**Math 176 Test 5** **Name:** _____

***No Calculators or Computing Devices allowed! Use Algebraic Notation AND Show All of Your Work.***

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1. (6 points) Use Gaussian elimination to find the complete solution of the system, or show that no solution exists.

$$\begin{cases} x - y + 2z = 0 \\ 2x - 4y + 5z = -5 \\ 2y - 3z = 5 \end{cases}$$

1. _____

2. (a) (2 points) Write a matrix equation equivalent to the following system.

$$\begin{cases} 4x - 3y = 10 \\ 3x - 2y = 30 \end{cases}$$

(a) _____

(b) (4 points) Find the inverse of the coefficient matrix, and use it to solve the system.

(b) _____

3. (5 points) Solve $\left\{ \begin{array}{r} x - y = 1 \\ 4x + 3y = 18 \end{array} \right\}$ using Cramer's Rule.

3. _____

4. Let $A = \begin{bmatrix} 1 & -5 \\ -3 & 7 \end{bmatrix}$, $B = \begin{bmatrix} -2 & -6 \\ 2 & 7 \\ 1 & 0 \end{bmatrix}$, $C = \begin{bmatrix} 1 & 3 & 1 \\ -2 & 7 & 2 \\ 0 & 2 & 4 \end{bmatrix}$

Carry out the indicated operation, or explain, using complete sentences, why it cannot be performed.

(a) (2 points) $A + B$

(b) (2 points) $AB$

(c) (2 points) $BA - 3A$

(d) (2 points) $B^{-1}$

(e) (2 points) $\det(B)$

5. (6 points) Find the inverse of $C = \begin{bmatrix} 1 & 0 & 4 \\ -1 & 1 & 2 \\ 0 & 1 & 3 \end{bmatrix}$ if it exists.

5. _____

6. (6 points) Find the partial fraction decomposition of $\dfrac{2x-3}{x^3+3x}$.

6. _____

7. Only one of the following two matrices has an inverse.

$$A = \begin{bmatrix} -2 & 5 & -2 \\ 0 & 7 & 0 \\ -2 & 1 & -2 \end{bmatrix}, \qquad B = \begin{bmatrix} 4 & 5 & 6 \\ 2 & 7 & 1 \\ -7 & 1 & -3 \end{bmatrix}$$

(a) (5 points) Find the determinant of each matrix. (a) _____

(b) (1 point) Use the determinants from part (a) to identify which matrix has an inverse.

(b) _____

8. (3 points) Find the first three terms of the sequence $a_n = 2n^2 - 1$

8. _____

9. (3 points) Find the third partial sum of the sequence $a_n = 2n^2 - 1$

9. _____

10. (3 points) A sequence is defined recursively by $a_{n+1} = 2a_n - 3n$, with $a_1 = 2$. Find the first 4 terms of the sequence.

10. _____

11. An arithmetic sequence begins with 6, 13, 20, 27, ... .

(a) (1 point) Find the common difference, $d$, for this sequence.

(a) _____

(b) (2 points) Find a formula for the $n^{\text{th}}$ term, $a_n$, of the sequence.

(b) _____

(c) (2 points) Find the $36^{\text{th}}$ term, $a_{36}$, of the sequence.

(c) _____

12. A geometric sequence begins with 12, 3, 3/4, 3/16, 3/64, ... .

(a) (1 point) Find the common ratio, $r$, for this sequence.

(a) _____

(b) (2 points) Find a formula for the $n^{\text{th}}$ term, $a_n$, of the sequence.

(b) _____

(c) (2 points) Find the $10^{\text{th}}$ term, $a_{10}$, of the sequence.

(c) _____

13. (6 points) Expand $(2x-1)^4$

14. (6 points) Express the repeating decimal $0.\overline{051}$ as a fraction in lowest terms.

14. _____

15. (4 points) Write the sum using sigma notation. Do not evaluate.

$$3 + 6 + 9 + 12 + \cdots + 99$$

16. (3 points) ***EXTRA CREDIT*** Find the sum $\sum_{k=3}^{5}(k+1)^2$