Prerequisite Review 2

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- If we replace the variable in a linear equation with a value that results in a true statement, we say that the value is a **solution** or **root** to the equation or that it **satisfies** the equation.
- Ex: How do you solve $2x - 4 = 0$ for x?

  use the addn./subt. and mult./div. props of equality—Section 1.1

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- A **CONTRADICTION** is a false equation, such as $4 = 9$.

Solving Linear Equations

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$$a \cdot x + b = c$$

where a, b and c are constants (numbers) and $a \neq 0$.

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For example, $2 \cdot x + 3 = -1$ is a linear equation in one variable.

$2x$, 3 and -1 are called the **terms** of the equation.

$2x$ is a **variable term** and 3 and -1 are **constant terms**.

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Solution:

$$4x - 2 = 10$$

$$4 \cdot (x) - 2 = 10 \quad \text{Associative Property of Multiplication}$$

$$4 \cdot (3) - 2 = 10 \quad \text{Replace } x \text{ with } 3$$

$$12 - 2 = 10 \quad \text{The Order of Operations says that we multiply before subtracting.}$$

Definition

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Example $2x + 3 = -1$, $x = -2$, $2x = -4$, and $x + 2 = 0$ are all equivalent equations since the solution set for each is $\{-2\}$.

Properties of Equality

Addition Property of Equality

For any three algebraic expressions A, B and C,

$$\text{If} \quad A = B$$

$$\text{then} \quad A + C = B + C$$

In words: Adding the same quantity to both sides of an equation will not change the solution set.

Properties of Equality

Multiplication Property of Equality

For any three algebraic expressions A, B and C, where $C \neq 0$,

$$\text{If} \quad A = B,$$

$$\text{then} \quad A \cdot C = B \cdot C$$

In words: Multiplying both sides of an equation by the same non-zero quantity will not change the solution set.

Classroom Examples: Solve the following equations for x.

1. $\frac{2}{3}x + 4 = -8$

2. $3x - 3 = -5x + 9$

3. $-x = 1$

4. $\frac{3}{5}x + \frac{1}{3} = -\frac{5}{6}$

5. $0.08x + 0.10(8000 - x) = 680$

Solving Linear Equations in One Variable

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Step 3: Use the Multiplication Property of Equality to get the variable by itself on one side of the equation.

Step 4: Check your solution in the original equation to be sure that you have not made a mistake in the solution process.

Classroom Examples: Solve the following equations for x.

1. $6 - 2(5x - 1) + 4x = 20$

2. $3(5x + 1) = 10 + 15x$ Special Case 1

3. $-4 + 8x = 2(4x - 2)$ Special Case 2

Equations involving Absolute Value

- Ex1: Solve $|x| = -2$ for x.
- Ex2: Solve $|x| = 4$ for x.
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Theorem

Suppose k represent any positive real number. Then $|x| = k$ if and only if $x = \pm k$.

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Theorem

Suppose k represent any positive real number. Then $|x| = k$ if and only if $x = \pm k$.

Comment: the variable x in the above theorem can represent any algebraic expression.

Ex: Solve $|x - 5| = 4$ for x.

Definition

A quadratic equation is an equation of the form

$$ax^2 + bx + c = 0$$

where a, b, and c are real numbers, and $a \neq 0$.

The condition $a \neq 0$ in the definition ensures that the equation actually has an x^2 term.

Notice that the LHS is a second degree polynomial expression.

Theorem (Soln to a Quadratic Eqn)

Suppose a, b, and c are any real numbers, with the exception that $a \neq 0$. The quadratic equation:

$$a\,x^2 + b\,x + c = 0$$

has the two solutions:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Example Solve the following quadratic equation:

$$x^2 - 4\,x - 7 = 0$$

Using the Quadratic Eqn

Example Solve the following quadratic equation:

$$x^2 - 4x - 7 = 0$$

Here $a = 1$, $b = -4$ and $c = -7$. The quadratic formula says that the two solutions to this equation are:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(-7)}}{2(1)}$$

$$= \frac{4 \pm \sqrt{16 + 28}}{2} = \frac{4}{2} \pm \frac{\sqrt{44}}{2} = \frac{4}{2} \pm \frac{\sqrt{4 \cdot 11}}{2}$$

$$= 2 \pm \frac{\sqrt{4}\sqrt{11}}{2} = 2 \pm \frac{2\sqrt{11}}{2} = 2 \pm \sqrt{11}$$

so that $x = 2 \pm \sqrt{11}$ satisfies the given equation.

The Discriminant

Definition

The expression (number) $b^2 - 4ac$ is called the *discriminant*, because it determines the number and type of solutions to the quadratic equation.

Value of $b^2 - 4ac$	# of real solns	soln number type
positive	2	2 real, both rational provided $b^2 - 4ac$ is a perfect square; else 2 irrational solns.
zero	1	rational number
negative	0	2 complex conjugate solns

Using the Quadratic Eqn and Discriminant

Example Use the discriminant to determine the number of solutions there are to the following quadratic equation. What type of number(s) is (are) the solution(s)? Solve for x using the quadratic formula:

$$4x^2 + 9 = 12x$$

Example Use the discriminant to determine the number of solutions there are to the following quadratic equation. What type of number(s) is (are) the solution(s)? Solve for x using the quadratic formula:

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Theorem (zero product)

Suppose A and B are algebraic expressions, then the equation $AB = 0$ is equivalent to the compound statement $A = 0$ or $B = 0$ (or both).

Note: The zero product theorem is one of the most used and abused rules in all of mathematics; that is to say that it is a hugely important rule!!

Example: Solve the equation $x^2 - 5x = -6$ by factoring and using the zero product theorem.

Example: Solve the equation $23x + 10 = -6x^2$ by factoring and using the zero product theorem.

Theorem (on Square Roots)

For any real number k, the equation $x^2 = k$ is equivalent to $x = \pm\sqrt{k}$

Examples: Solve the following equations by using the square root property.

1. $x^2 - 9 = 0$
2. $2x^2 - 1 = 0$
3. $(2x - 1)^2 = 0$
4. $(x - 3)^2 + 8 = 0$

The Sqrt Thm in Action

Theorem (on Square Roots)

For any real number k, the equation $x^2 = k$ is equivalent to $x = \pm\sqrt{k}$

Examples: Solve the following equations by first completing the square; then use the sqrt thm.

1. $x^2 + 6x + 7 = 0$
2. $2x^2 - 3x = 4$
3. $3x^2 - 4x = 5$

Linear Inequalities in One Variable

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Linear Inequalities in One Variable

Definition

Replacing the equal sign in the general linear equation $a \cdot x + b = c$ by any of the symbols $<, \leq, >$ or $\geq$ gives a **linear inequality in one variable**.

For example, $2 \cdot x - 1 \leq 0$ and $3x + 5 > 8$ are two different linear inequalities in a single variable, x.

Solving Linear Inequalities

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For example, $\{x \mid x < -2\}$ is shorthand notation for the set of real numbers less than -2.

$$\{x \;\mid\; x < -2\}$$

$$\swarrow \quad \downarrow \quad \searrow$$

x $\overbrace{\text{such that}}$ $\overbrace{x \text{ is any real number less than } -2}$

Addition Property for Inequalities

For any three algebraic expressions A, B and C,

$$\text{If} \quad A < B$$

$$\text{then} \quad A + C < B + C$$

In words: Adding the same quantity to both sides of an inequality will not change the solution set.

We can use the Addn. Prop. to write ***equivalent inequalities***.

Example 1 Solve the inequality, $5x + 4 < 4x + 2$, then graph the solution.

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$$5x + 4 < 4x + 2$$

$$5x + 4 + (-4) < 4x + 2 + (-4) \qquad \text{Addition Prop. of Inequalities}$$

$$5x + \Big(4 + (-4)\Big) < 4x + \Big(2 + (-4)\Big) \qquad \text{Associative Prop. of Addition}$$

$$5x + 0 < 4x + (-2) \qquad \text{Additive Inverse \& Closure Props.}$$

$$5x < 4x - 2 \qquad \text{Additive Identity \& the Defn. of Subtraction}$$

Example 1 Solve the inequality, $5x + 4 < 4x + 2$, then graph the solution.

Solution:

$$5x < 4x - 2$$

$$5x + (-4x) < 4x - 2 + (-4x)$$ Addition Prop. of Inequalities

$$5x + (-4x) < 4x + (-4x) - 2$$ Commutative Prop. of Addn.

$$\left(5x + (-4x)\right) < \left(4x + (-4x)\right) - 2$$ Associative Prop. of Addn.

$$(5 - 4) \cdot x < 0 - 2$$ Distributive & Additive Inverse Props.

$$1 \cdot x < -2$$ Closure & Additive Identity Props.

$$x < -2$$ Multiplicative Identity Prop.

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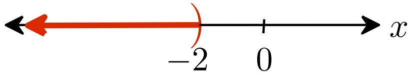
Graph: We can shade the number line to the left of -2 to give a graphical description of the solution set.



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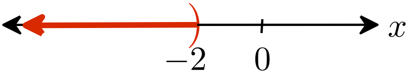

We use a left-opening parenthesis at -2 to indicate that -2 is not part of the solution set.

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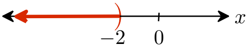
Line Graph



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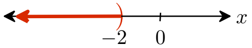

Interval Notation

$(-\infty, -2)$

Properties of Inequalities

Multiplication Property of Inequalities

For any three algebraic expressions A, B and C, where $C \neq 0$,

If	$A < B$,	
then	$A \cdot C < B \cdot C$	if C is positive $(C > 0)$
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In words: Multiplying both sides of an inequality by a positive quantity always produces an equivalent inequality. Multiplying both sides of an inequality by a negative number produces an equivalent inequality BUT it reverses the direction of the inequality symbol.

Example 2 Determine what set is the solution to $-2x - 3 \leq 3$

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Solution:

$$-2x - 3 \leq 3$$

$$-2x - 3 + 3 < 3 + 3 \qquad \text{Addition Prop. of Inequalities}$$

$$-2x \leq 6 \qquad \text{Additive Inverse \& Identity Props}$$

$$\left(-\frac{1}{2}\right) \cdot (-2x) \geq \left(-\frac{1}{2}\right) \cdot 6 \qquad \text{Multiplication Prop. of Inequalities}$$

$$x \geq -3 \qquad \text{Closure}$$

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Solution:

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$-2x - 3 + 3 < 3 + 3$	Addition Prop. of Inequalities
$-2x \leq 6$	Additive Inverse & Identity Props
$\left(-\frac{1}{2}\right) \cdot (-2x) \geq \left(-\frac{1}{2}\right) \cdot 6$	Multiplication Prop. of Inequalities
$x \geq -3$	Closure

Set Notation
$\{x \mid x \geq -3\}$

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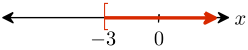

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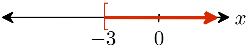
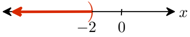
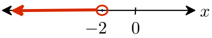
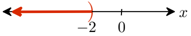
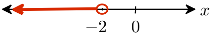
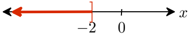
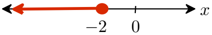


Interval Notation
$[-3, \infty)$

Interval Notation and Graphing

Inequality Notation	Interval Notation	Graph Using Parenthesis/Brackets	Graph using open and closed circles
$x < -2$	$(-\infty, -2)$		

Interval Notation and Graphing

Inequality Notation	Interval Notation	Graph Using Parenthesis/Brackets	Graph using open and closed circles
$x < -2$	$(-\infty, -2)$		
$x \leq -2$	$(-\infty, -2]$		

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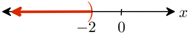
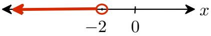
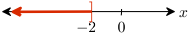
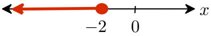
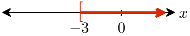
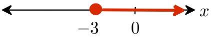
$x < -2$	$(-\infty, -2)$		
$x \leq -2$	$(-\infty, -2]$		
$x \geq -3$	$[-3, \infty)$		

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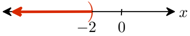
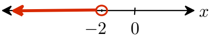
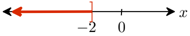
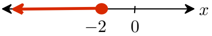
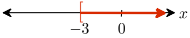
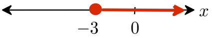
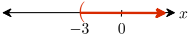
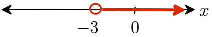
$x < -2$	$(-\infty, -2)$		
$x \leq -2$	$(-\infty, -2]$		
$x \geq -3$	$[-3, \infty)$		
$x > -3$	$(-3, \infty)$		

Linear Inequalities in One Variable

Classroom Example: Solve the following inequality.

- $3(2x + 5) \leq -3x$

Linear Inequalities in One Variable

Classroom Examples: Take the next five minutes to work these 6 problems. Graph the solution set to the given inequality, then write the solution set using interval notation.

- $x \leq -6$
- $x > 5$
- $x \geq -1$
- $x > 10$

Classroom Examples: Solve each inequality. Graph the solution set, then write the solution set using interval notation.

- $2x - 1 \leq -6$
- $-3x < 2x - 6$

Linear Inequalities in One Variable

Definition

A compound inequality is two or more simple inequalities {sets} joined by the terms 'and' or 'or' .

For Example, the set $\left\{ x \;\middle|\; 3x - 6 \leq -3 \quad \text{or} \quad 3x - 6 \geq 3 \right\}$ is a compound inequality.

The inequality statement $-7 < x < 7$ is to be read "x is in between −7 and 7."

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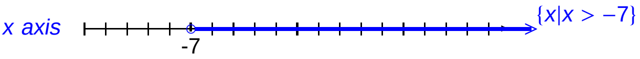

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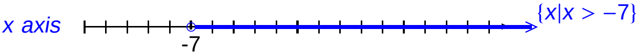

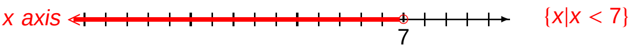

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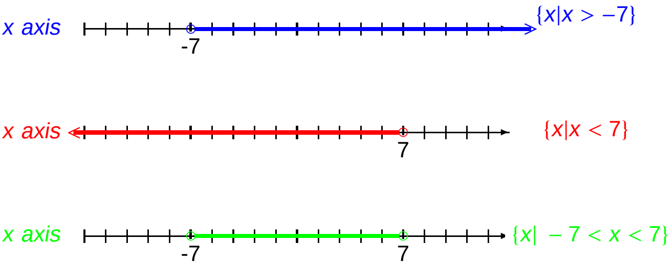

Linear Inequalities in One Variable

Classroom Examples: Solve the following compound inequalities. Graph the solution set on a number line, then write the solution set using interval notation.

- $-7 \leq 2x + 1 \leq 7$
- $3x - 6 \leq -3$ or $3x - 6 \geq 3$

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Ordered Pairs

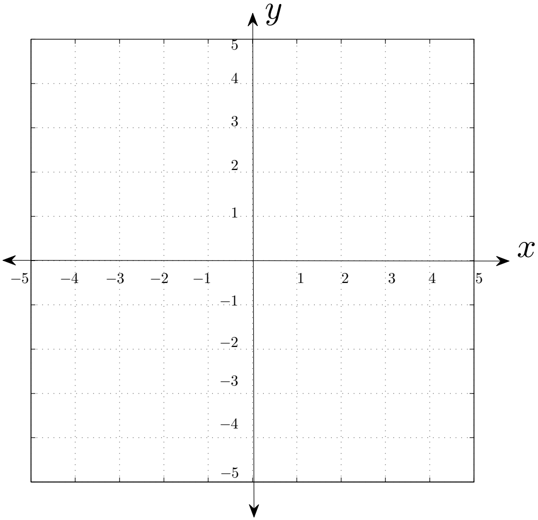
We now turn our attention to equations containing two variables, x and y. Paired data plays an important role in these type of equations.

Ordered Pairs

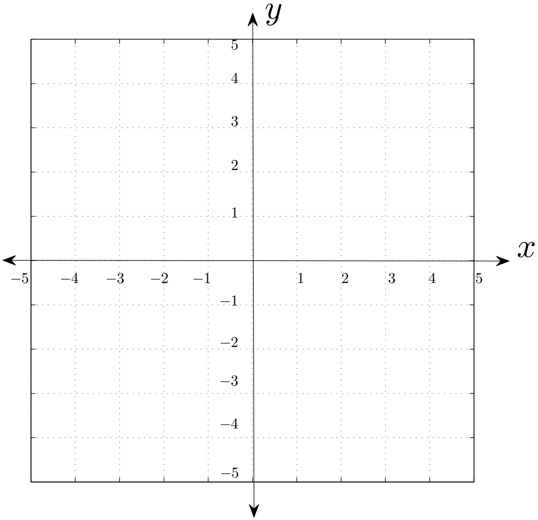
Definition

A pair of numbers enclosed in parenthesis and separated by a comma, such as $(-2, 1)$, is called an ordered pair of numbers. The first number in the pair is called the x-coordinate of the ordered pair; the second number is called the y-coordinate. For the ordered pair $(-2, 1)$, the x-coordinate is -2 and the y-coordinate is 1.

We use a rectangular coordinate system to visualize ordered pairs.



A rectangular coordinate system is made by drawing two real number lines at right angles to each other.



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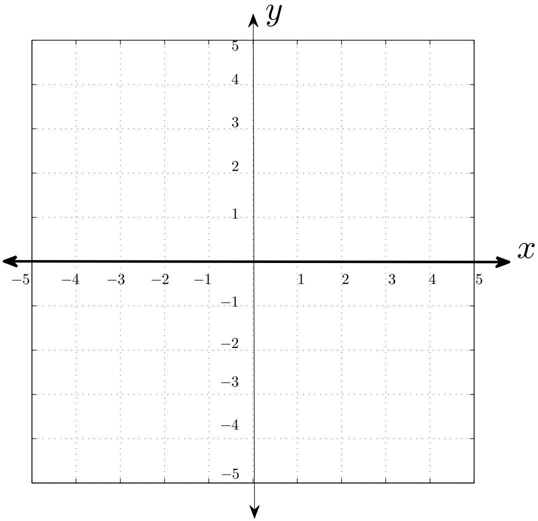

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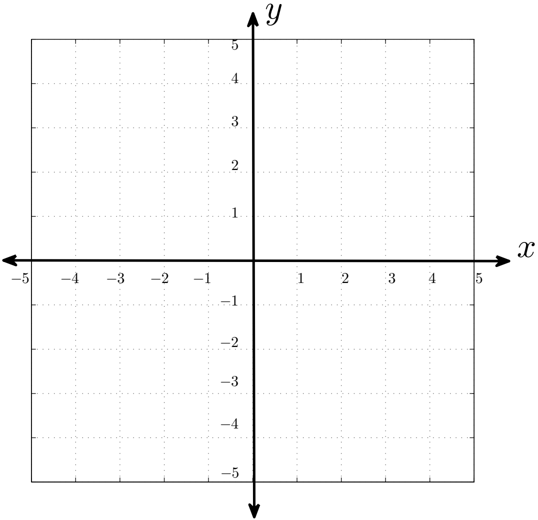
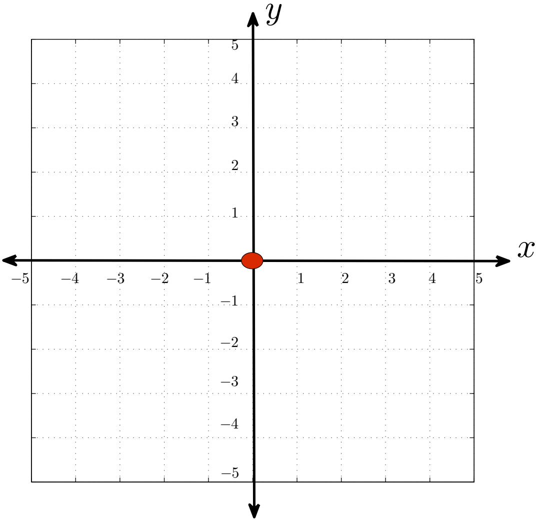
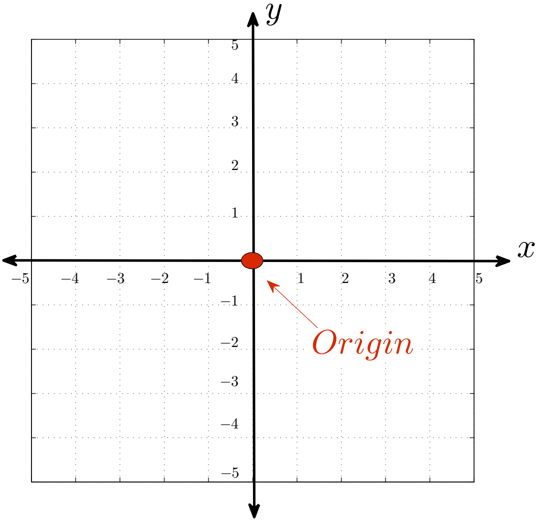


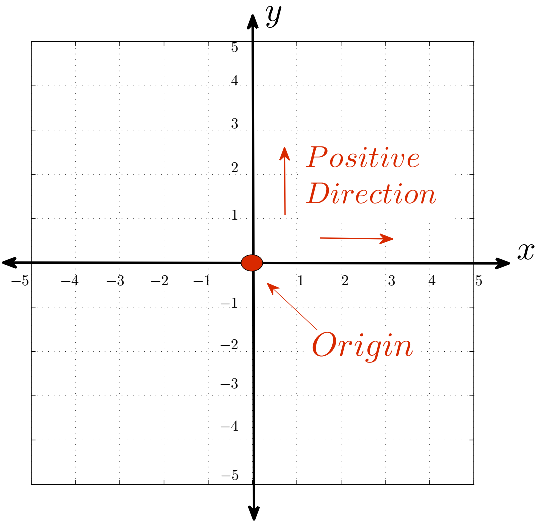
Two number lines, called axes, cross each other at zero.



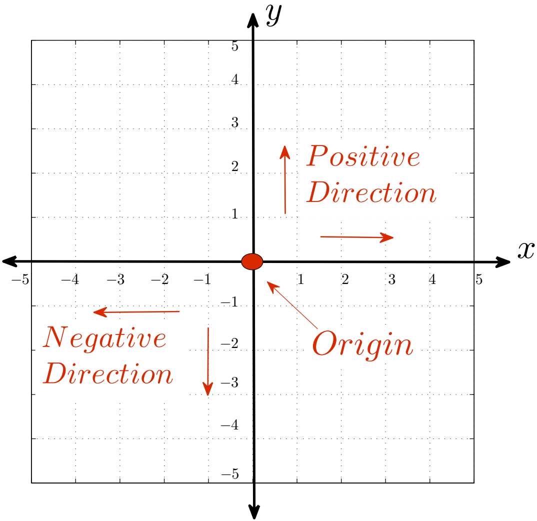
Two number lines, called axes, cross each other at zero. This point is called the *origin*.



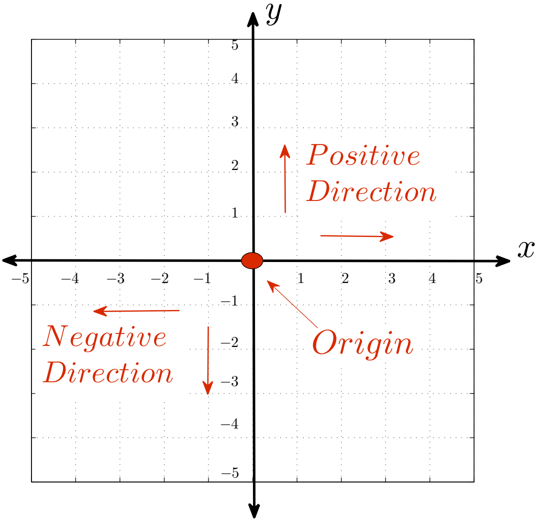
Relative to the origin, positive directions are to the right and up.



Negative directions are to the left and down.



The horizontal number line is called the *x*-axis



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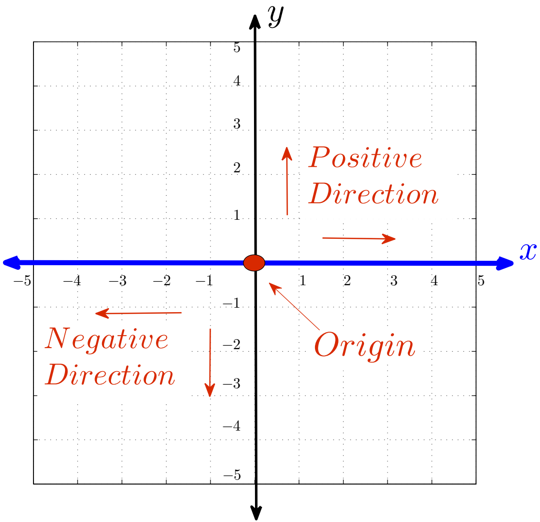
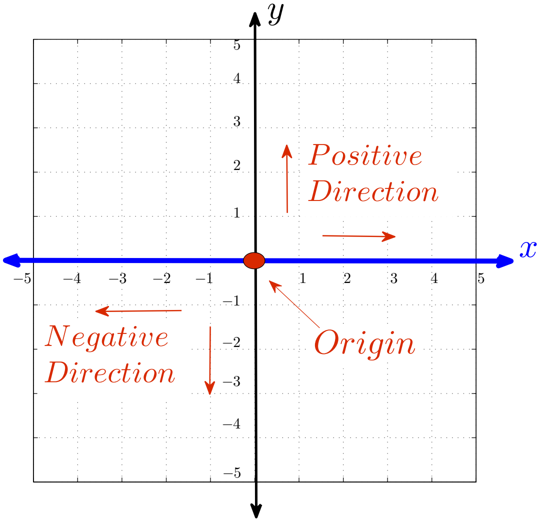


The horizontal number line is called the *x*-axis and the vertical number line is called the *y*-axis.



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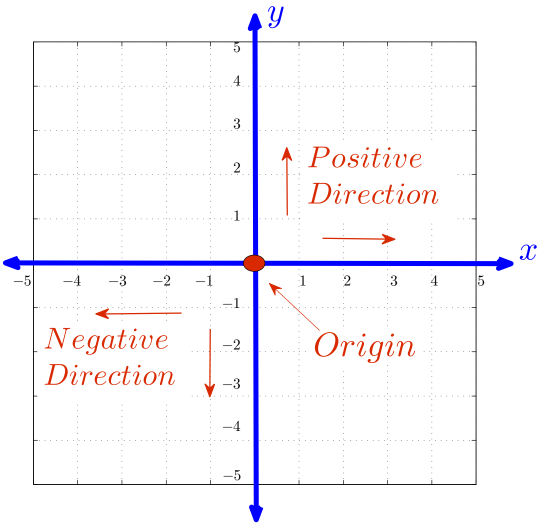
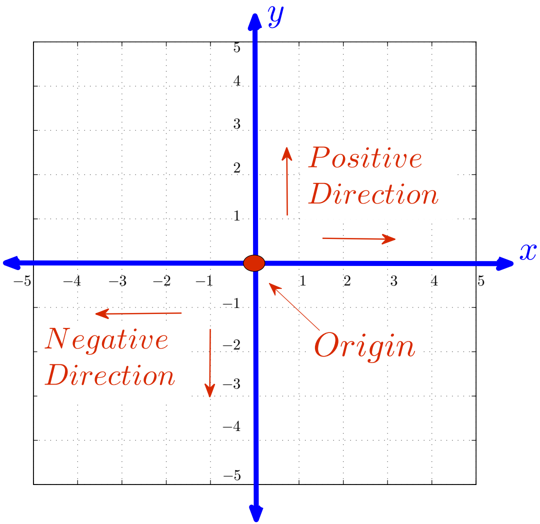


The two number lines divide the coordinate system into four quadrants, which we number I through IV in a counterclockwise direction.



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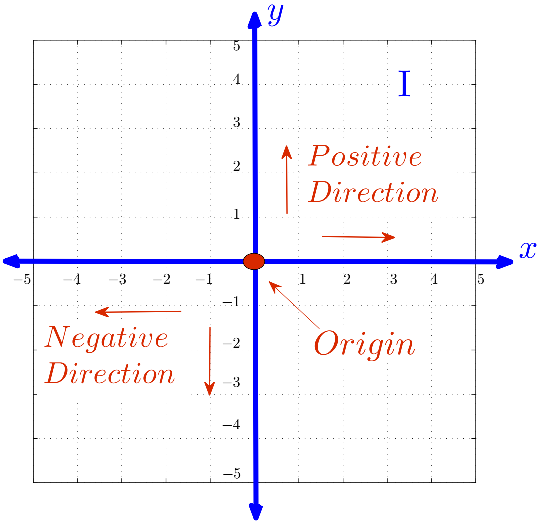

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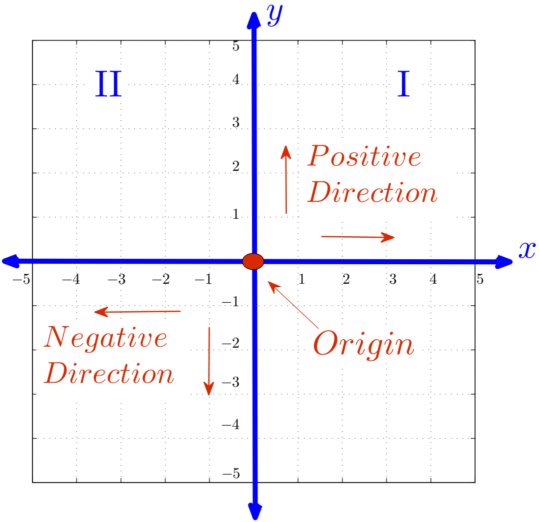

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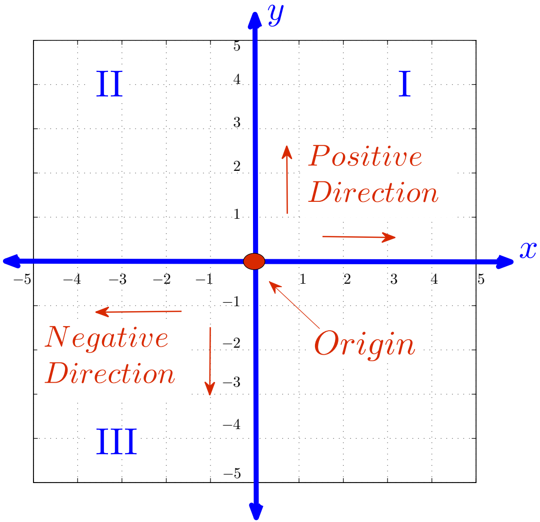

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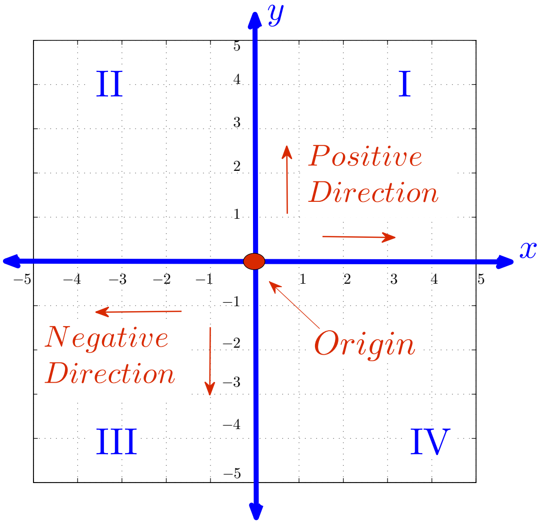

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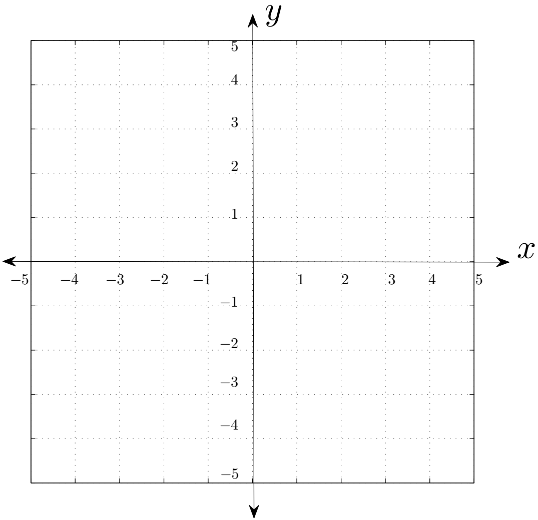
1. begin at the origin and move along the x-axis a units right or a units left (right if a is positive and left if a is negative).
2. From that point we move b units up or down (up if b is positive and down if b is negative).
3. The point where we end up is the graph of the ordered pair.

Example 1: Plot (graph) the following ordered pairs:

$$(2,3),\ (-2,3),\ (-2,-3),\ (2,-3),$$



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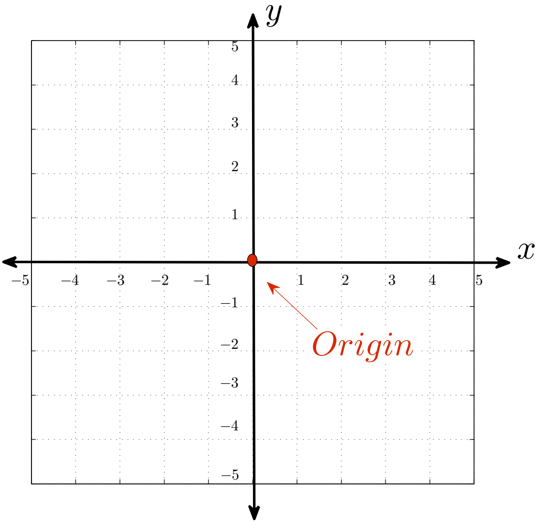

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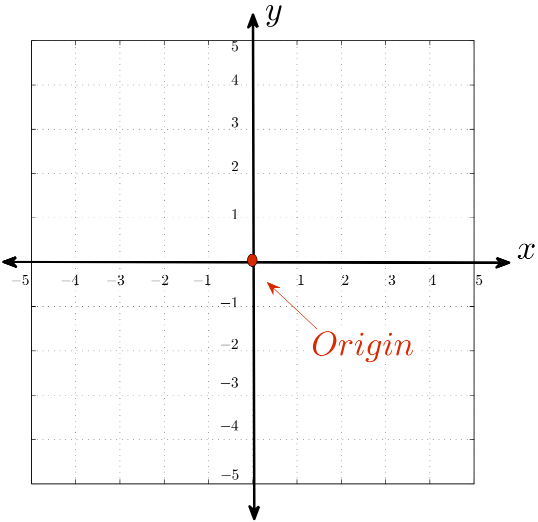

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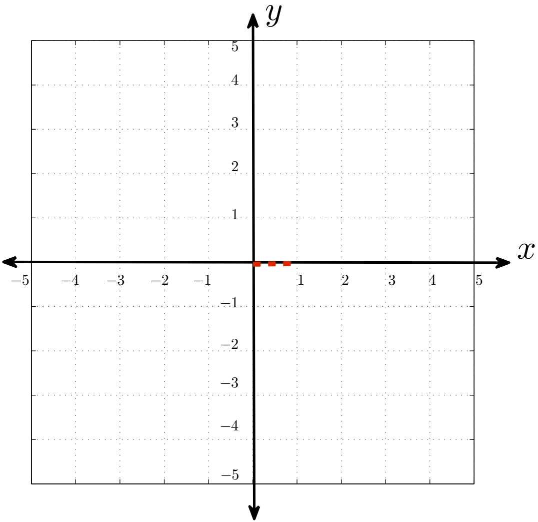

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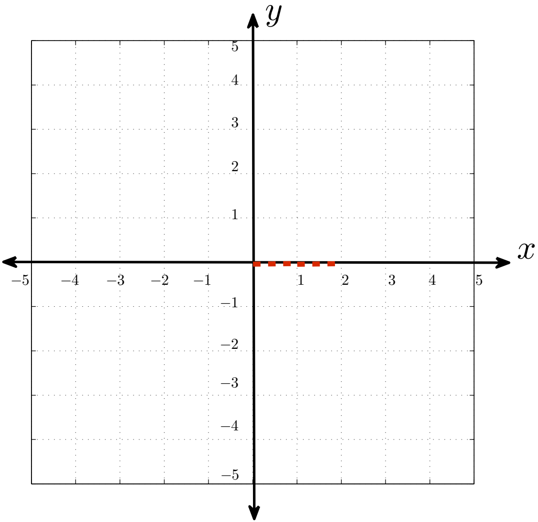

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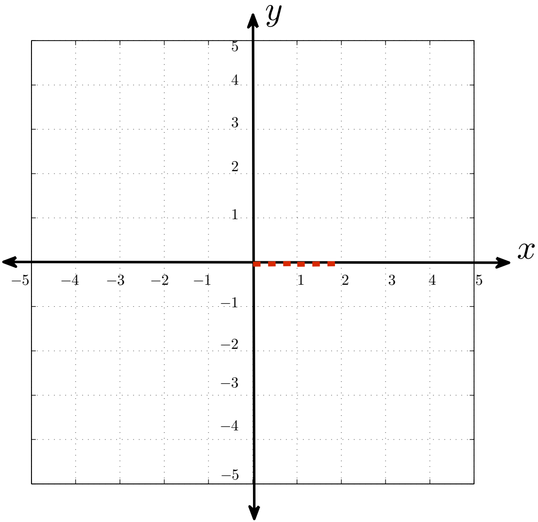
From that point, move in the upwards (positive y) direction 3 units.



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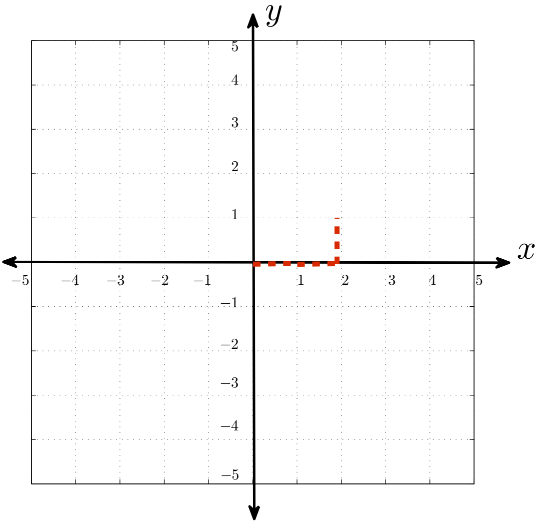

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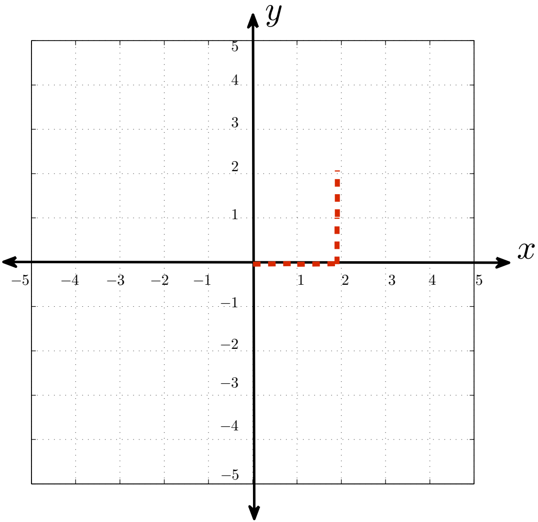

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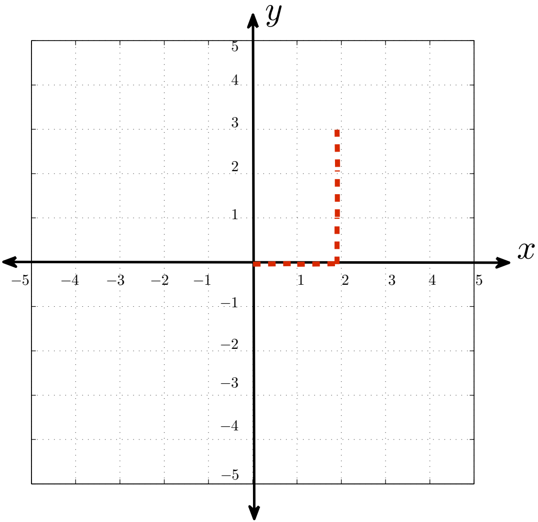

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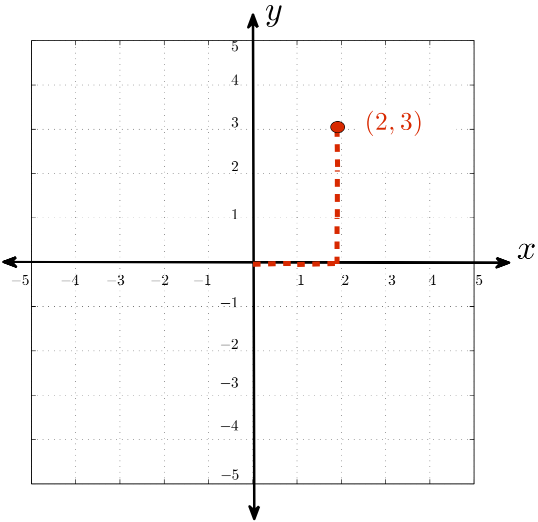

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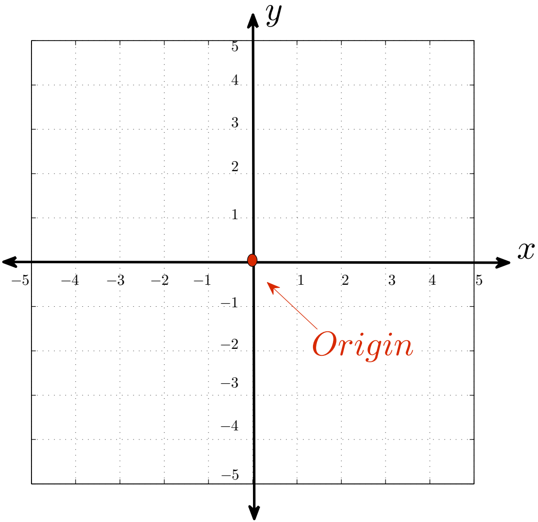

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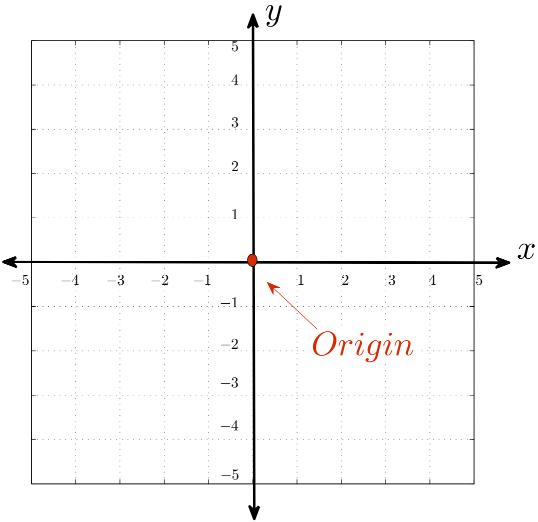

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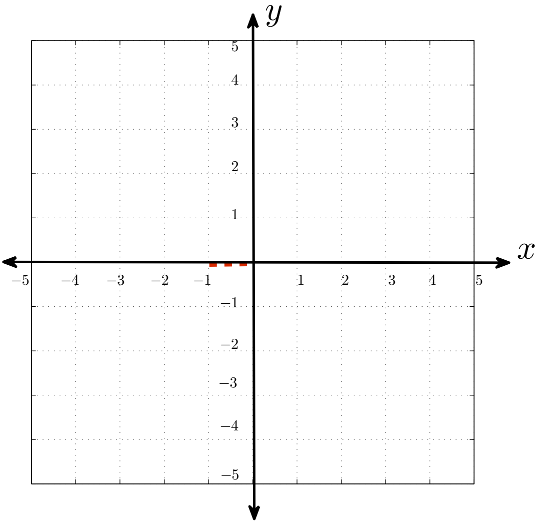

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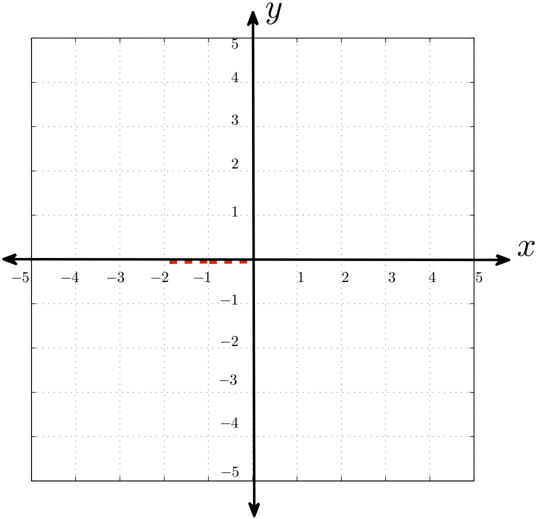
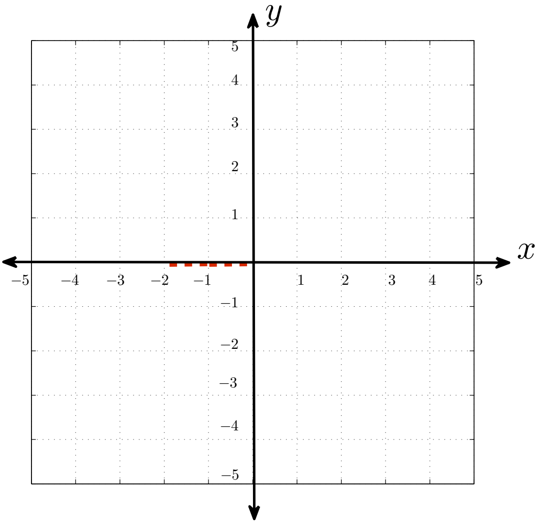
To plot $(-2, 3)$, begin at the origin. Travel along the x-axis 2 units left (in the negative direction).



Example 1: Plot (graph) the following ordered pairs:

$(2, 3)$, $(-2, 3)$, $(-2, -3)$, $(2, -3)$,

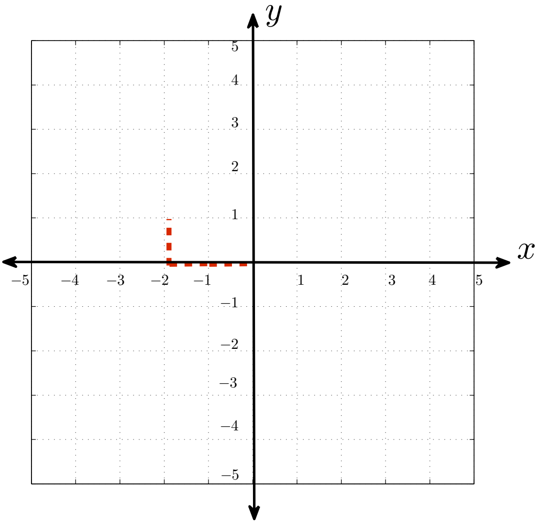
From that point, move in the upwards (positive y) direction 3 units.



Example 1: Plot (graph) the following ordered pairs:

$(2, 3)$, $(-2, 3)$, $(-2, -3)$, $(2, -3)$,

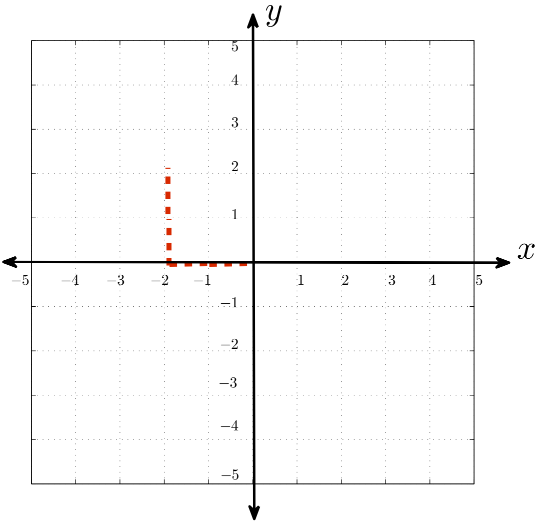
From that point, move in the upwards (positive y) direction 3 units.



Example 1: Plot (graph) the following ordered pairs:

$(2, 3)$, $(-2, 3)$, $(-2, -3)$, $(2, -3)$,

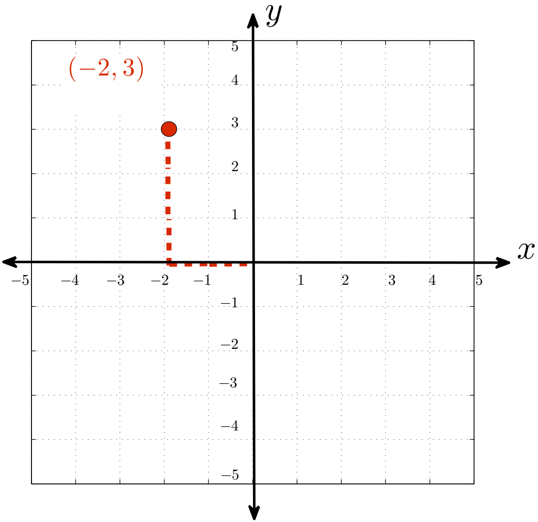
From that point, move in the upwards (positive y) direction 3 units.



Example 1: Plot (graph) the following ordered pairs:

$(2, 3)$, $(-2, 3)$, $(-2, -3)$, $(2, -3)$,

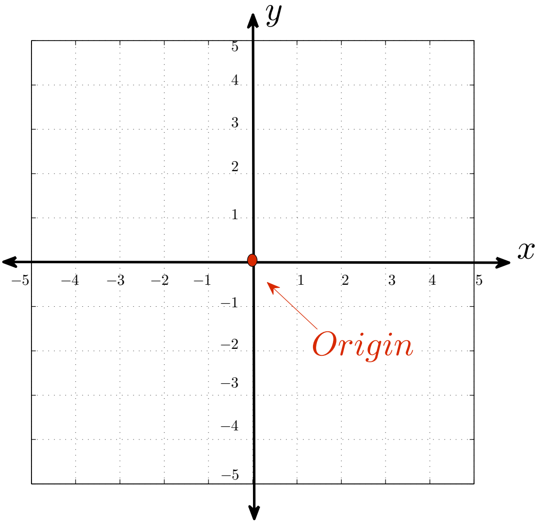
From that point, move in the upwards (positive y) direction 3 units.



Example 1: Plot (graph) the following ordered pairs:

$(2, 3)$, $(-2, 3)$, $(-2, -3)$, $(2, -3)$,

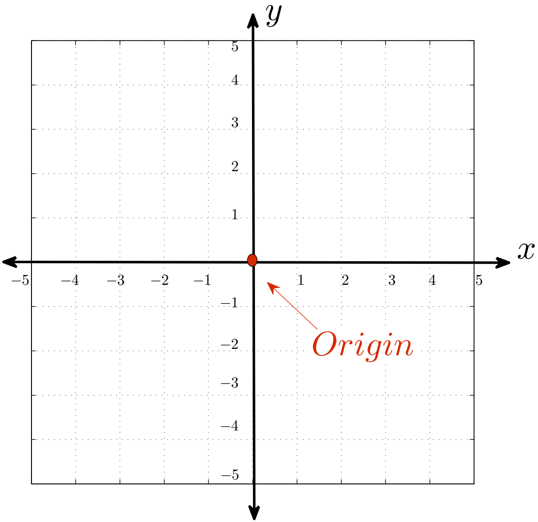
To plot $(-2, -3)$, begin at the origin.



Example 1: Plot (graph) the following ordered pairs:

$(2, 3)$, $(-2, 3)$, $(-2, -3)$, $(2, -3)$,

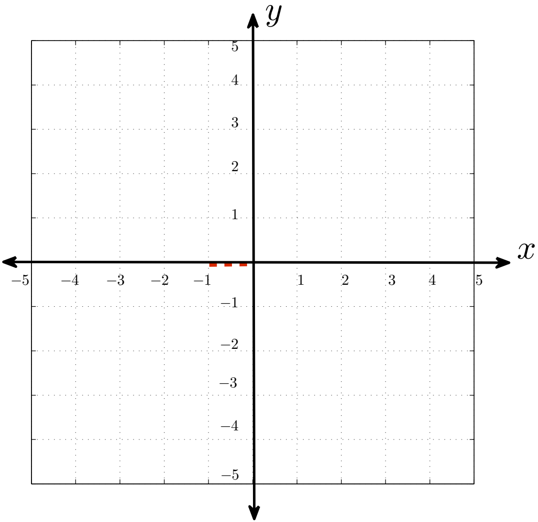
To plot $(-2, -3)$, begin at the origin. Travel along the x-axis 2 units left (in the negative direction).



Example 1: Plot (graph) the following ordered pairs:

$(2, 3)$, $(-2, 3)$, $(-2, -3)$, $(2, -3)$,

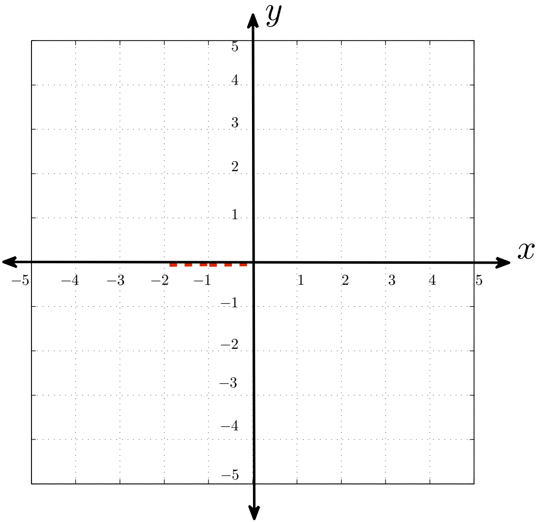
To plot $(-2, -3)$, begin at the origin. Travel along the *x*-axis 2 units left (in the negative direction).



Example 1: Plot (graph) the following ordered pairs:

$(2,3)$, $(-2,3)$, $(-2,-3)$, $(2,-3)$,

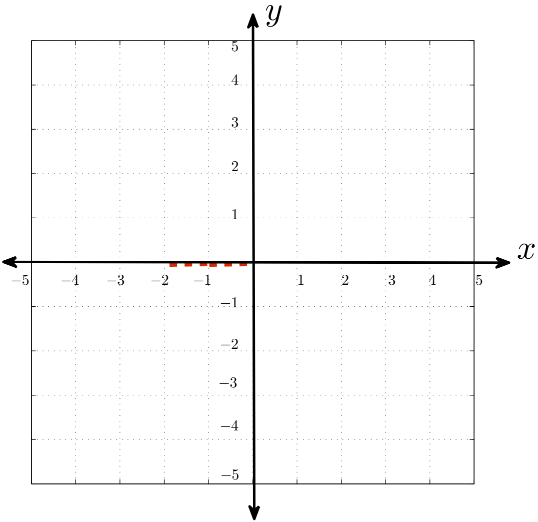
To plot $(-2,-3)$, begin at the origin. Travel along the x-axis 2 units left (in the negative direction).



Example 1: Plot (graph) the following ordered pairs:

$(2, 3)$, $(-2, 3)$, $(-2, -3)$, $(2, -3)$,

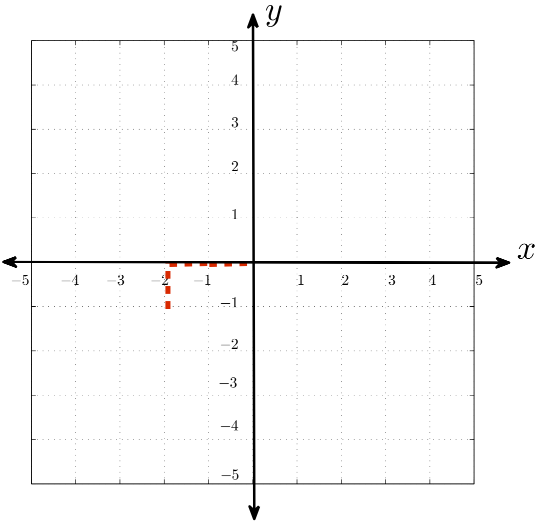
From that point, move in the downwards (negative y) direction 3 units.



Example 1: Plot (graph) the following ordered pairs:

$(2,3)$, $(-2,3)$, $(-2,-3)$, $(2,-3)$,

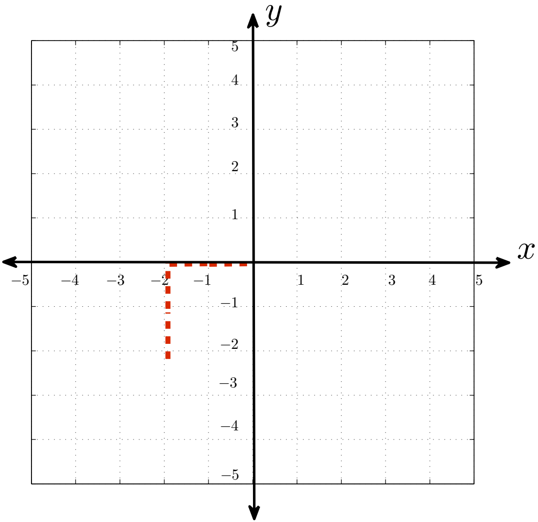
From that point, move in the downwards (negative y) direction 3 units.



Example 1: Plot (graph) the following ordered pairs:

$(2, 3)$, $(-2, 3)$, $(-2, -3)$, $(2, -3)$,

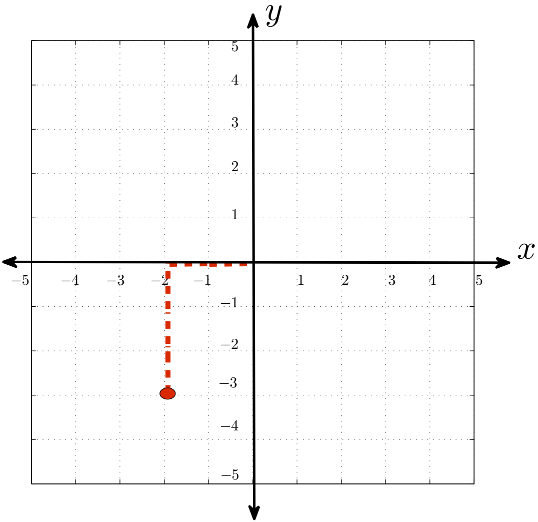
From that point, move in the downwards (negative y) direction 3 units.



Example 1: Plot (graph) the following ordered pairs:

$(2, 3)$, $(-2, 3)$, $(-2, -3)$, $(2, -3)$,

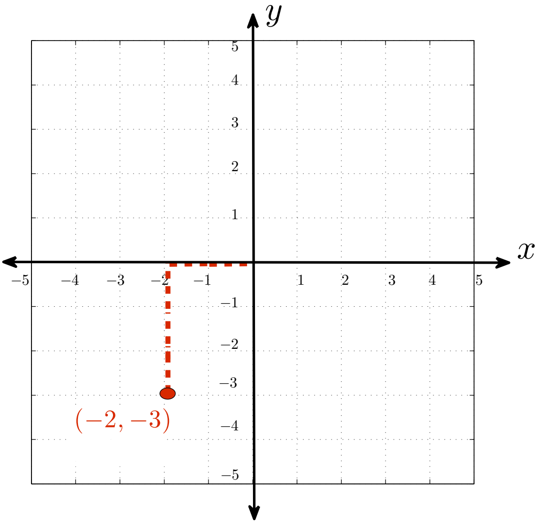
From that point, move in the downwards (negative y) direction 3 units.



Example 1: Plot (graph) the following ordered pairs:

$(2,3)$, $(-2,3)$, $(-2,-3)$, $(2,-3)$,

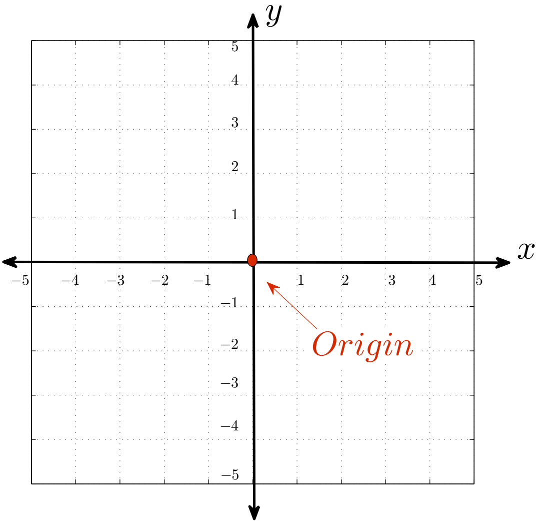
From that point, move in the downwards (negative y) direction 3 units.



Example 1: Plot (graph) the following ordered pairs:

$(2, 3)$, $(-2, 3)$, $(-2, -3)$, $(2, -3)$,

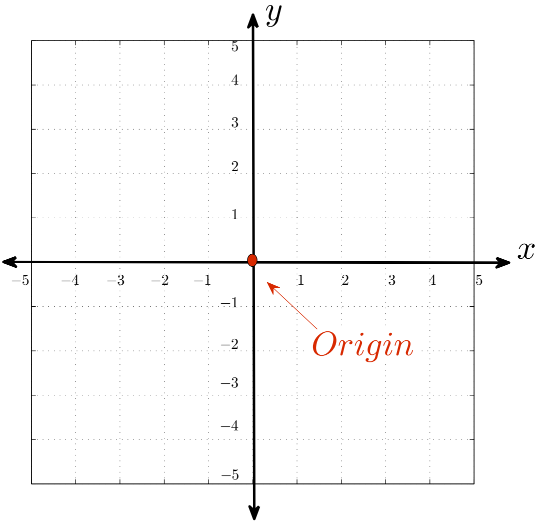
To plot $(2, -3)$, begin at the origin.



Example 1: Plot (graph) the following ordered pairs:

$(2, 3)$, $(-2, 3)$, $(-2, -3)$, $(2, -3)$,

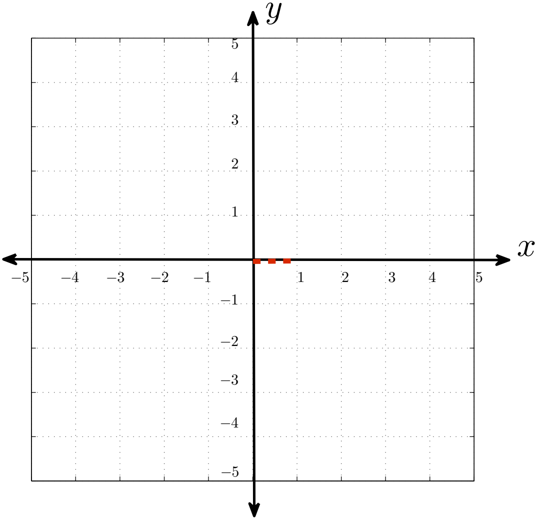
To plot $(2, -3)$, begin at the origin. Travel along the x-axis 2 units right (in the positive direction).



Example 1: Plot (graph) the following ordered pairs:

$(2, 3),\ (-2, 3),\ (-2, -3),\ (2, -3),$

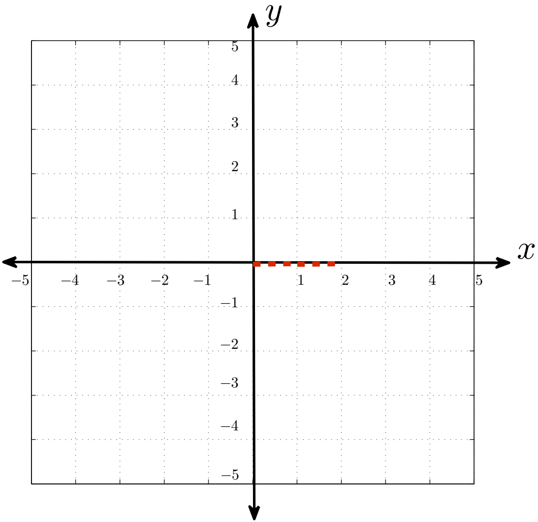
To plot $(2, -3)$, begin at the origin. Travel along the x-axis 2 units right (in the positive direction).



Example 1: Plot (graph) the following ordered pairs:

$(2, 3)$, $(-2, 3)$, $(-2, -3)$, $(2, -3)$,

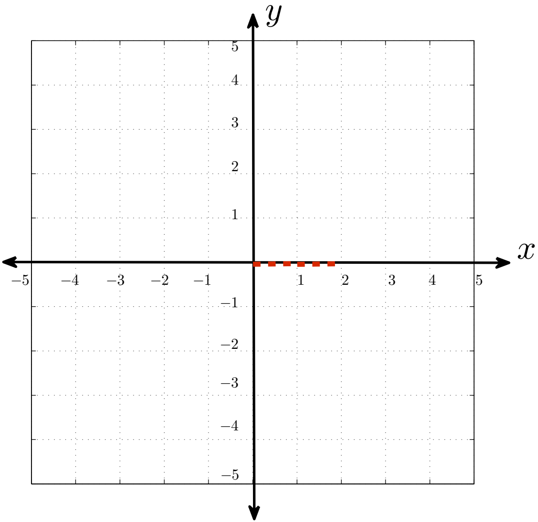
To plot $(2, -3)$, begin at the origin. Travel along the x-axis 2 units right (in the positive direction).



Example 1: Plot (graph) the following ordered pairs:

$(2, 3), \ (-2, 3), \ (-2, -3), \ (2, -3),$

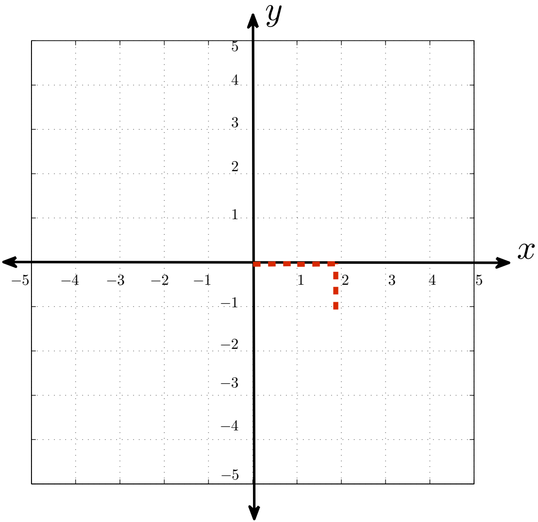
From that point, move in the downwards (negative y) direction 3 units.



Example 1: Plot (graph) the following ordered pairs:

$(2, 3)$, $(-2, 3)$, $(-2, -3)$, $(2, -3)$,

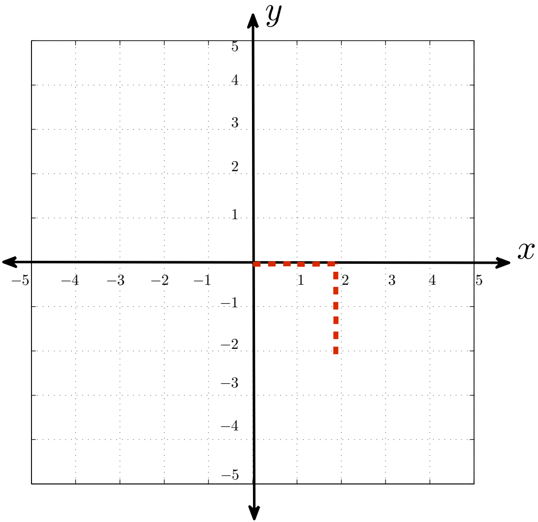
From that point, move in the downwards (negative y) direction 3 units.



Example 1: Plot (graph) the following ordered pairs:

$(2, 3)$, $(-2, 3)$, $(-2, -3)$, $(2, -3)$,

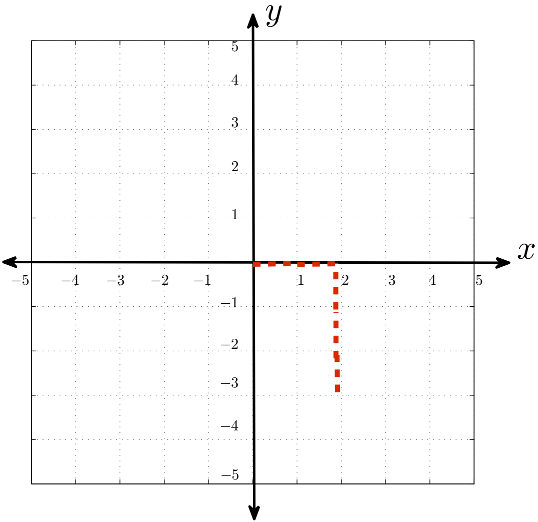
From that point, move in the downwards (negative y) direction 3 units.



Example 1: Plot (graph) the following ordered pairs:

$(2,3),\ (-2,3),\ (-2,-3),\ (2,-3),$

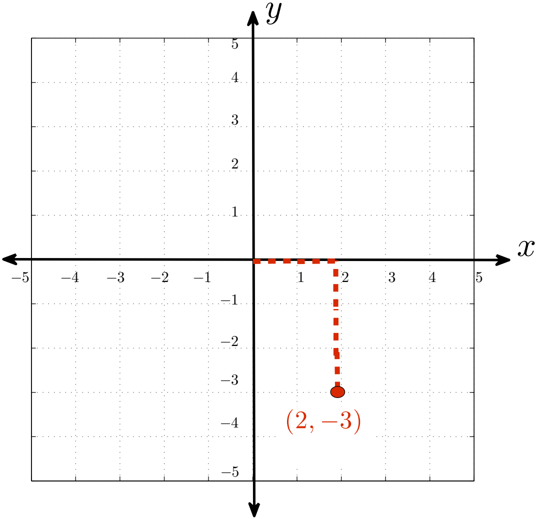
From that point, move in the downwards (negative y) direction 3 units.



Example 1: Plot (graph) the following ordered pairs:

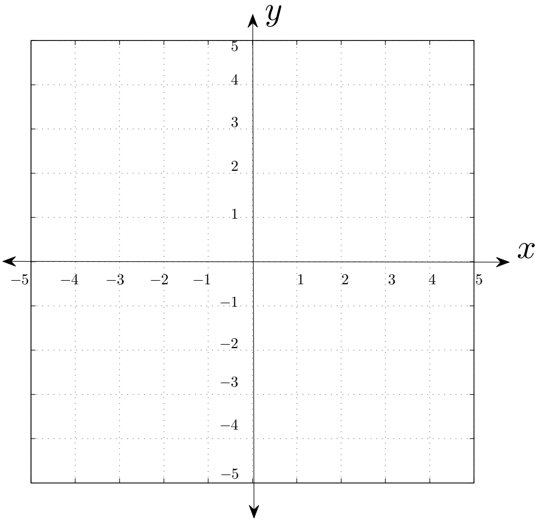
$(2, 3),\ (-2, 3),\ (-2, -3),\ (2, -3),$

From that point, move in the downwards (negative *y*) direction 3 units.



Example 2: Plot (graph) the following ordered pairs:

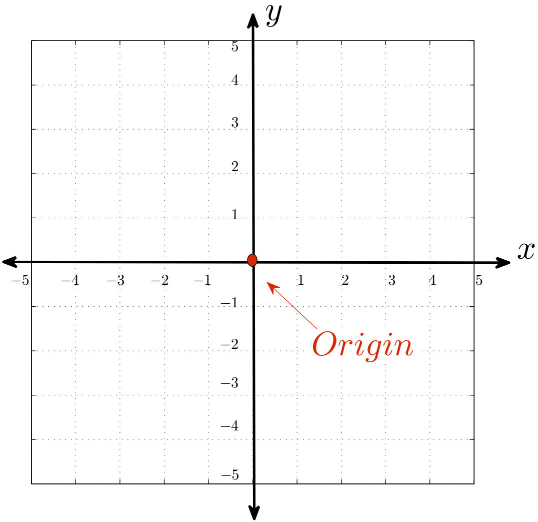
$$(3, 0),\ (0, 2),\ (-3, 0),\ (0, -2),$$



Example 2: Plot (graph) the following ordered pairs:

$(3,0)$, $(0,2)$, $(-3,0)$, $(0,-2)$,

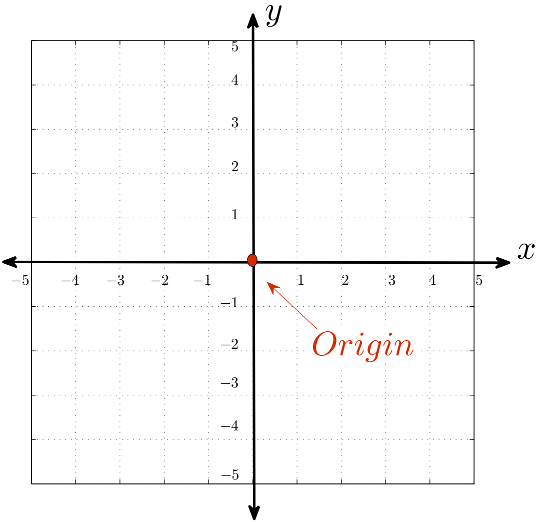
To plot $(3,0)$, begin at the origin.



Example 2: Plot (graph) the following ordered pairs:

$(3, 0)$, $(0, 2)$, $(-3, 0)$, $(0, -2)$,

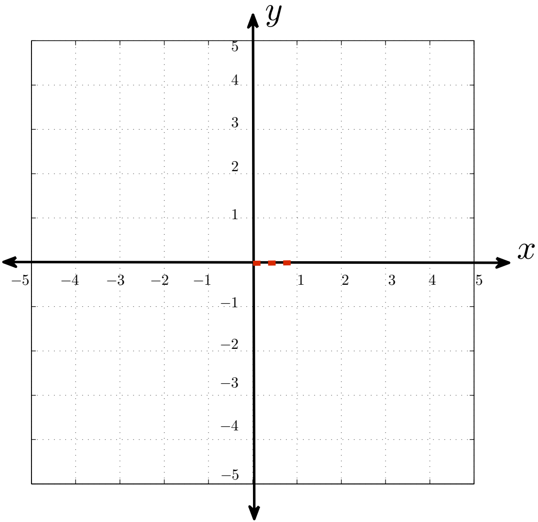
To plot $(3, 0)$, begin at the origin. Travel along the x-axis 3 units right, in the positive direction.



Example 2: Plot (graph) the following ordered pairs:

$(3,0)$, $(0,2)$, $(-3,0)$, $(0,-2)$,

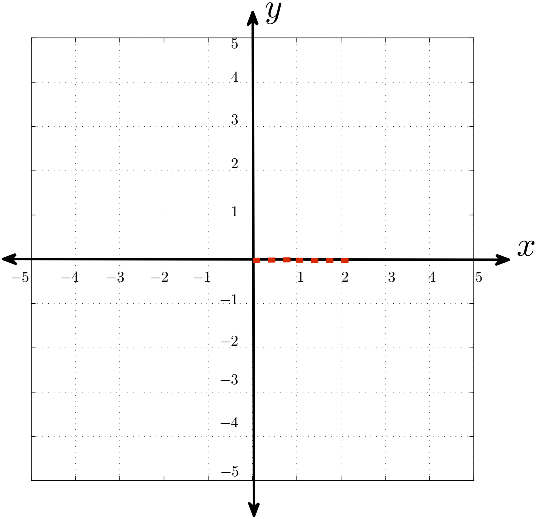
To plot $(3,0)$, begin at the origin. Travel along the x-axis 3 units right, in the positive direction.



Example 2: Plot (graph) the following ordered pairs:

$(3, 0)$, $(0, 2)$, $(-3, 0)$, $(0, -2)$,

To plot $(3, 0)$, begin at the origin. Travel along the x-axis 3 units right, in the positive direction.



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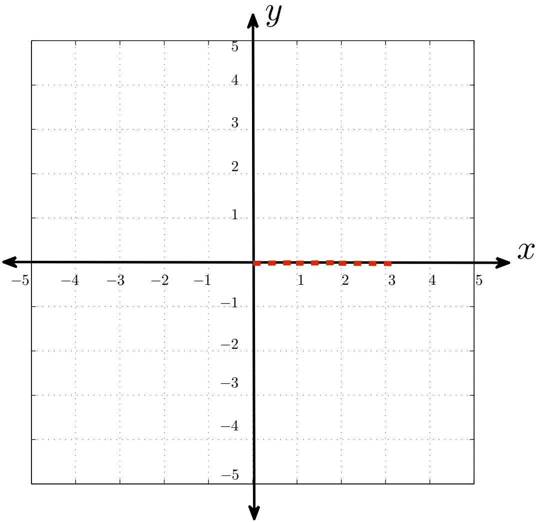

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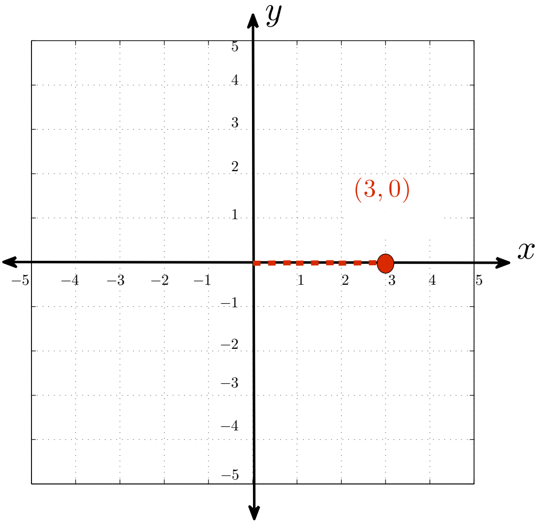
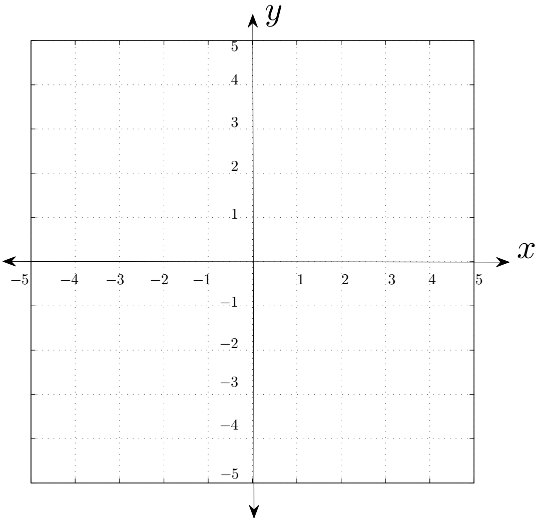


Example 2: Plot (graph) the following ordered pairs:

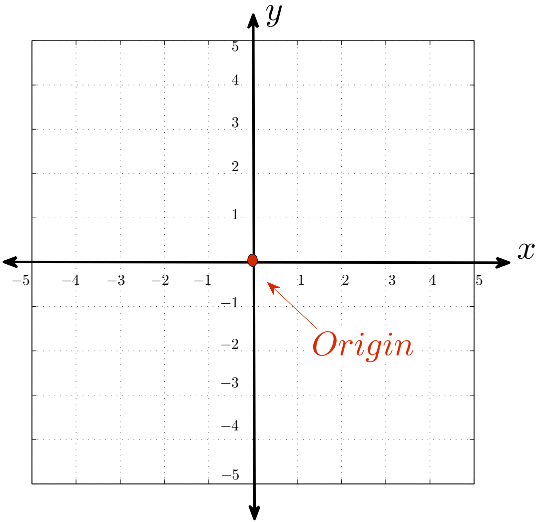
$(3, 0)$, $(0, 2)$, $(-3, 0)$, $(0, -2)$,



Example 2: Plot (graph) the following ordered pairs:

$(3, 0)$, $(0, 2)$, $(-3, 0)$, $(0, -2)$,

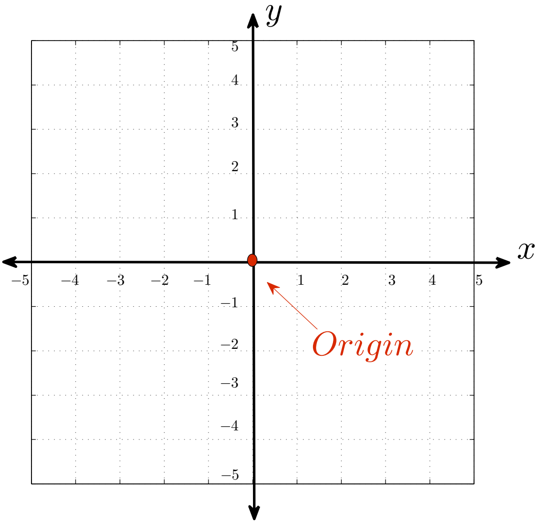
To plot $(0, 2)$, begin at the origin. Travel along the x-axis 0 units.



Example 2: Plot (graph) the following ordered pairs:

$(3, 0)$, $(0, 2)$, $(-3, 0)$, $(0, -2)$,

From that point (the origin), move up 2 spaces in the positive y direction.



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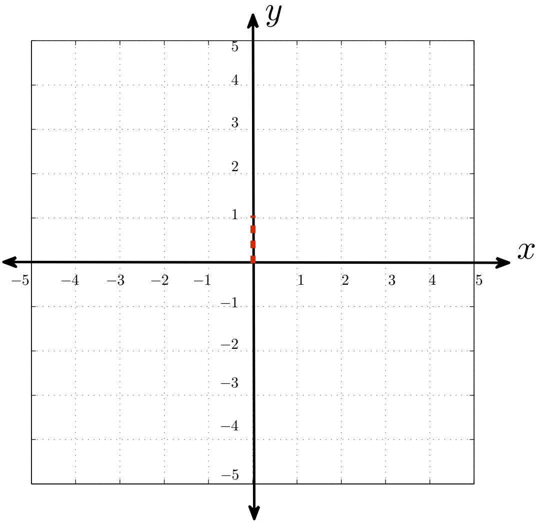

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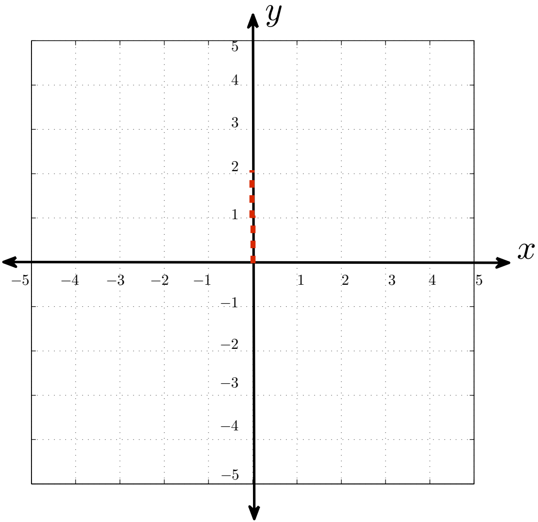

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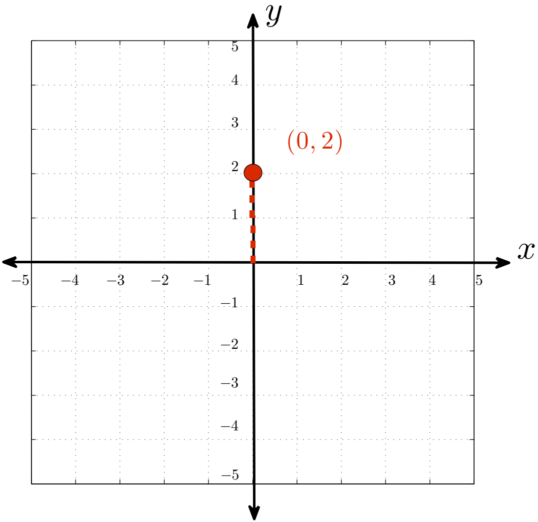
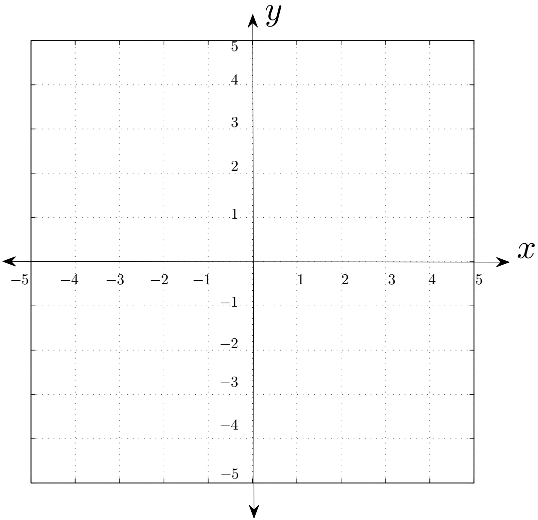


Example 2: Plot (graph) the following ordered pairs:

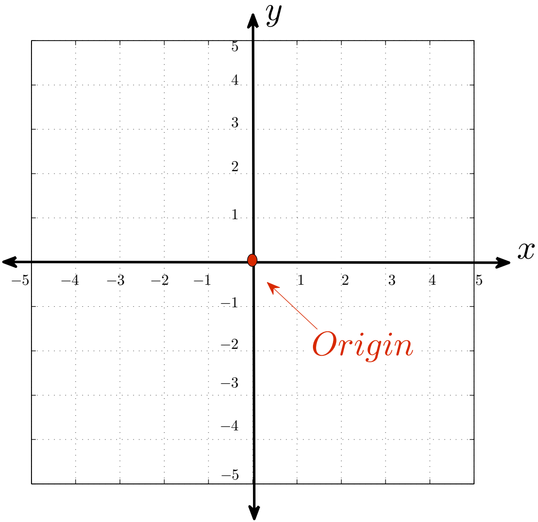
$(3, 0)$, $(0, 2)$, $(-3, 0)$, $(0, -2)$,



Example 2: Plot (graph) the following ordered pairs:

$(3, 0)$, $(0, 2)$, $(-3, 0)$, $(0, -2)$,

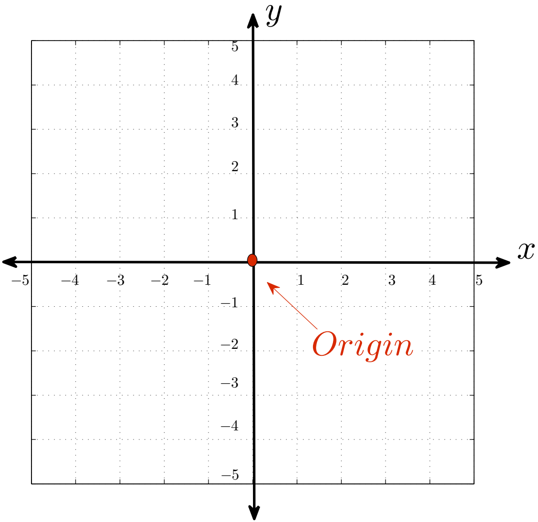
To plot $(-3, 0)$, begin at the origin.



Example 2: Plot (graph) the following ordered pairs:

$(3, 0)$, $(0, 2)$, $(-3, 0)$, $(0, -2)$,

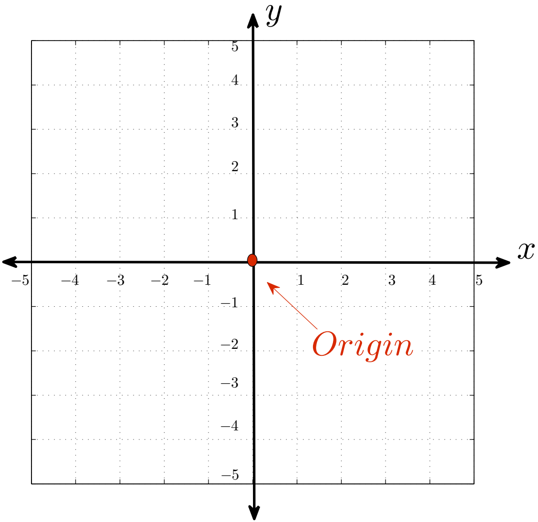
To plot $(-3, 0)$, begin at the origin. Travel along the *x*-axis 3 units left (the negative *x* direction).



Example 2: Plot (graph) the following ordered pairs:

$(3, 0)$, $(0, 2)$, $(-3, 0)$, $(0, -2)$,

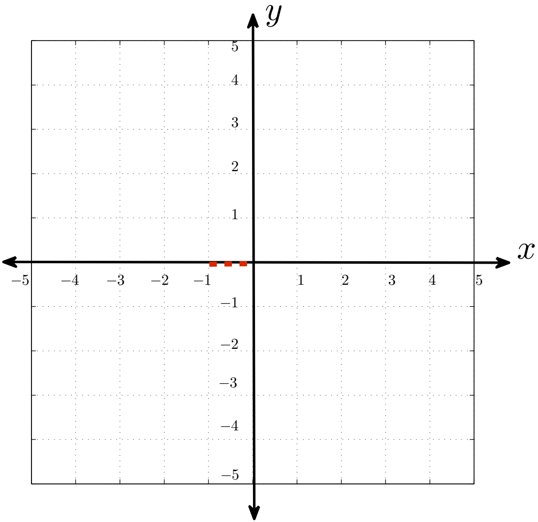
To plot $(-3, 0)$, begin at the origin. Travel along the x-axis 3 units left (the negative x direction).



Example 2: Plot (graph) the following ordered pairs:

$(3, 0)$, $(0, 2)$, $(-3, 0)$, $(0, -2)$,

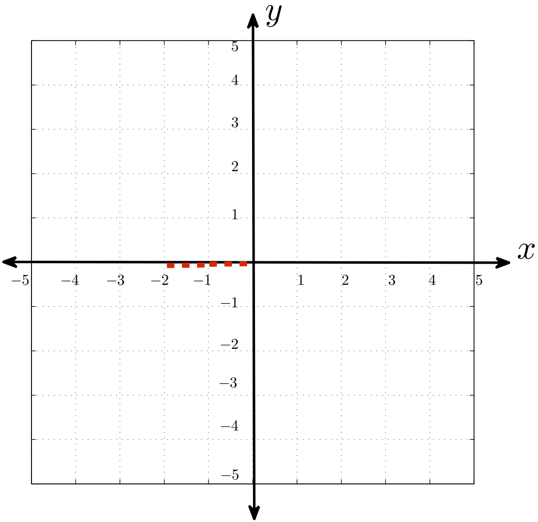
To plot $(-3, 0)$, begin at the origin. Travel along the x-axis 3 units left (the negative x direction).



Example 2: Plot (graph) the following ordered pairs:

$(3, 0)$, $(0, 2)$, $(-3, 0)$, $(0, -2)$,

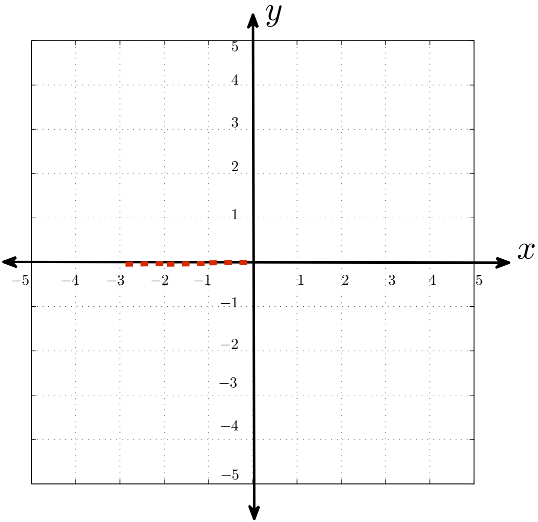
To plot $(-3, 0)$, begin at the origin. Travel along the x-axis 3 units left (the negative x direction).



Example 2: Plot (graph) the following ordered pairs:

$(3, 0)$, $(0, 2)$, $(-3, 0)$, $(0, -2)$,

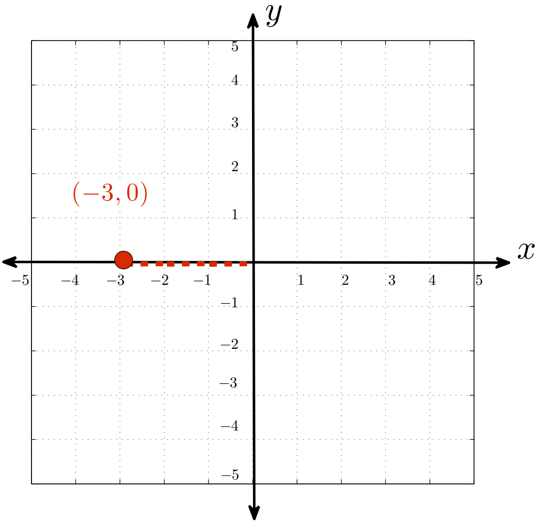
To plot $(-3, 0)$, begin at the origin. Travel along the x-axis 3 units left (the negative x direction).



Example 2: Plot (graph) the following ordered pairs:

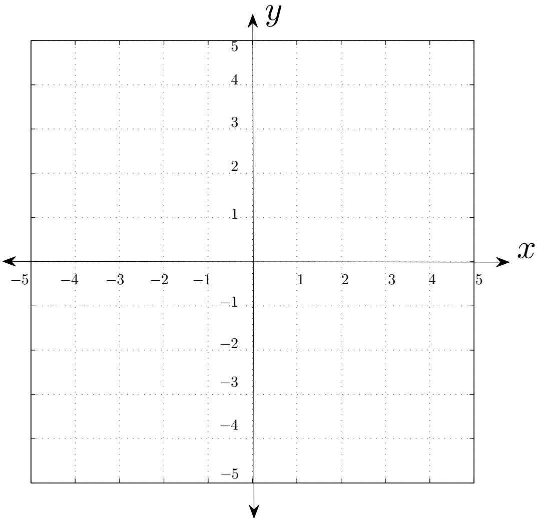
$(3, 0)$, $(0, 2)$, $(-3, 0)$, $(0, -2)$,

From that point (the origin), move up 0 spaces in the y direction.



Example 2: Plot (graph) the following ordered pairs:

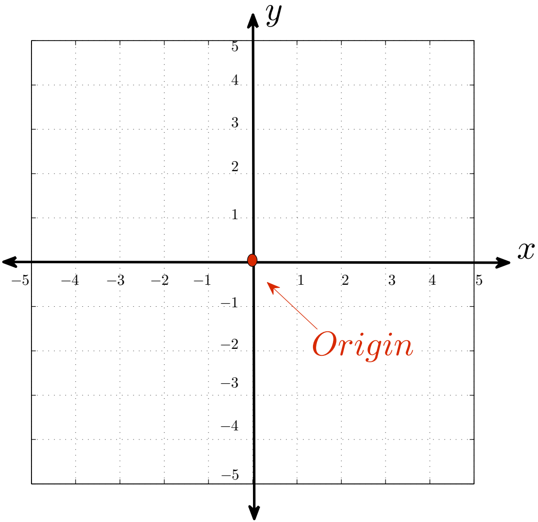
$(3, 0)$, $(0, 2)$, $(-3, 0)$, $(0, -2)$,



Example 2: Plot (graph) the following ordered pairs:

$(3, 0)$, $(0, 2)$, $(-3, 0)$, $(0, -2)$,

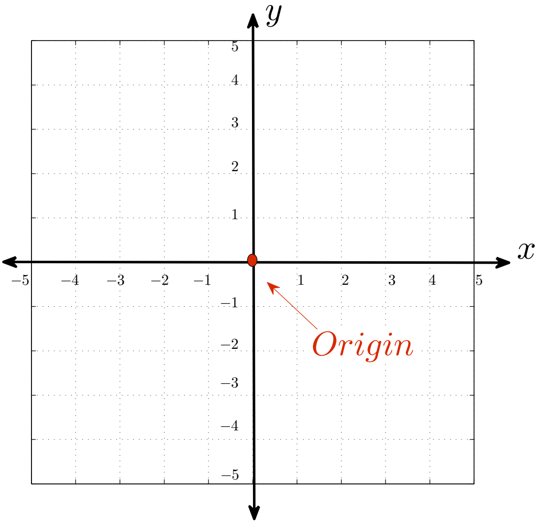
To plot $(0, -2)$, begin at the origin. Travel along the x-axis 0 units.



Example 2: Plot (graph) the following ordered pairs:

$(3, 0)$, $(0, 2)$, $(-3, 0)$, $(0, -2)$,

From that point (the origin), move up 2 spaces in the negative y direction (downwards).



Example 2: Plot (graph) the following ordered pairs:

$(3, 0)$, $(0, 2)$, $(-3, 0)$, $(0, -2)$,

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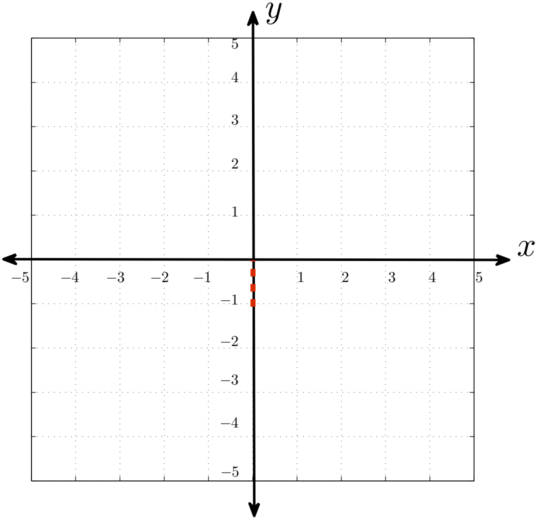

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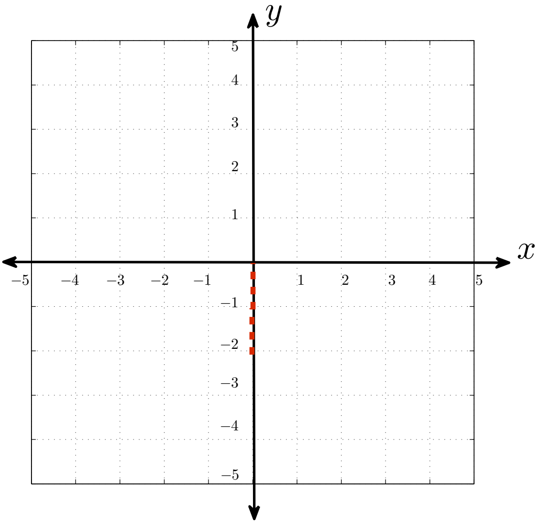
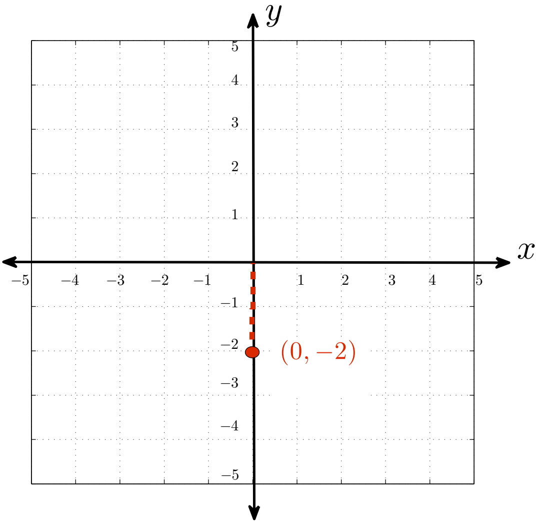


Example 2: Plot (graph) the following ordered pairs:

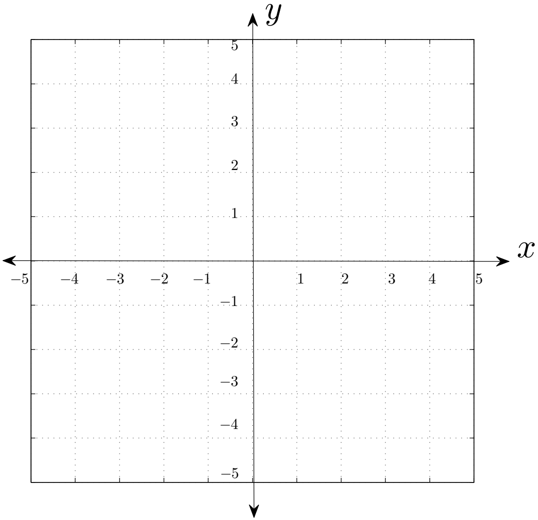
$(3,0)$, $(0,2)$, $(-3,0)$, $(0,-2)$,

From that point (the origin), move up 2 spaces in the negative y direction (downwards).



Concept Check

Plot (graph) $(3,4)$, $(-4,3)$, $(-1,-4)$ and $(5,-4)$



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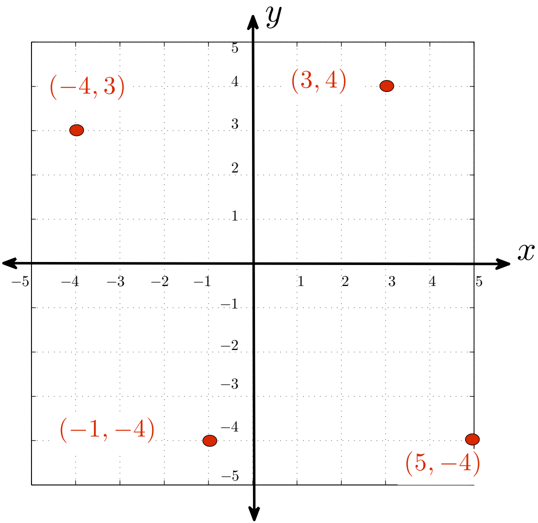
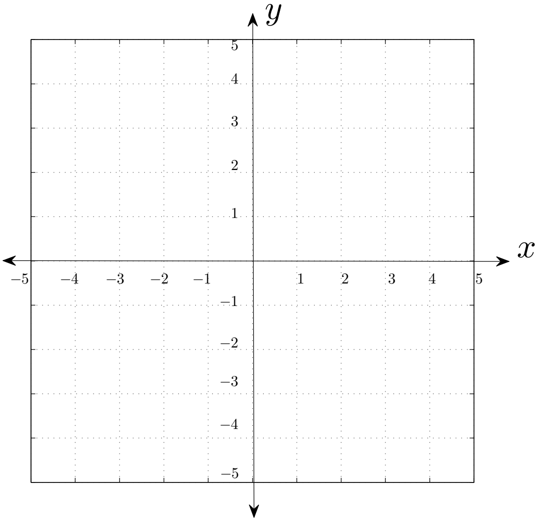


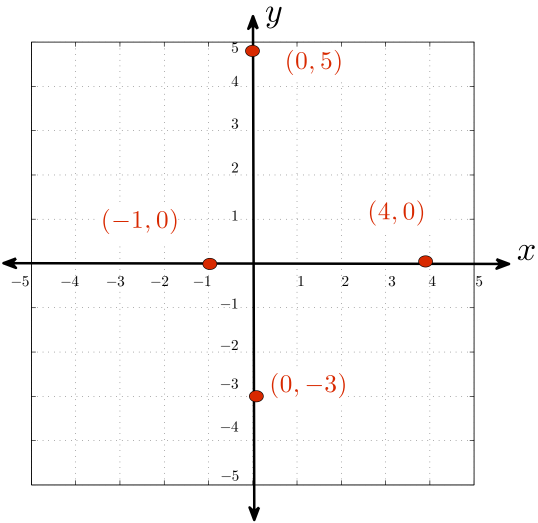
Concept Check

Plot (graph) $(4, 0)$, $(0, -3)$, $(-1, 0)$, and $(0, 5)$



Concept Check

Plot (graph) $(4, 0)$, $(0, -3)$, $(-1, 0)$, and $(0, 5)$



Definition

Suppose A, B and C represent any real numbers. A **linear equation in two variables** is an equation having the *form*

$$A\,x + B\,y = C,$$

For example, $2\,x + 3\,y = 1$ is a linear equation in the two variables x and y.

Solutions of a linear equation in two variables

Any linear equation in two variables always has in infinite number of solutions, and solutions come in the form of ordered pairs.

Solutions of a linear equation in two variables

Any linear equation in two variables always has in infinite number of solutions, and solutions come in the form of ordered pairs.

Terminology	Definition	Illustration
Solution of an equation in x and y	An ordered pair (a, b) that yields a true statement if $x = a$ and $y = b$	(1,4) is a solution of $y = 5x - 1$, since substituting x=1 and $y = 4$ renders the LHS = 4 and the RHS $= 5(1) - 1 = 4$

LHS is an abbreviation for "left-hand side" (of the equation)

RHS is an abbreviation for "right-hand side" (of the equation)

Equations and Graphs

Definition

For each ordered-pair solution, (a, b), of an equation in x and y there is a point (a, b) in a rectangular coordinate plane. The set of all such points is called a **graph of the equation**.

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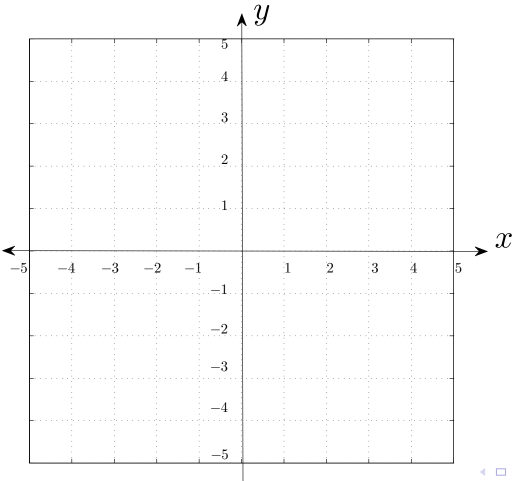
We can graph a linear equation by finding 3 ordered-pair solutions of the equation, plot the corresponding points on the rectangular grid, then draw a line between the three points.

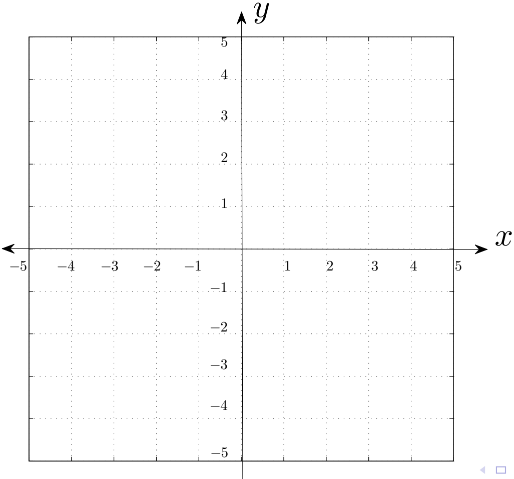
We use the third point for "insurance." If all three points line up in a straight we have not made a mistake!

Example 3: Graph the linear equation $y = -\frac{1}{2}x - 3$



Example 3: Graph the linear equation $y = -\frac{1}{2}x - 3$

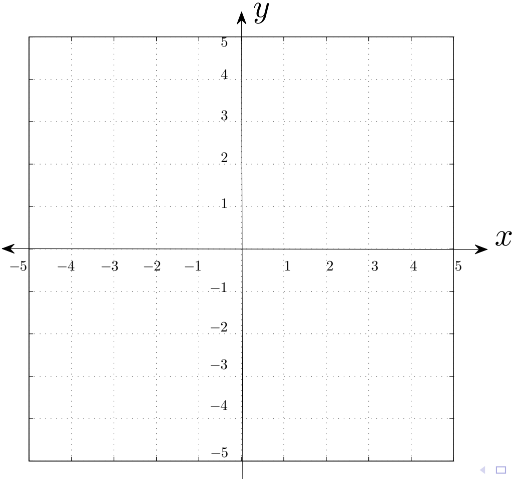
We begin by a making a table that summarizes x and y values. Since every value of x we substitute into the equation will be multiplied by $-\frac{1}{2}$, we use numbers for x that are divisible by 2.



x	y	$(x, y$
-2		
0		
2		

Example 3: Graph the linear equation $y = -\frac{1}{2}x - 3$

That way, when we multiply by $-\frac{1}{2}$, the result will be an integer.



x	y	(x, y)
-2		
0		
2		

Example 3: Graph the linear equation $y = -\frac{1}{2}x - 3$

We let $x = -2$ in the equation to find the y-value of the ordered pai
which is associated with x-coordinate -2.

x	y	$(x, y$
-2		
0		
2		

Example 3: Graph the linear equation $y = -\frac{1}{2}x - 3$

We let $x = -2$ in the equation to find the y-value of the ordered pai
which is associated with x-coordinate -2.

$$y = -\frac{1}{2} \cdot (x) - 3$$

$$= -\frac{1}{2} \cdot (-2) - 3$$

$$= 1 - 3$$

$$= -2$$

x	y	$(x, y$
-2		
0		
2		

Example 3: Graph the linear equation $y = -\frac{1}{2}x - 3$

Upon simplification, we get the ordered pair solution (-2,-2)

$$y = -\frac{1}{2} \cdot (x) - 3$$

$$= -\frac{1}{2} \cdot (-2) - 3$$

$$= 1 - 3$$

$$= -2$$

x	y	$(x, y$
-2	-2	(-2,-
0		
2		

Example 3: Graph the linear equation $y = -\frac{1}{2}x - 3$

Next, we let $x = 0$ in the equation to find the y-value of the ordered pair which is associated with x-coordinate 0.

$$y = -\frac{1}{2} \cdot (x) - 3$$

$$= -\frac{1}{2} \cdot (0) - 3$$

$$= 0 - 3$$

$$= -3$$

x	y	$(x, y$
-2	-2	(-2,-
0		
2		

Example 3: Graph the linear equation $y = -\frac{1}{2}x - 3$

This gives us the ordered pair solution (0,-3)

$$y = -\frac{1}{2} \cdot (x) - 3$$
$$= -\frac{1}{2} \cdot (0) - 3$$
$$= 0 - 3$$
$$= -3$$

x	y	$(x, y$
-2	-2	(-2,-
0	-3	(0,-3
2		

Example 3: Graph the linear equation $y = -\frac{1}{2}x - 3$

Afterwards, we let $x = 2$ in the equation.

$$y = -\frac{1}{2} \cdot (x) - 3$$

$$= -\frac{1}{2} \cdot (2) - 3$$

$$= -1 - 3$$

$$= -4$$

x	y	$(x, y$
-2	-2	(-2,-
0	-3	(0,-3
2		

Example 3: Graph the linear equation $y = -\frac{1}{2}x - 3$

Upon simplification, we get the ordered pair solution (2,-4)

$$y = -\frac{1}{2} \cdot (x) - 3$$

$$= -\frac{1}{2} \cdot (2) - 3$$

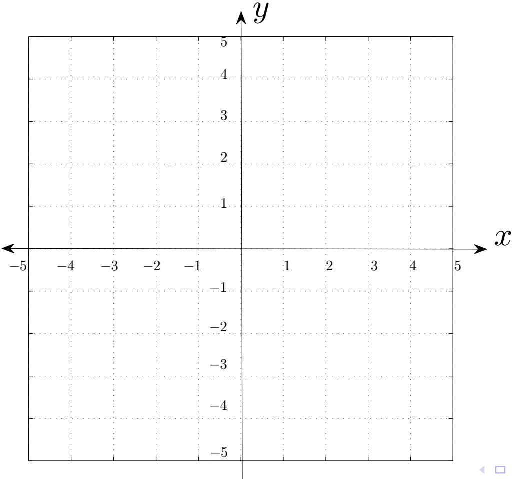
$$= -1 - 3$$

$$= -4$$

x	y	$(x, y$
-2	-2	(-2,-
0	-3	(0,-3
2	-4	(2,-4

Example 3: Graph the linear equation $y = -\frac{1}{2}x - 3$

We now locate the three ordered pair solutions (points) on the rectangular coordinate grid, then draw a line through the solutions.



x	y	$(x, y$
-2	-2	(-2, -
0	-3	(0,-3
2	-5	(2, -

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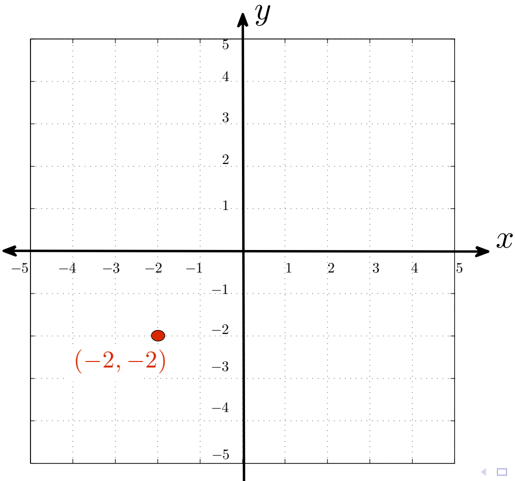

x	y	$(x, y$
-2	-2	(-2, -
0	-3	(0,-3
2	-5	(2, -

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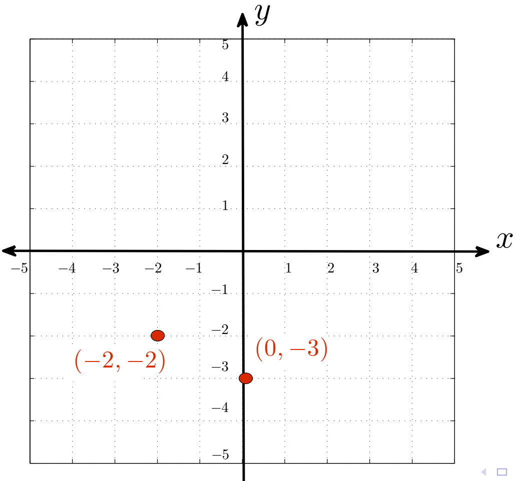

x	y	$(x, y$
-2	-2	(-2, -
0	-3	(0,-3
2	-5	(2, -

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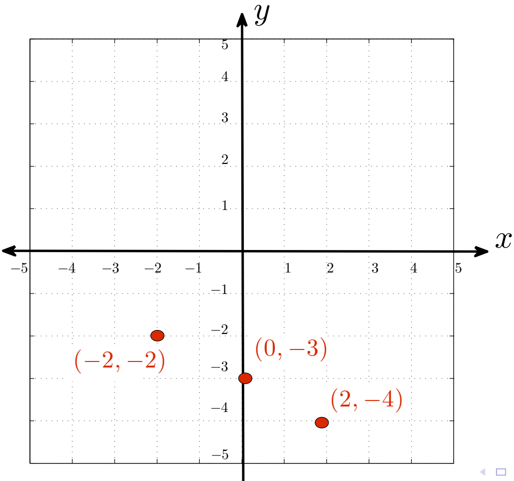

x	y	$(x, y$
-2	-2	(-2, -
0	-3	(0,-3
2	-5	(2, -

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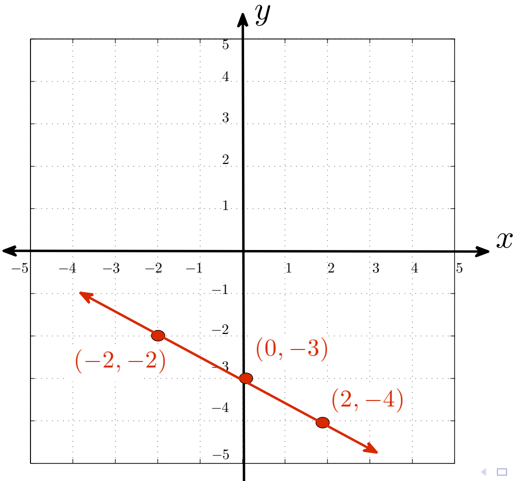
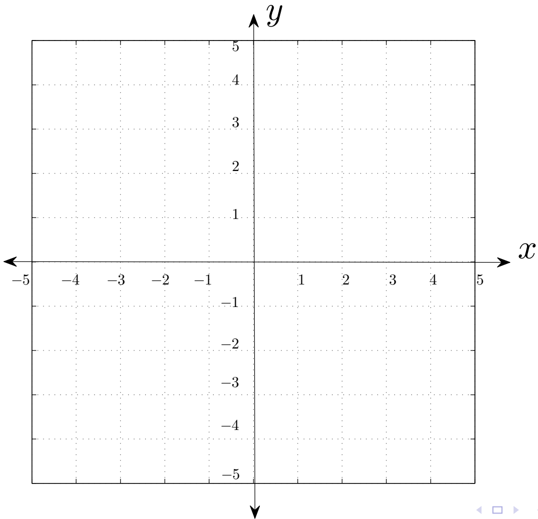


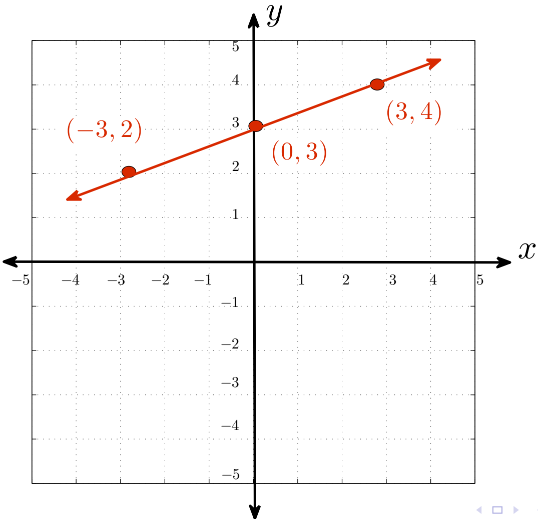
x	y	$(x, y$
-2	-2	(-2, -
0	-3	(0,-3
2	-5	(2, -

Concept Check: Graph $y = \frac{1}{3}x + 3$



x	y	$(x, y$

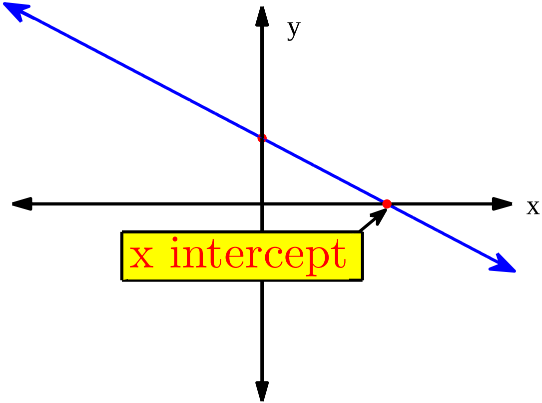
Concept Check: Graph $y = \frac{1}{3}x + 3$



x	y	$(x, y$
-3	2	(-3,2
0	3	(0,3)
3	4	(3,4)

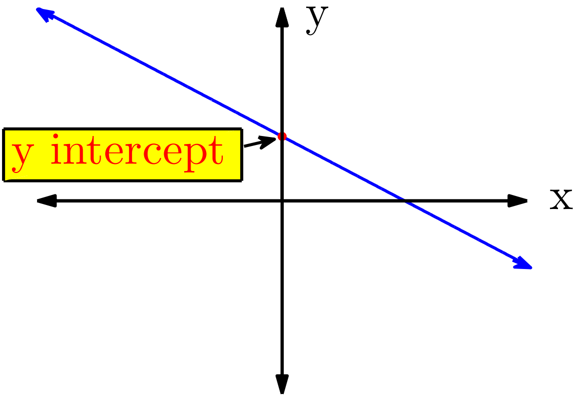
Definition

The graph of an equation has an **x-intercept** whenever the graph of the equation crosses the x axis. The x intercept always occurs when the value of y is equal to zero.



Definition

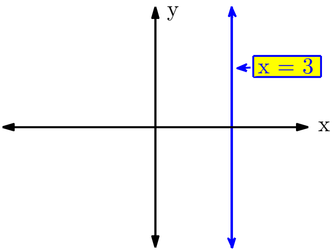
The graph of an equation has an **y-intercept** whenever the graph of the equation crosses the y axis. The y intercept always occurs when the value of x is equal to zero.



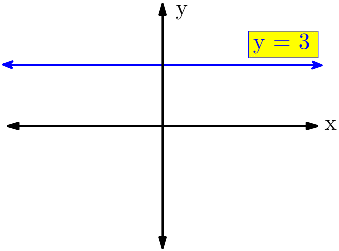
Example 4 Find the x- and y-intercepts for $5x - 7y = -35$, then graph the solution set.

Theorem

Suppose a and b are real numbers. Graphs of linear equations of the form $x = a$ are vertical lines and graphs of linear equations of the form $y = b$ are horizontal lines.



graph of $x = 3$



graph of $y = 3$

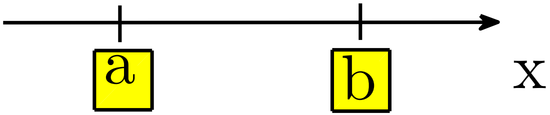
More Classroom Examples

Graph each of the following lines:

- $y = \frac{1}{2}x$
- $x = -2$
- $y = -4$

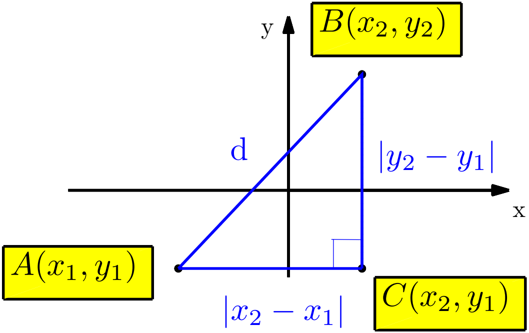
Theorem (Distance Formula: 1 dimension)

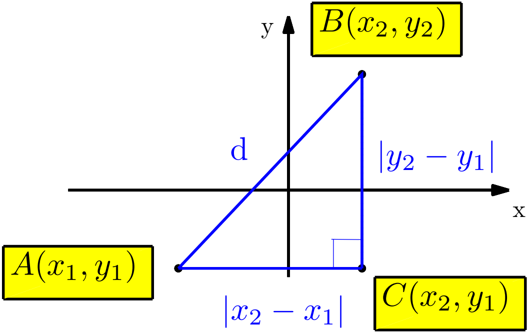
If a and b are real numbers, then the distance between them on a number line is $|a - b|$.



Distance Formula: 2 dimensions

Consider the points $A(x_1, y_1)$ and $B(x_2, y_2)$ in the figure below. Let d be the distance between points A and B (the HYPOTENUSE LENGTH of the right triangle). Since A and C lie on a horizontal line, the distance between them is $|x_2 - x_1|$. Likewise, $\overline{CB} = |y_2 - y_1|$.





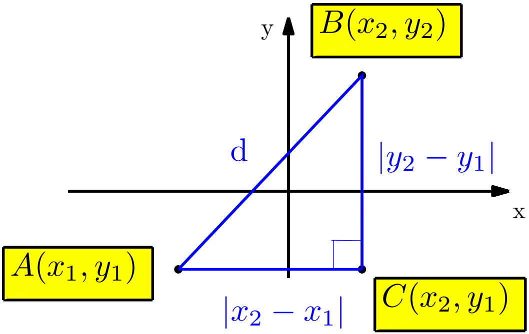
Since the sum of the squares of the legs of a right triangle is equal to the square of the hypotenuse (Pyth. thm), then from the diagram

$$d^2 = |x_2 - x_1|^2 + |y_2 - y_1|^2$$

Theorem (Distance Formula: 2 dimensions)

The distance d between the points (x_1, y_1) and (x_2, y_2) is given by the formula

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$



Example: Find the exact distance between the points $(5, -3)$ and $(-1, -6)$

Solution

Let $(x_1, y_1) = (5, -$

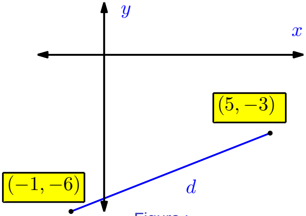


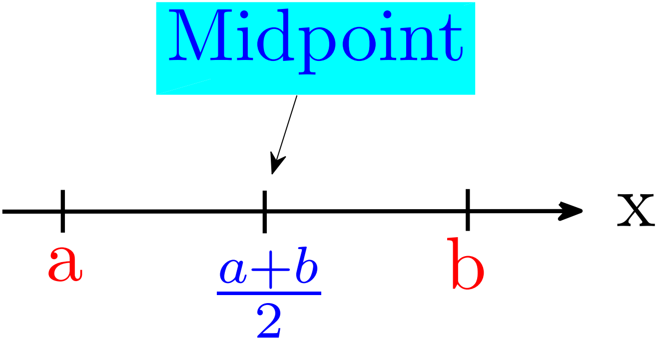
Figure :

Then

$$
\begin{aligned}
d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\
&= \sqrt{(-1 - 5)^2 + (-6 - (-3))^2} \\
&= \sqrt{(-6)^2 + (-3)^2} \\
&= \sqrt{36 + 9} = \sqrt{45} = 3\sqrt{5}
\end{aligned}
$$

Theorem (Midpoint Formula: 1 dimension)

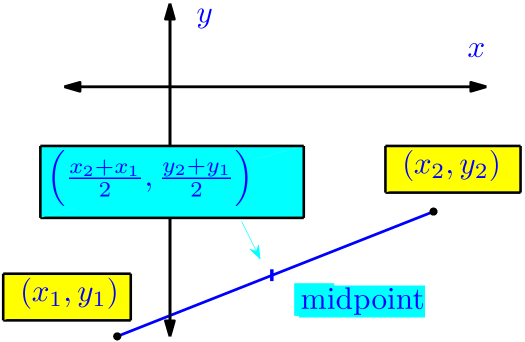
If a and b are real numbers, then the midpoint between them on a number line is $\frac{a+b}{2}$.



Theorem (Midpoint Formula: 2 dimensions)

Suppose (x_1, y_1) *and* (x_2, y_2) *are any two points in two-dimensional space. Then the midpoint of the line segment that joins them is:*

$$m = \left(\frac{(x_2 + x_1)}{2}, \frac{(y_2 + y_1)}{2} \right).$$



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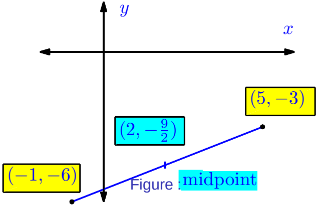

Figure :

Then

$$\begin{aligned} m &= \left(\frac{(x_2 + x_1)}{2}, \frac{(y_2 + y_1)}{2}\right) \\ &= \left(\frac{5 + (-1)}{2}, \frac{-3 + (-6)}{2}\right) \\ &= \left(\frac{4}{2}, \frac{-9}{2}\right) \\ &= \left(2, -\frac{9}{2}\right) \end{aligned}$$

The Circle

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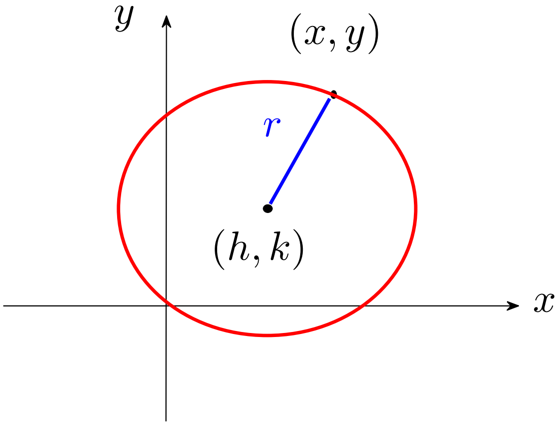
- The **graph of (the solution set to) an equation** in two variables is a two-dimensional geometric object that gives us a visual image of an algebraic object.

Definition (Circle)

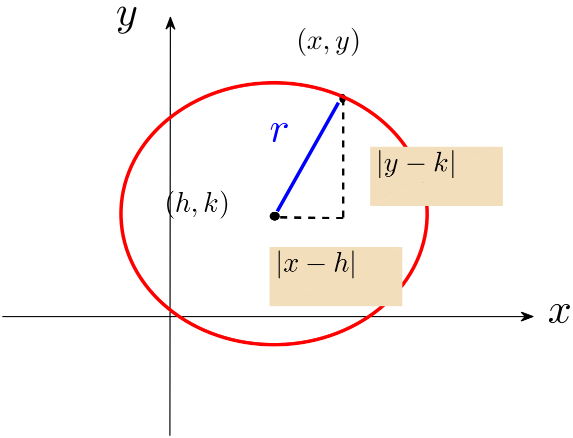
A *circle* is defined by the set of all points in the xy plane that lie a fixed distance from a given point (the center). The fixed distance is called the *radius*, and the given point is the center.

The distance formula can be used to write an equation for a circle with center (h, k) and radius r for $r > 0$.

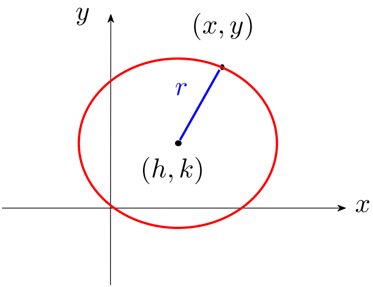


A point (x, y) is on the circle if and only if it satisfies the equation

$$\sqrt{(x-h)^2 + (y-k)^2} = r.$$



Since both sides of the equation (previous slide) are positive, we can square each side to get the standard form for the equation of a circle.



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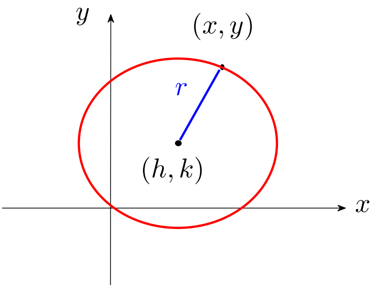


Theorem (Equation for a Circle in Standard Form)

The equation for a circle with center (h, k) and radius r (where $r > 0$) is

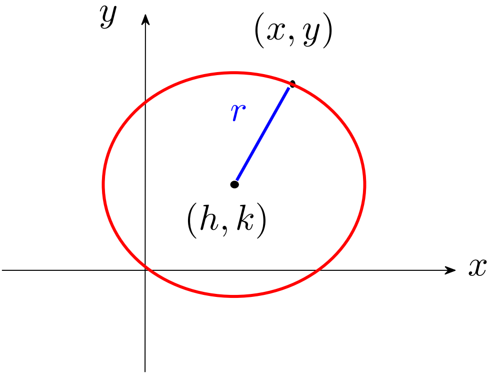
$$(x - h)^2 + (y - k)^2 = r^2$$

A circle centered at the origin has equation $x^2 + y^2 = r^2$.

Example: Sketch the graph of the equation

$$(x-1)^2+(y+2)^2=3$$

.



Perfect Square Trinomials

Recall (chapter p.4) that algebraic expressions of the form $a^2 + 2ab + b^2$ have factorization $(a + b)^2$ and are called perfect square trinomials.

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is an example of a perfect square trinomial. The expression is called a trinomial because it is a polynomial (see chapter p.4) with three terms. But $x^2 + 10x + 25$ also has a perfect square factorization $(x + 5)^2$; hence the name (or classification) of $x^2 + 10x + 25$ as a perfect square trinomial.

Completing the Square

Completing the Square means *finding* the third term of a perfect square trinomial when given the first two.

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$$x^2 + 10x.$$

If we add 25 to the given expression, well them we have completed the square since $x^2 + 10x + 25 = (x + 5)^2$. The generalized rule for completing a square is as follows:

Theorem (Completing the square)

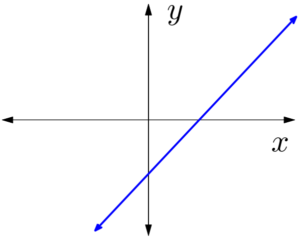
Let b be any real number. Given the first two terms of a quadratic expression $x^2 + bx$, the third term that must be added to the expression in order to render the expression a perfect square trinomial is $\left(\frac{b}{2}\right)^2$. That is,

$$x^2 + bx + \left(\frac{b}{2}\right)^2 = \left(x + \frac{b}{2}\right)^2$$

Definition

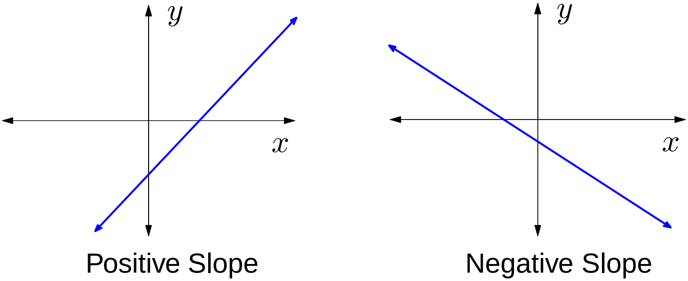
The slope of a line is a measure of the steepness of the line.

A line that rises from left to right has positive slope.

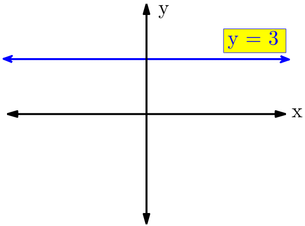


Positive Slope

A line that falls from left to right has negative slope.

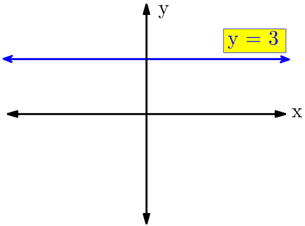


Horizontal lines have zero slope

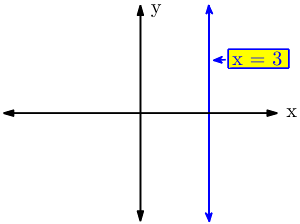


graph of $x = 3$

Horizontal lines have zero slope, and vertical lines have no slope.

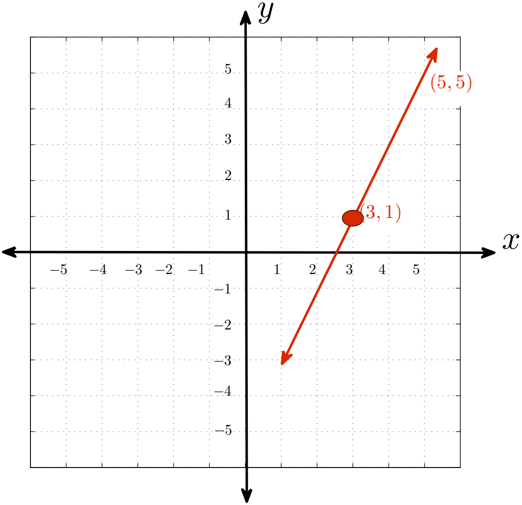


graph of $x = 3$

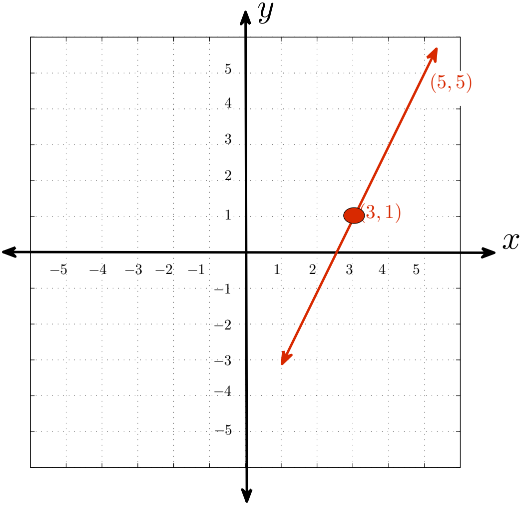


graph of $y = 3$

We can define the slope of a line as the ratio of the vertical change to the horizontal change when moving from one point to another on the line.



We can define the slope of a line as the ratio of the vertical change to the horizontal change when moving from one point to another on the line.



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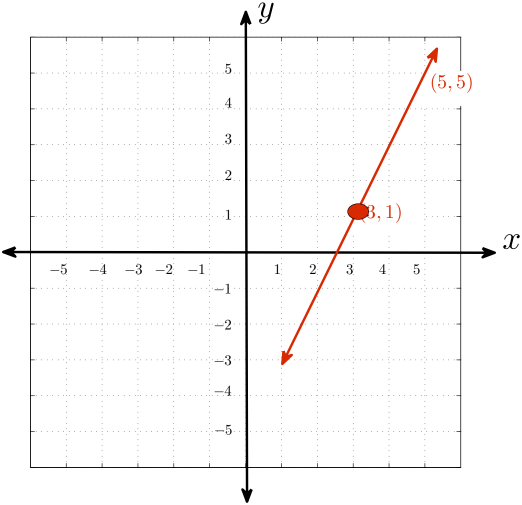

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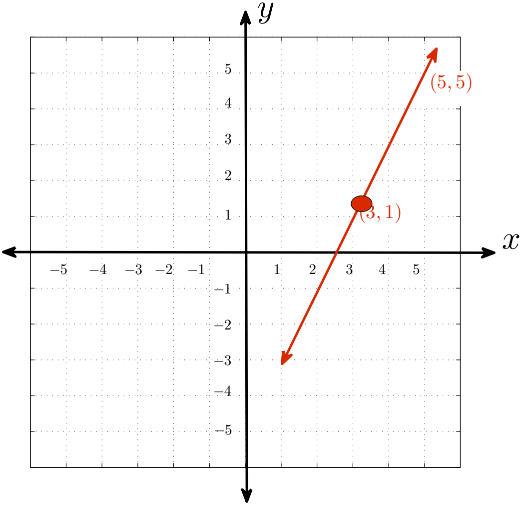

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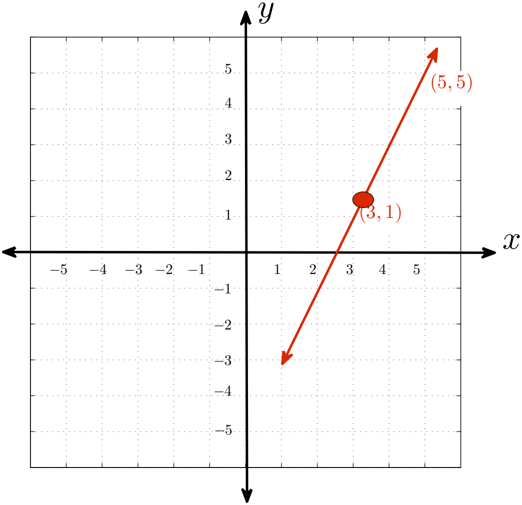

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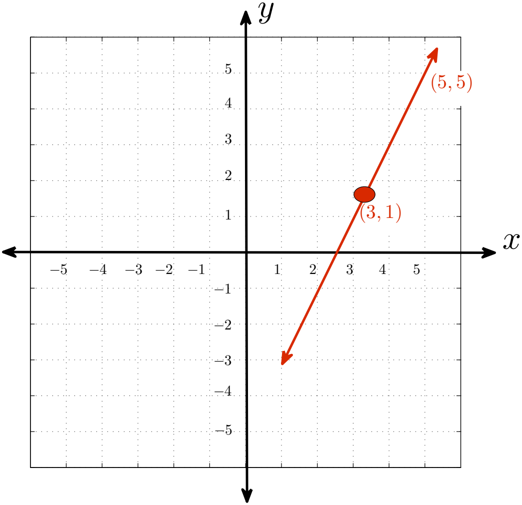
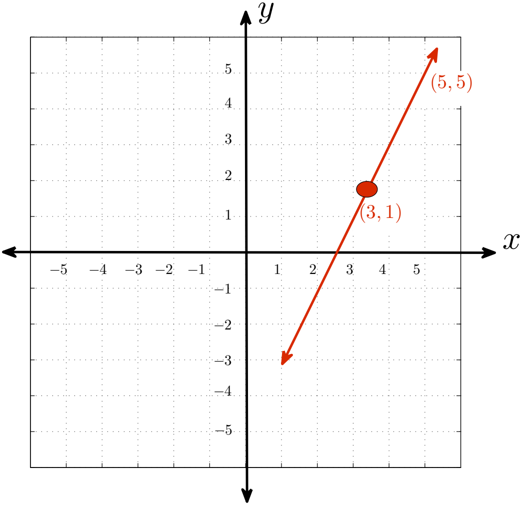


We can define the slope of a line as the ratio of the vertical change to the horizontal change when moving from one point to another on the line.



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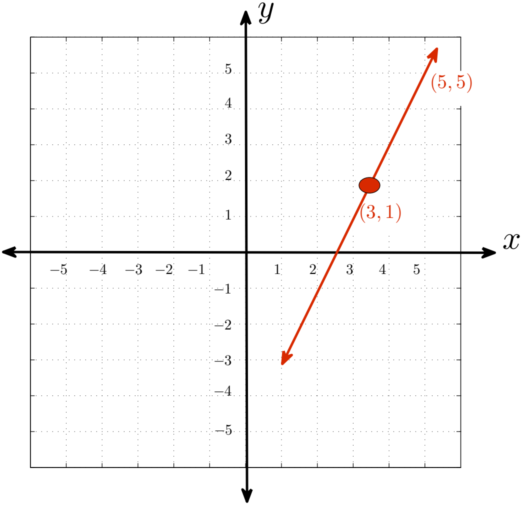

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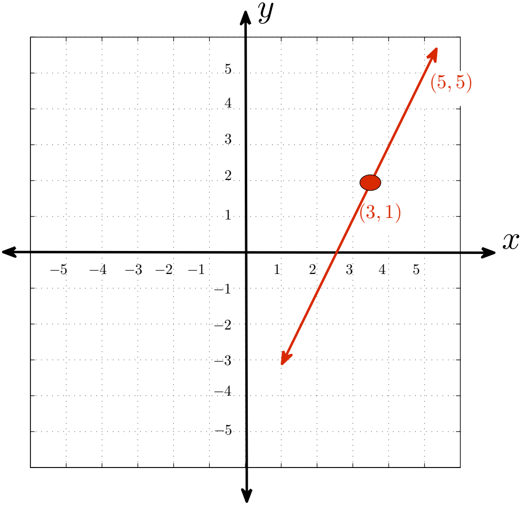

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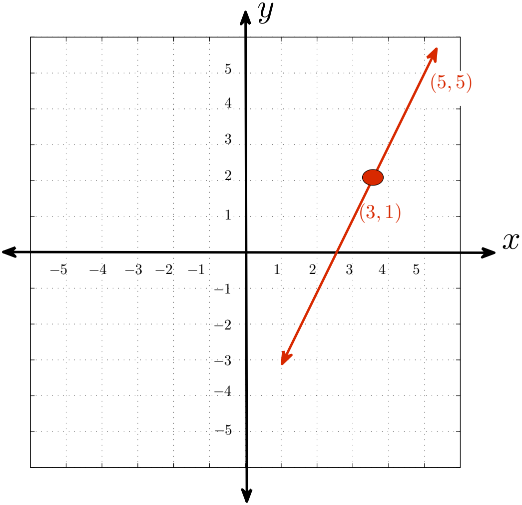

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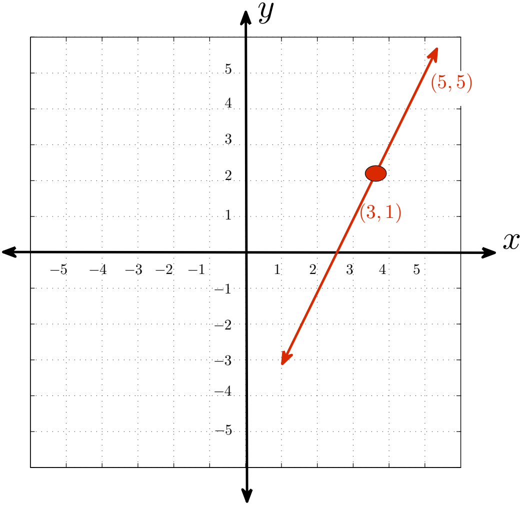

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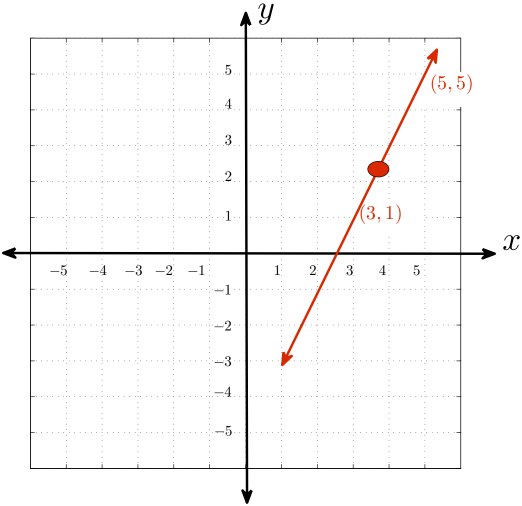

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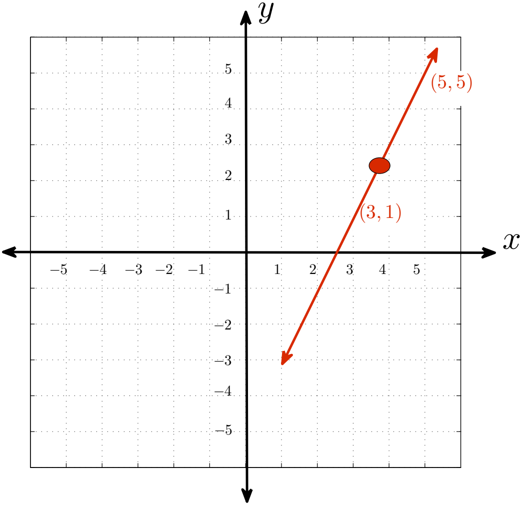

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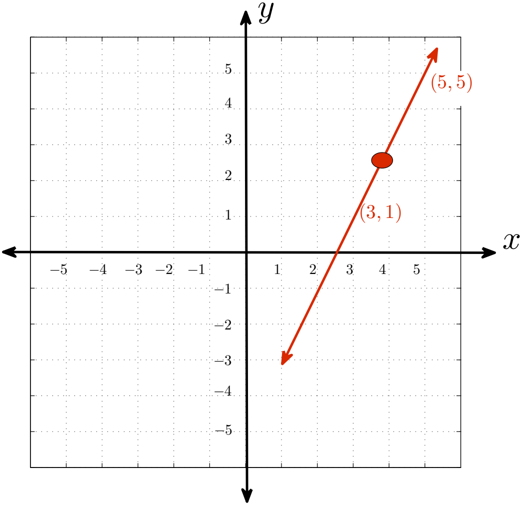

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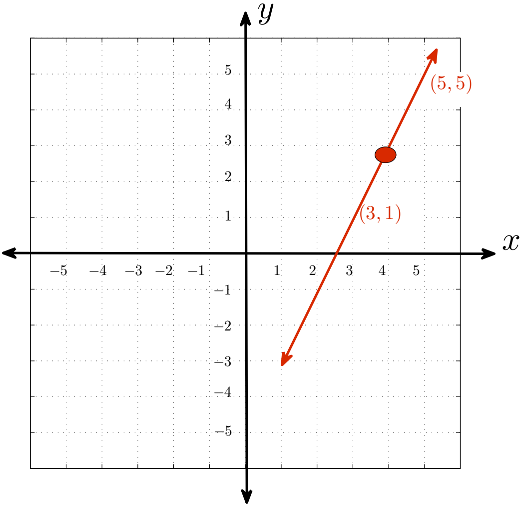

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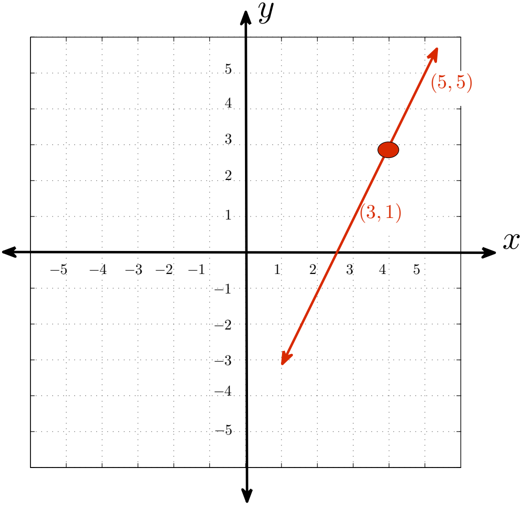

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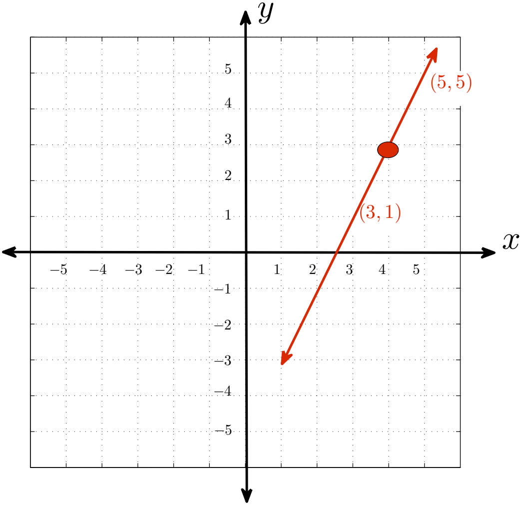

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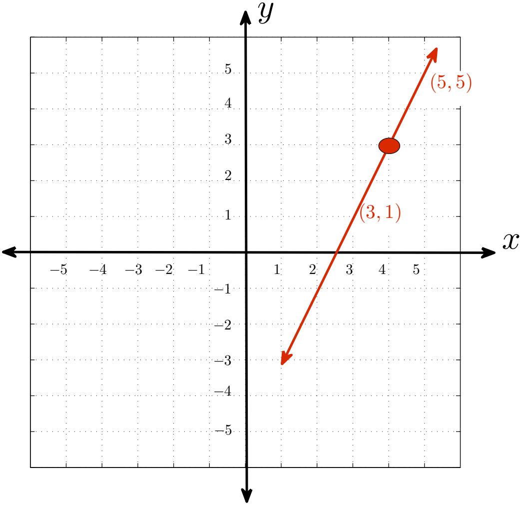

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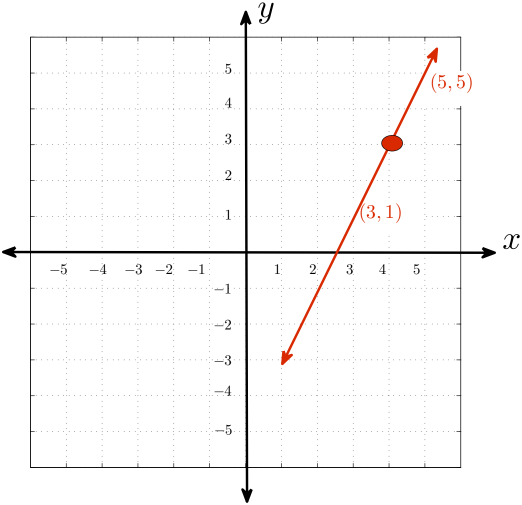

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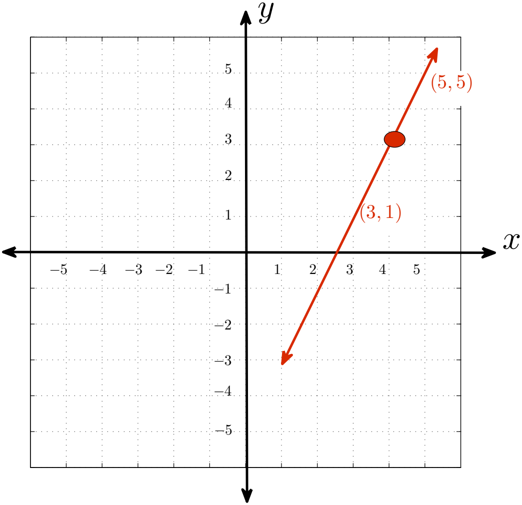

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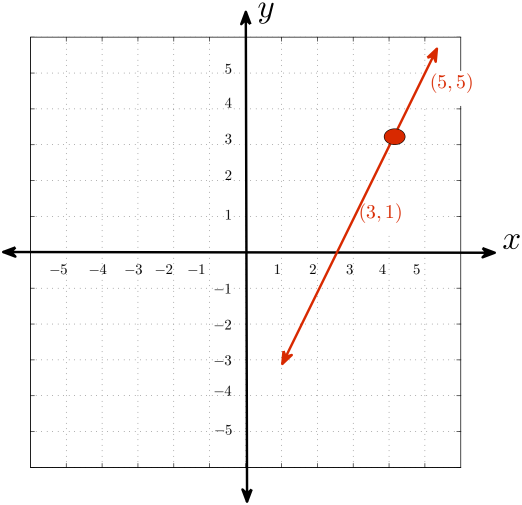

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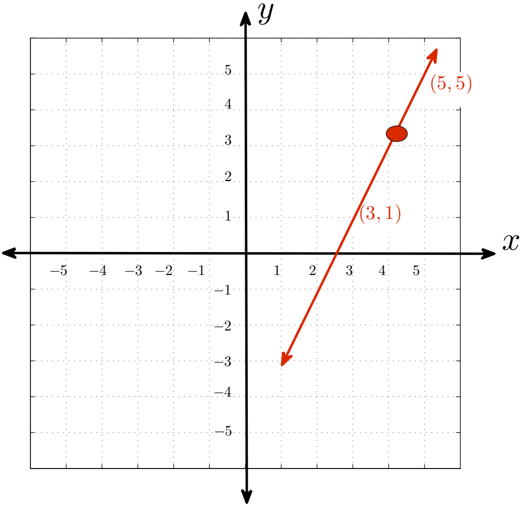

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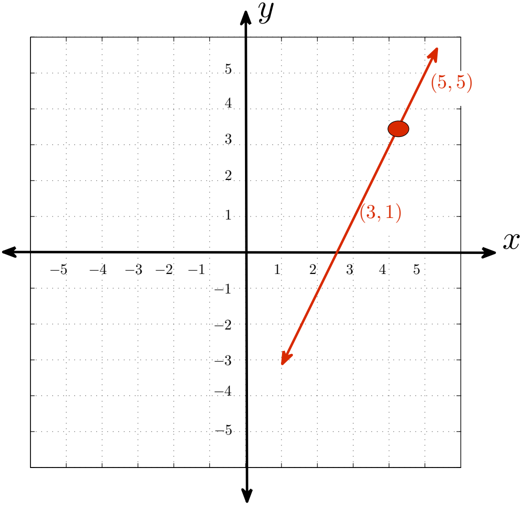

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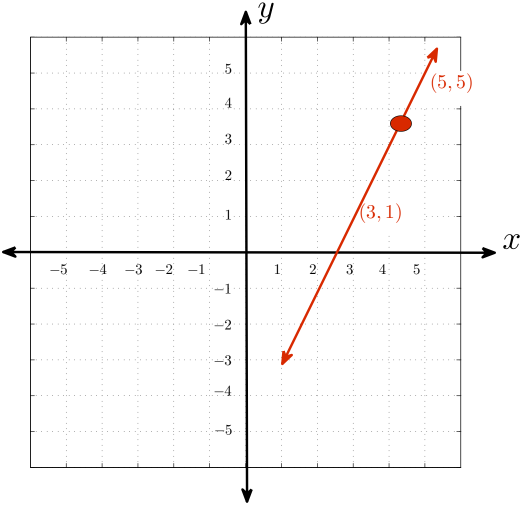

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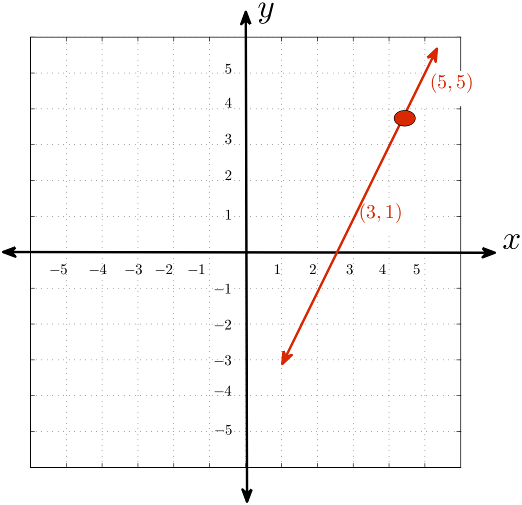

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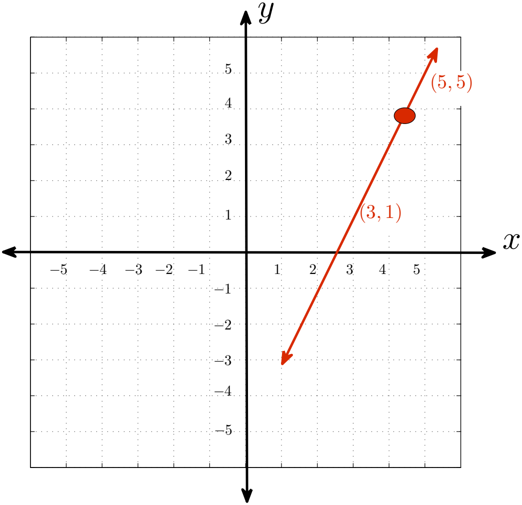

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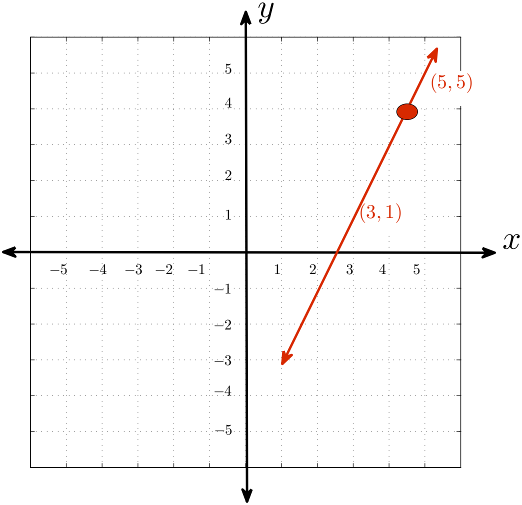

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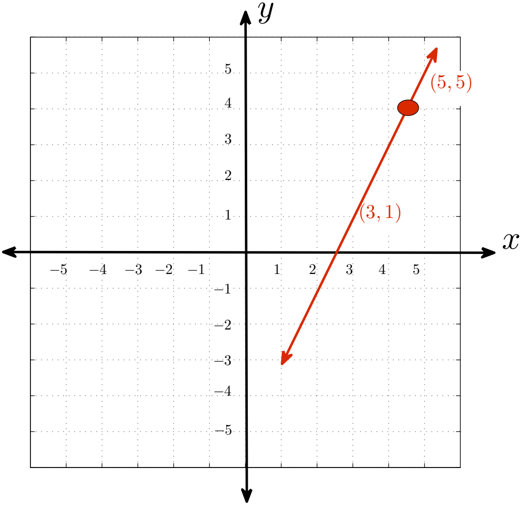

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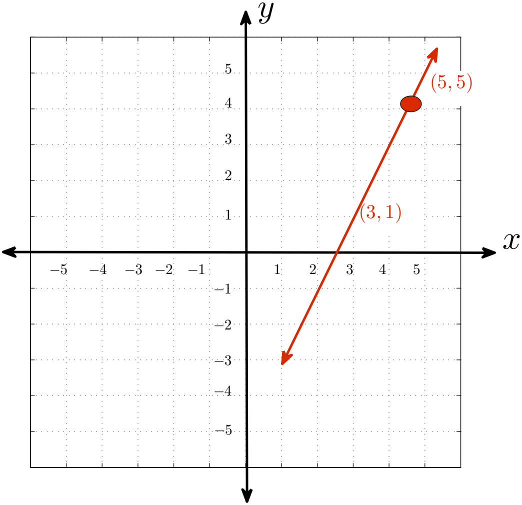

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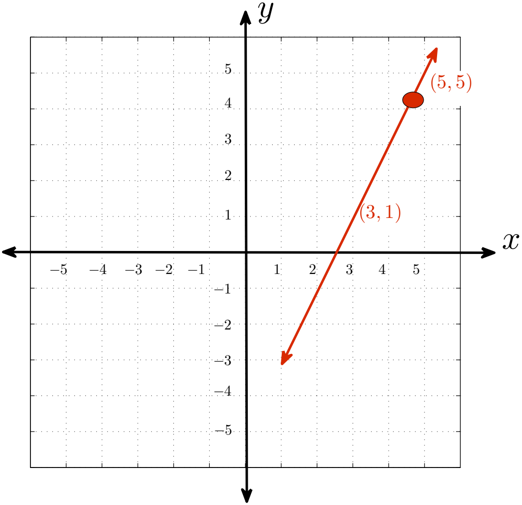

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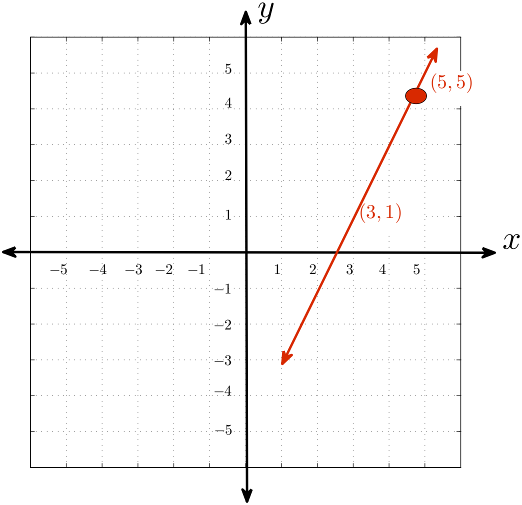

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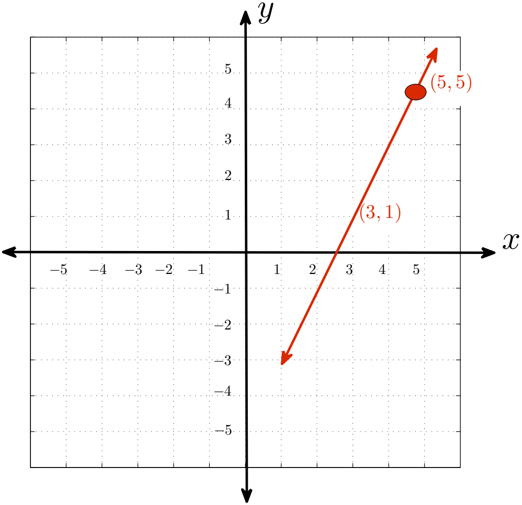

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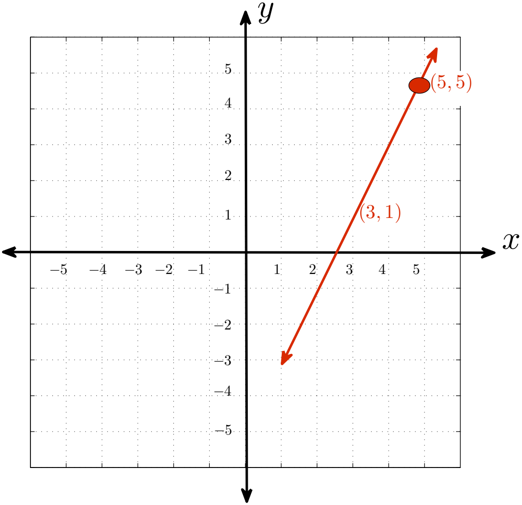

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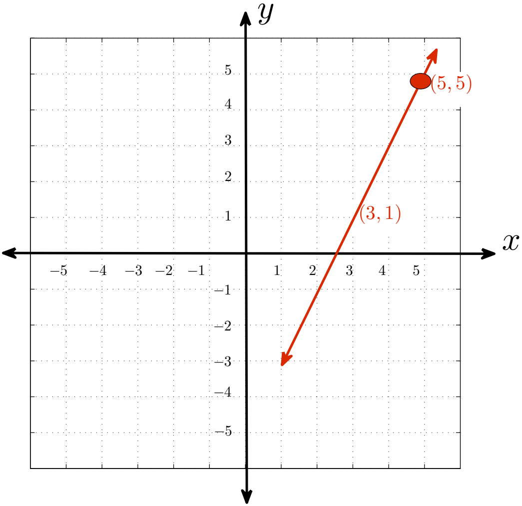

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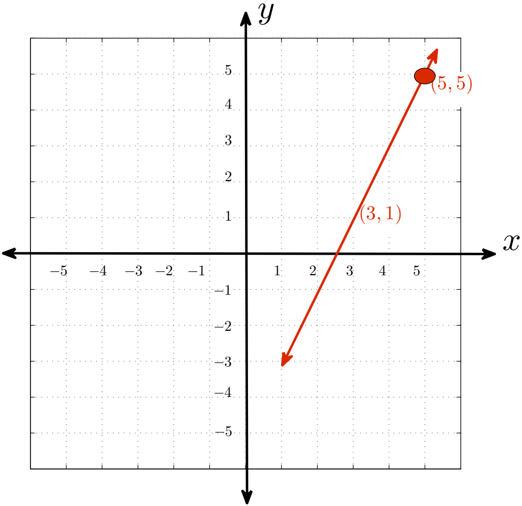

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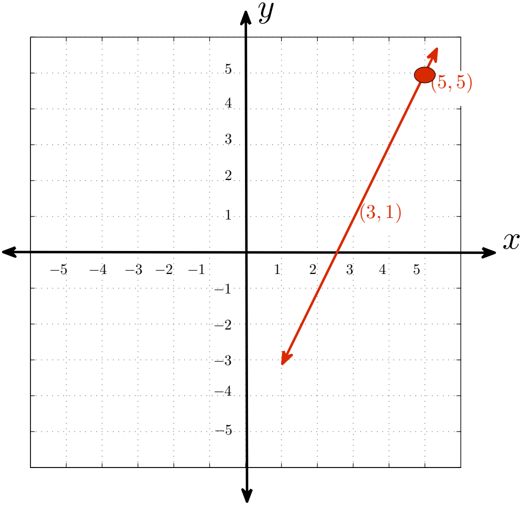
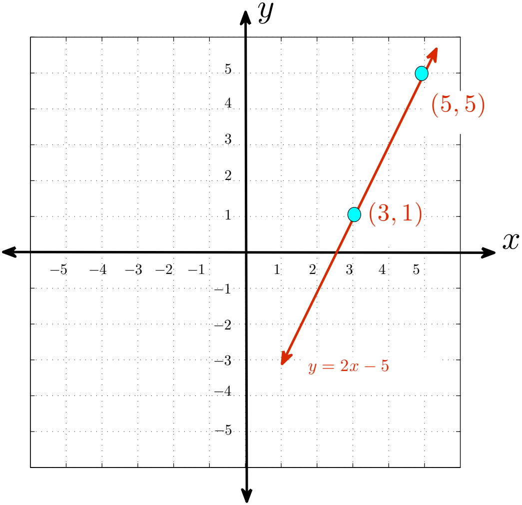


Geometrically, we define the slope of a line as the ratio of the vertical change to the horizontal change when moving from one point to another on the line.



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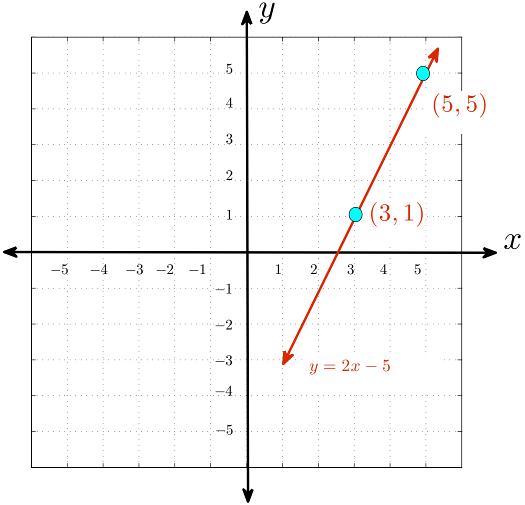

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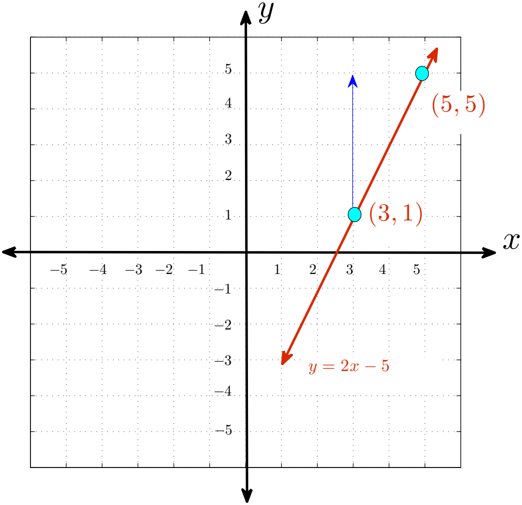

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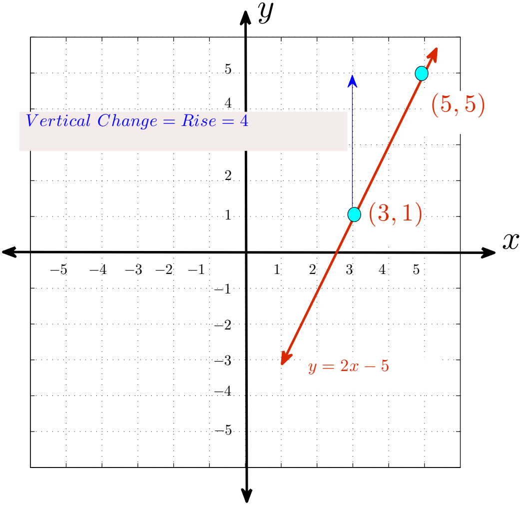

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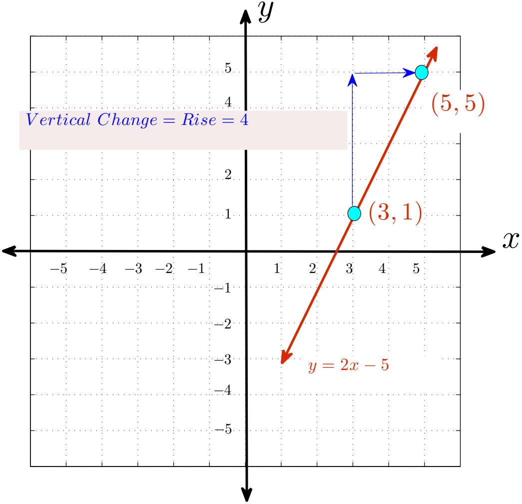

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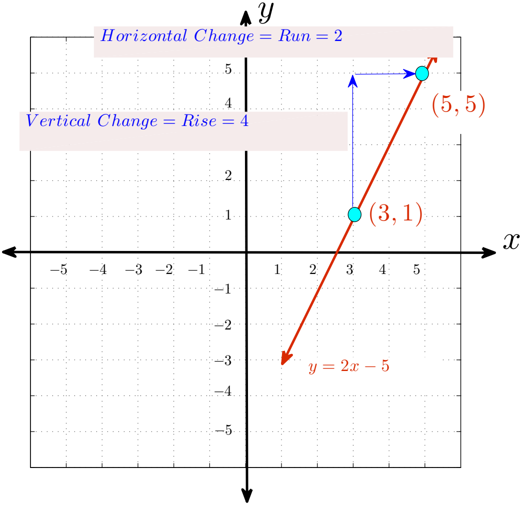
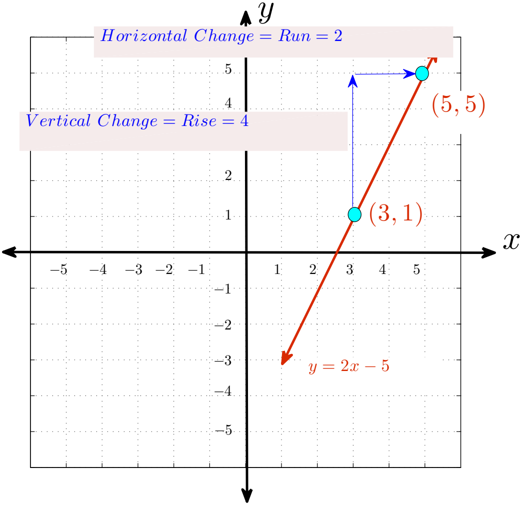
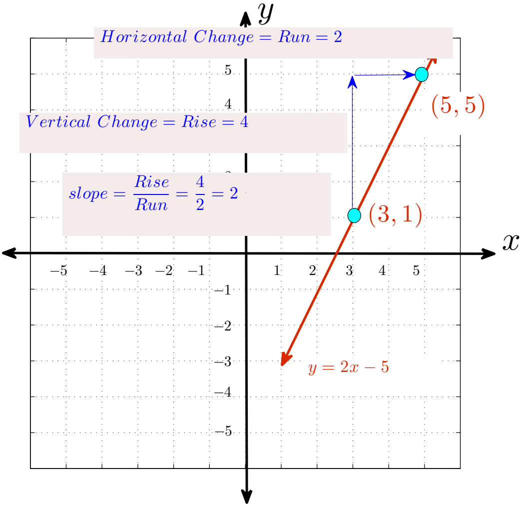


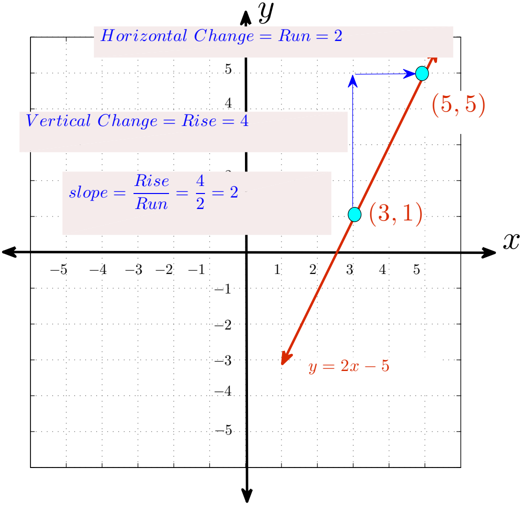
$$slope = \frac{Rise}{Run} = \frac{4}{2} = 2$$



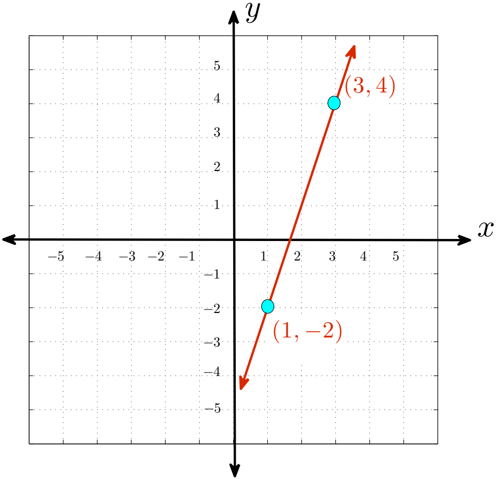
Notice that the vertical change is measured by subtracting the y-coordinates of the two points, $5 - 1 = 4$.



Notice that the vertical change is measured by subtracting the y-coordinates of the two points, $5 - 1 = 4$. The horizontal change is the difference between the x-coordinates, $5 - 3 = 2$.

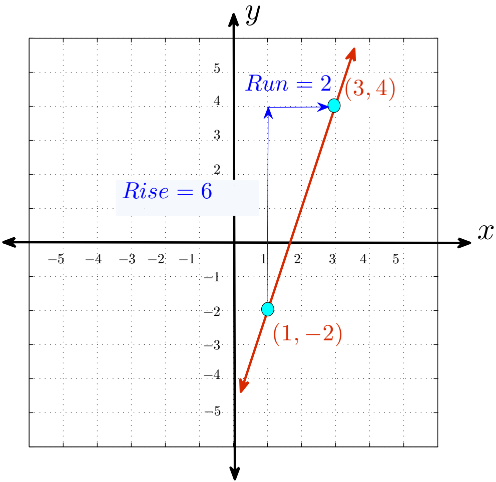


Ex. Find the slope of a line through $(3, 4)$ and $(1, -2)$.



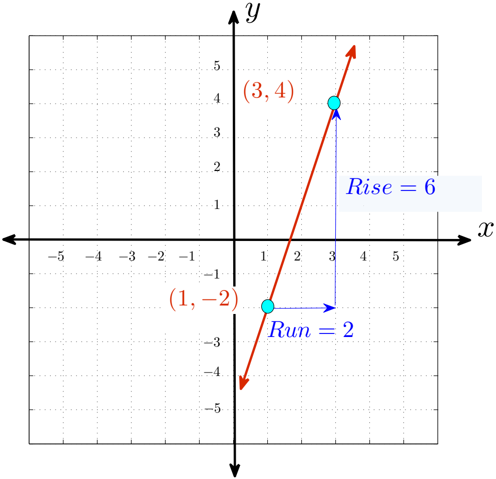
Ex. Find the slope of a line through $(3, 4)$ and $(1, -2)$.

$$\text{slope} = \frac{\text{Rise}}{\text{Run}} = \frac{6}{2} = 3$$



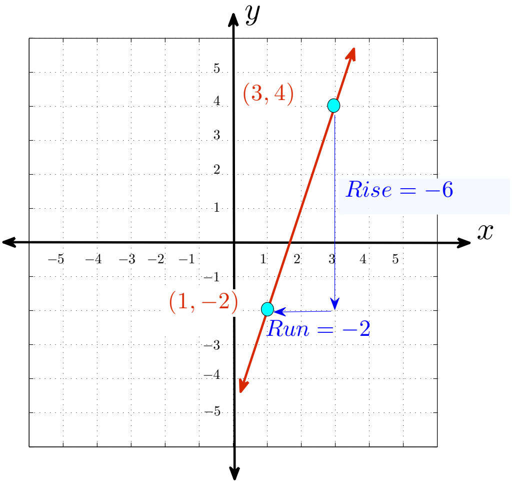
Ex. Find the slope of a line through $(3, 4)$ and $(1, -2)$.

$$slope = \frac{Rise}{Run} = \frac{6}{2} = 3$$



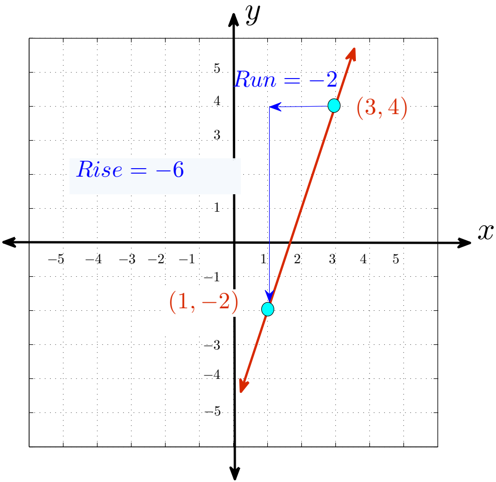
Ex. Find the slope of a line through $(3, 4)$ and $(1, -2)$.

$$slope = \frac{Rise}{Run} = \frac{-6}{-2} = 3$$

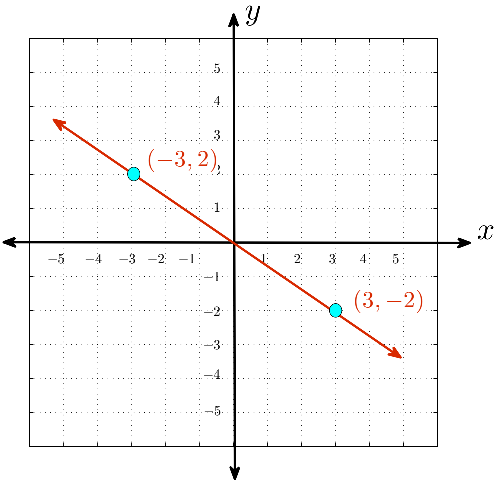


Ex. Find the slope of a line through $(3, 4)$ and $(1, -2)$.

$$\textit{slope} = \frac{\textit{Rise}}{\textit{Run}} = \frac{-6}{-2} = 3$$

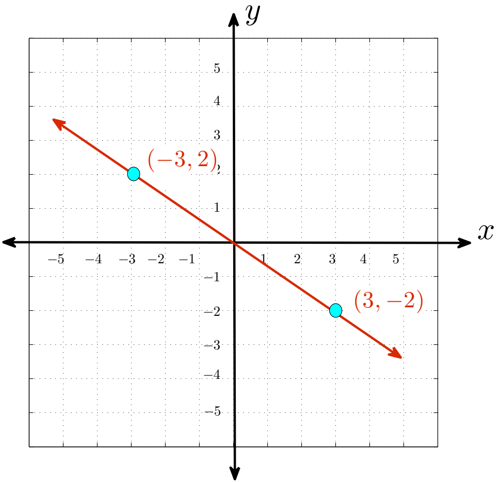


Ex. Find the slope of a line through $(-3, 2)$ and $(3, -2)$.



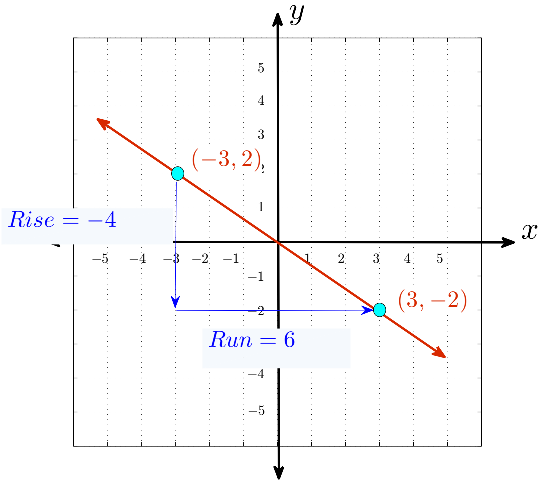
Ex. Find the slope of a line through $(-3, 2)$ and $(3, -2)$

We expect a negative slope for the solution because the graph of the line falls from left to right.



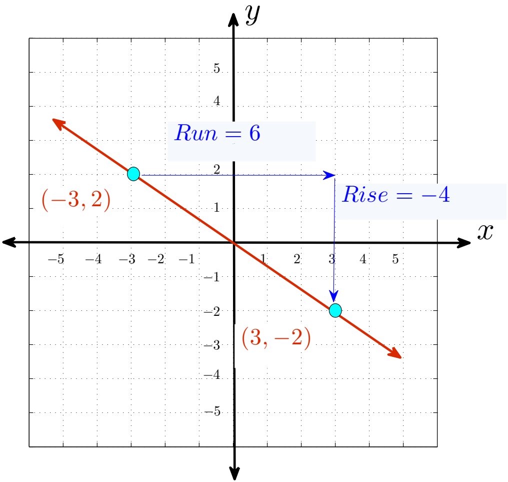
Ex. Find the slope of a line through $(-3, 2)$ and $(3, -2)$.

$$slope = \frac{Rise}{Run} = \frac{-4}{6} = \frac{2 \cdot (-2)}{2 \cdot 3} = \frac{\not{2} \cdot (-2)}{\not{2} \cdot 3} = -\frac{2}{3}$$



Ex. Find the slope of a line through $(-3, 2)$ and $(3, -2)$.

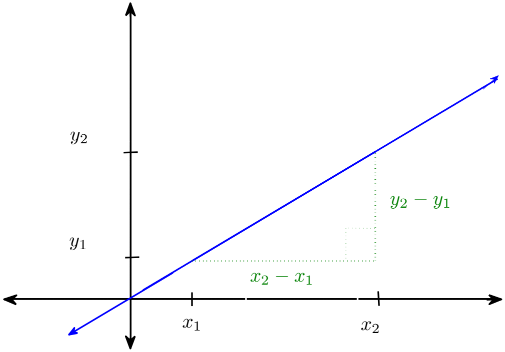
$$\textit{slope} = \frac{\textit{Rise}}{\textit{Run}} = \frac{-4}{6} = \frac{2\cdot(-2)}{2\cdot\ \ 3} = \frac{\cancel{2}\cdot(-2)}{\cancel{2}\cdot\ \ 3} = -\frac{2}{3}$$



Definition

Let (x_1, y_1) and (x_2, y_2) be any two points on the rectangular coordinate plane. The **SLOPE** of a line which passes through the points (x_1, y_1) and (x_2, y_2) is m, where m is given by the formula:

$$m = \frac{(y_2 - y_1)}{(x_2 - x_1)}$$



Classroom Examples: Take some time out to work these 2 problems.

Use the slope formula, $m = \dfrac{(y_2 - y_1)}{(x_2 - x_1)}$, to find the slope of a line containing the given points.

- $(7, -4)$ and $(4, 2)$
- $(2, -3)$ and $(-1, -3)$

The Equation of a Line

Suppose, now, that line l has slope m and y-intercept b, then what is the equation for l? Because the y-intercept is b, we know that point $(0, b)$ is on the line.

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$$\frac{y - b}{x - 0} = m$$

Multiplying both sides of this equation by x, then adding b to both sides gives

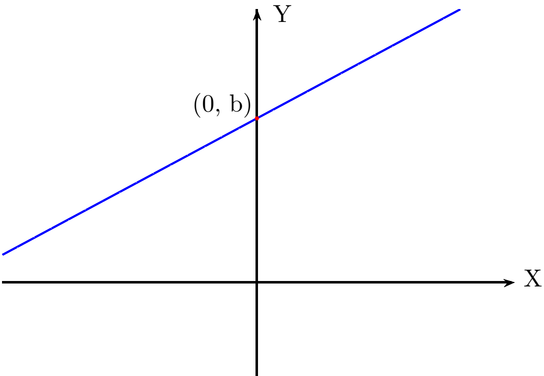
$$y = mx + b$$

Definition (The Equation of a Line: The Slope-Intercept Form)

Suppose m and b are real numbers (constants). Then another form for the equation of a line is

$$y = mx + b$$

where m represents the slope of the line and b represents the y-intercept.



Classroom Examples: Take some time out to work these 2 problems.

5. Determine the slope and y-intercept for the line $4x - 5y = 7$.

6. Graph the line $-3x + 2y = -6$ using the slope-intercept form ($y = mx + b$).

Suppose m and b represent particular, but arbitrarily chosen real numbers (fixed constants). A **linear function**, $f(x)$, has the form

$$f(x) = m \cdot x + b$$

where m represents the slope of the line and b represents the y intercept.

Example: Suppose $f(x) = 2x - 1$
What is the slope of f and what is it's y intercept.
Explain the geometric interpretation of $f(3)$.

Theorem (Point-Slope Equation of a line)

The line with slope m that passes through the point (x_1, y_1) is given by the equation:

$$y - y_1 = m(x - x_1)$$

We call this the ***Point-Slope Equation of a line****.*

Classroom Examples

7. Find the equation of the line with slope 3 that contains the point $(-1, 2)$. Use the Point-Slope Form, $y - y_1 = m(x - x_1)$.

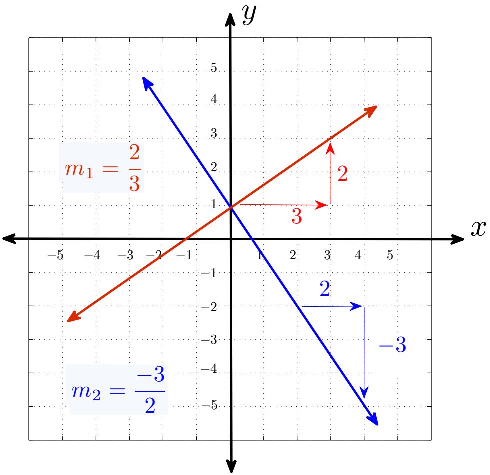
8. Find the equation of the line through $(2, 5)$ and $(6, -3)$.

9. Find the equation of the line that goes through the point (3,2), and is perpendicular to the graph of $3y - y = 2$.

Theorem

If line L_1 has slope m_1 and line L_2 has slope m_2, then

L_1 is perpendicular to L_2 if and only if $m_1 \cdot m_2 = -1$ $\left(\text{or } m_1 = -\frac{1}{m_2}\right)$

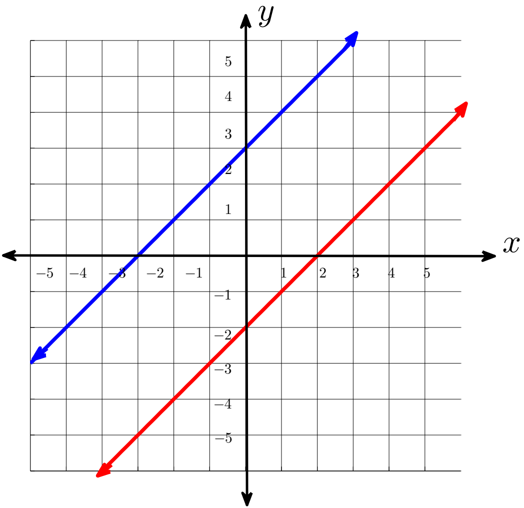


Classroom Example

9. Find the equation of the line that goes through the point (3,2), and is perpendicular to the graph of $3y - y = 2$.

Theorem

Two lines have the same slope if and only if they are parallel.



Classroom Example

10. Find the equation of the line that goes through the point (3,2), and is parallel to the graph of $3y - y = 2$.

Definition (The Equation of a Line: Standard Form)

Suppose A, B and C represent any real numbers. A **linear equation in two variables** is an equation having the *form*

$$A\,x + B\,y = C,$$

For example, $2\,x + 3\,y = 1$ is a linear equation in the two variables x and y which is written in standard form.

Determine whether or not the following equations are **linear equation in two variables**. If not, why?

11. $y = 2x - 1$
12. $2y - x = 4$
13. $2x^2 + y = 1$