sqrt{2x+4} = 6$$

$$81. \sqrt{x-5} + 9 = 4$$

$$82. \sqrt{2x-3} + x = 3$$

$$83. \sqrt{x-4} + \sqrt{x+1} = 5$$

$$84. (x^2 + 6x)^{\frac{1}{3}} + 2 = 0$$

85. The bar graph shows the percentage of U.S. college freshmen who described their health as “above average” for six selected years.



Source: John Macionis, *Sociology, Fourteenth Edition*, Pearson, 2012.

The function

$$f(x) = -1.6\sqrt{x} + 54$$

models the percentage of freshmen women who described their health as above average, $f(x)$, x years after 1985.

a. Find and interpret $f(20)$. Round to the nearest tenth of a percent. How does this rounded value compare with the percentage of women displayed by the graph?

b. According to the model, in which year will 44.4% of freshmen women describe their health as above average?