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# Chapter 2

# Professor Tim Busken

Grossmont College Mathematics Department

August 28, 2013

# Tim Busken

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# The graph of a function *f* is the graph of the equation y = f(x). A function is called **continuous** if its graph has no breaks or holes.

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We can read the value of f(x) from the graph as being the height of the graph above a point *x*.

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# Definition

A function is a special type of relation. A <u>FUNCTION</u> is a correspondence between a first set, called the *domain*, and a second set, called the *range*, such that each member of the domain corresponds to *exactly one* member of the range.

However, different elements of the domain are allowed to have a correspondence with the same value in the range.

DOMAIN RANGE	DOMAIN RANGE	Domain Range
11	11	0
24 39	<b>1</b> 57	

Figure : F is a FUNCTION (left), R is a relation but NOT A FUNCTION (center) & an example of a function (right) whose two different domain elements are associated with the same range element.

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# Theorem (VERTICAL LINE TEST (VLT))

A curve in the coordinate plane is the graph of a function if and only if there is no vertical line that crosses the graph more than once.



Figure : GRAPHS OF  $y = x^2$  and  $x = y^2$ 

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# Does the equation $x^2 + y^2 = 16$ define y as a function of x?

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# $p(x) = x^n$ is called a **power function**.

If *n* is even, the graph of  $f(x) = x^n$  is similar to the parabola  $y = x^2$ . If *n* is odd, the graph of  $f(x) = x^n$  is similar to the cubic  $y = x^3$ .

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The graph of a function has origin symmetry when for any point (x,y) on the graph, there is also a point (-x,-y) on the graph.



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The graph of a function has y-axis symmetry if for every point (x,y), there is also a point (-x,y) on the graph.



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# Definition

The graph of a relation has x-axis symmetry if for every point (x, y) on the graph, the point (x, -y) is also on the graph.



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Can a function have x-axis symmetry?

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# Definition

- A function f(x) can be classified as (one of the following):
  - Even
  - Odd
  - 3 Neither Even Nor Odd



Figure : A function that is neither:  $f(x) = x(x-2)^2$ 

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# How to Test for Symmetry

- A function is <u>EVEN</u> if its graph has *y*-axis symmetry. If substitution of -x for *x* leads to the same equation, i.e., If f(-x) = f(x), then *f* is an even function.
- A function is <u>ODD</u> if its graph has origin symmetry. If substitution of -x for x leads to the negative version of f, i.e.,

If f(-x) = -f(x), then *f* is an odd function.

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If a, b, c, and d are real numbers with  $a \neq 0$ , then  $y = a \cdot f(bx - c) + d$  is called a **linear** transformation of the function y = f(x).

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# Vertical Shifts of Graphs

Suppose d > 0. The graph of y = f(x) + d is the graph of y = f(x) shifted vertically *upward* d units.



The graphs of  $f(x) = x(x-2)^2$  and f(x) + d (left); and the graphs of  $f(x) = x^2$  and  $g(x) = x^2 + 2$  (right).

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# Vertical Shifts of Graphs

Suppose d > 0. The graph of y = f(x) - d is the graph of y = f(x) shifted vertically *downward* d units.



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# Horizontal Shifts of Graphs

Let c > 0. The graph of y = f(x + c) is the graph of y = f(x) shifted to the *left* c units.



The graphs of  $f(x) = x(x-2)^2$  and f(x+c) are given in the left panel; and the graphs of  $f(x) = x^2$  and  $g(x) = (x+2)^2$  are presented in the right panel above.

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# Horizontal Shifts of Graphs

Let c > 0. The graph of y = f(x - c) is the graph of y = f(x) shifted to the *right* c units.



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# Combining Horizontal and Vertical Shifts



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# Combining Horizontal and Vertical Shifts



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To graph y = -f(x) reflect the graph of f(x) about the *x*-axis.





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# To graph y = f(-x) **reflect** the graph of f(x) about the *y*-axis.



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# Vertical Stretching and Shrinking

Graphing  $y = a \cdot f(x)$ 

If a > 1, stretch the graph of y = f(x) vertically by a factor of a.



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# Vertical Stretching and Shrinking

# Graphing $y = a \cdot f(x)$

If a > 1, stretch the graph of y = f(x) vertically by a factor of a.

If 0 < a < 1, shrink the graph of y = f(x) vertically by a factor of a.



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# Example: $f(x) = a \cdot x^2$

- If a > 1, stretch the graph of y = f(x) vertically by a factor of a.
- If 0 < a < 1, shrink the graph of y = f(x) vertically by a factor of a.



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# Example: $f(x) = a \cdot x^2$

- If a > 1, stretch the graph of y = f(x) vertically by a factor of a.
- If 0 < a < 1, shrink the graph of y = f(x) vertically by a factor of a.



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# Reflection and Vertical Shrinking & Stretching $y = x^2$



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# Combining Shifting, Stretching and Reflecting



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# Horizontal Stretching and Shrinking

Graphing  $y = f(b \cdot x)$ 

If b > 1, shrink the graph of y = f(x) horizontally by a factor of 1/b.



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# Horizontal Stretching and Shrinking

Graphing  $y = f(b \cdot x)$ 

If b > 1, shrink the graph of y = f(x) horizontally by a factor of 1/b.

If 0 < b < 1, stretch the graph of y = f(x) horizontally by a factor of 1/b.



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If *a*, *b*, *c*, and *d* are real numbers with  $a \neq 0$ , then  $y = a \cdot f(bx - c) + d$  is called a **linear transformation** of y = f(x).



All of the transformations of a function **form a family of functions**. For example,  $y = -3\sqrt[3]{x-1} + 1$  (graph below) is in the cube root family of functions.



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# Linear Transformation

$$y = \mathbf{a} \cdot f(\mathbf{b}x - \mathbf{c}) + \mathbf{d}$$

- a represents the reflection and vertical shrinking or stretching of *f*.
- *b* represents the horizontal shrinking or stretching of *f*.
- c represents the horizontal translation of f.
- *d* represents the vertical translation of *f*.

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# **Graphing using Translations**

$$y = \mathbf{a} \cdot f(\mathbf{b}x - \mathbf{c}) + \mathbf{d}$$

- 1.) Identify and graph f(x). Use symmetry, if possible.
- 2.) Horizontal Shift
- 3.) Reflection and horizontal and/or vertical shrinking or stretching
- 4.) Vertical Shift

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Graph  $y = -3 \cdot \sqrt[3]{x-1} + 1$ 

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1.)  $f(x) = \sqrt[3]{x}$ 

x	$f(x) = \sqrt[3]{x}$
-8	-2
-1	-1
0	0
1	1
8	2

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1.)  $f(x) = \sqrt[3]{x}$ 

# 2.) Horizontal Shift


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- 1.)  $f(x) = \sqrt[3]{x}$
- 2.) Horizontal Shift
- 3.) Reflection and Vertical Stretching

x	$y = \sqrt[3]{x-1}$	$y = -3\sqrt[3]{x-1}$
-7	-2	6
0	-1	3
1	0	0
2	1	-3
9	2	-6

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- 1.)  $f(x) = \sqrt[3]{x}$
- 2.) Horizontal Shift
- 3.) Reflection and Vertical Stretching
- 4.) Vertical Shift



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- 1.)  $f(x) = \sqrt[3]{x}$
- 2.) Horizontal Shift
- 3.) Reflection and Vertical Stretching
- 4.) Vertical Shift

 $y = -3 \cdot \sqrt[3]{x-1} + 1 = -3 \cdot f(x-1) + 1$ 

# has the following characteristics:

- domain:  $x \in (-\infty, \infty)$
- range:  $y \in (-\infty, \infty)$
- g(x) is a decreasing function.
   g(x) ↓ for x ∈ (-∞, ∞)



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# Set Intersection

The <u>intersection</u> of two sets A and B, written  $A \cap B$ , is the set of all elements (numbers) that are in both A and B. The  $\cap$  symbol means the word "and."

**Example:** Suppose A =  $\{1, 2, 3, 4\}$  and B =  $\{2, 4, 20\}$ . Then A  $\cap$  B =  $\{2, 4\}$ 



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**Example:** Suppose A =  $\{1, 2, 3, 4\}$  and B =  $\{2, 4, 20\}$ . Then A  $\cap$  B =  $\{2, 4\}$ 

**Example:** A =  $[0, \infty)$  and B =  $(-\infty, \infty)$ .



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**Example:** Suppose A =  $\{1, 2, 3, 4\}$  and B =  $\{2, 4, 20\}$ . Then A  $\cap$  B =  $\{2, 4\}$ 

**Example:** A =  $[0, \infty)$  and B =  $(-\infty, \infty)$ .



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# Algebra of Functions

Let *f* and *g* be functions with domains A and B. Then the functions f + g, f - g, fg, and f/g are defined as follows:

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(f+g)(x)	= f(x) + g(x)	domain $A \cap B$
(f-g)(x)	= f(x) - g(x)	domain $A \cap B$
$(f \cdot g)(x)$	$= f(x) \cdot g(x)$	domain $A \cap B$
$\left(\frac{f}{g}\right)(x)$	$= \frac{f(x)}{g(x)}$	domain $\{x \in A \cap B \mid g(x) \neq 0\}$

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# Algebra of Functions

Let *f* and *g* be functions with domains A and B. Then the functions f + g, f - g, fg, and f/g are defined as follows:

$\left( f+g ight) \left( x ight)$	= f(x) + g(x)	domain $A \cap B$
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$(f \cdot g)(x)$	$= f(x) \cdot g(x)$	domain $A \cap B$
$\left(\frac{f}{g}\right)(x)$	$= \frac{f(x)}{g(x)}$	domain $\{x \in A \cap B \mid g(x) \neq 0\}$

**Example**: Suppose  $f(x) = \sqrt{x}$ ,  $g(x) = x^2$  and  $h(x) = (f+g)(x) = \sqrt{x} + x^2$ .

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# Algebra of Functions

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,  $g(x) = x^2$  and  $h(x) = (f + g)(x) = \sqrt{x} + x^2$ .

xaxis 
$$\leftarrow dom(g) \equiv (-\infty, \infty)$$

xaxis  $dom(h) \equiv dom(f) \cap dom(g) \equiv [0, \infty)$ 

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# **Composition of Functions**

If f and g are two functions, the composition of f and g, written  $f \circ g$  is defined by the equation

$$f \circ g = f(g(x)),$$

provided that g(x) is in the domain of f.

**Example:** Suppose  $f(x) = \sqrt{x}$  and g(x) = 2x + 1. Then  $f(g(x)) = f(2x + 1) = \sqrt{2x + 1}$ .

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**Example**: Suppose  $g \equiv \{(1, 2), (3, 4), (5, 6)\}$  and  $f \equiv \{(2, 8), (4, 9), (1, 1)\}$ . Find  $f \circ g$ .

**Solution:** Since g(1) = 2 and f(2) = 8, then f(g(1)) = 8, and (1, 8) is an ordered pair in  $f \circ g$ . Also since g(3) = 4 and f(4) = 9, then f(g(3)) = 9, and (3, 9) is an ordered pair in  $f \circ g$ . Now g(5) = 6 but 6 is not in the domain of f. So there are only two ordered pairs in  $f \circ g$ , namely  $f \circ g \equiv \{(1, 8), (3, 9)\}$ 

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**Comment:** the domain of g is {1, 3, 5} while the domain of  $f \circ g$  is {1, 3}. In order to find the domain of  $f \circ g$  we remove from the domain of g any number x such that g(x) is not in the domain of f.

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# One to one functions have inverses!

A function f with domain D and range R is a <u>one to one function</u> if *either* of the following equivalent conditions is satisfied:

Whenever  $x_1 \neq x_2$  in D, then  $f(x_1) \neq f(x_2)$  in R.

Whenever  $f(x_1) = f(x_2)$  in R, then  $x_1 = x_2$  in D.



**Example:**  $f(x) = x^2$  is *NOT* a one to one function since for  $x_1 = -2$  and  $x_2 = 2$ , it is true that  $x_1 \neq x_2$  and  $f(x_1) = f(x_2) = 4$ .

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# The Horizontal Line Test

<u>A function f is one to one</u> if and only if every horizontal line intersects the graph of f in at most one point.

 $f(x) = x^{2}$ is not one to one is one to one. yyyyxyxyxyxyxis one to one.

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# Inverse Function

Suppose *f* is a one to one function, with domain D and range R. The <u>inverse function</u> of *f* is the function denoted  $f^{-1}$  with domain R and range D provided that

 $f^{-1}(f(x)) = x$ 

Note: A function has an inverse (function) only when it is one to one.

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Note: A function has an inverse (function) only when it is one to one.

# **CAUTION:** $f^{-1}(x) \neq f(x)^{-1}$

•  $f^{-1}(x)$  is notation for the <u>function inverse</u> of a one to one function f

•  $f(x)^{-1} = (f(x))^{-1} = \frac{1}{f(x)}$  is the <u>multiplicative inverse</u> of the number f(x).

**Example**: Suppose *f* is one-to-one and f(-9) = 15, then  $f^{-1}(15) = -9$  and  $(f(-9))^{-1} = 1/15$ 

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# **Properties of Inverse Functions**

Suppose that f is a one to one function with domain D and range R. Then

- The inverse function f<sup>-1</sup> is unique.
- The domain of f<sup>-1</sup> is the range of f.
- The range of f<sup>-1</sup> is the domain of f.
- The statement f(x) = y is equivalent to  $f^{-1}(y) = x$

**Note:** The graph of  $y = t^{-1}(x)$  is the reflection of the graph of y = f(x) about the line y = x. For every point (a, b) on the graph of f(x) there is a corresponding point (b, a) on the graph of  $t^{-1}(x)$ .



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# **Inverse Function**

# How to find the inverse of a one to one function:

**1** Replace f(x) with y. Then interchange x and y.

- 2 Solve the resulting equation for y.
- 3 Replace y with  $f^{-1}(x)$ .