Math 176 — Exam 2 **Name:** ____________________

Directions: You may not use a calculator. The use of any other electronic devices are strictly prohibited. Show your work on ALL 33 questions (6 pages). You will not be allowed to leave to use the restroom. Pages 24, 43 and 44 of your workbook (the graphing algorithms) have been included at the end of this document along with a scratch piece of paper. I won't grade the scratch paper.

1. (3 points) Find the quotient and the remainder for $\dfrac{x^5 - 2x^3 + 2x + 1}{x^2 + 1}$

1. ____________

2. (3 points) Divide and simplify $\dfrac{2+3i}{3-5i}$. Write your solution in the form $a + bi$.

2. ____________

3. (3 points) Solve $\ln(x+5) = \ln(x-1) - \ln(x+1)$.

3. ____________

For questions 4 through 9, use $f(x) = x^4 + 10x^3 + 35x^2 + 50x + 24$.

4. (3 points) Find all the zeros of $f(x)$ 4. ____________

5. (1 point) What is the domain of $f(x)$? 5. ____________

6. (1 point) Find the y-intercept of $f(x)$ 6. ____________

7. (2 points) Write an end behavior description of $f(x)$

8. (2 points) Find the solution set to $f(x) < 0$ 8. ____________

9. (2 points) Use synthetic division to find the value of $f(-5)$ 9. ____________

For questions 10 through 16, use $f(x) = \dfrac{x^2 - 5x + 6}{3x^2 - 15x - 18}$

10. (2 points) Find the vertical asymptote(s) of $f(x)$ 10. ____________

11. (1 point) Find the domain of $f(x)$ 11. ____________

12. (2 points) Find the zeros of $f(x)$ 12. ____________

13. (1 point) Find the y-intercept of $f(x)$ 13. ____________

14. (2 points) Find the horizontal asymptote of $f(x)$ 14. ____________

15. (4 points) Describe the behavior of the graph of f around its vertical asymptote(s).

16. (2 points) Describe the end behavior of the graph of f.

17. (3 points) Find a 2nd-degree polynomial function with integer coefficients that has a zero at $x = 2 + 5i$. Write the polynomial in descending order (leaving your polynomial in factored form doesn't constitute a full credit answer).

17. ____________

For questions 18 through 23, use $f(x) = 2 - 3^{x+1}$

18. (2 points) Find the range of $f(x)$ 18. ____________

19. (2 points) Find the horizontal asymptote(s) of $f(x)$ 19. ____________

20. (1 point) Find the domain of $f(x)$ 20. ____________

21. (2 points) Find the zero(s) of $f(x)$ if there are any. You can leave your answer(s) in exact form.

21. ____________

22. (1 point) Find the y-intercept of $f(x)$ if there is one. 22. ____________

23. (2 points) Describe the end behavior of the graph of f.

For questions 24 through 29, use $f(x) = 2 - \log_3(x+1)$

24. (2 points) Find the range of $f(x)$ 24. ____________

25. (2 points) Find the vertical asymptote(s) of $f(x)$ 25. ____________

26. (1 point) Find the domain of $f(x)$ 26. ____________

27. (2 points) Find the zero(s) of $f(x)$ if there are any. You can leave your answer(s) in exact form.

27. ____________

28. (1 point) Find the y-intercept of $f(x)$ if there is one. 28. ____________

29. (2 points) Describe the end behavior of the graph of f.

30. (2 points) Find the slant asymptote(s) of $f(x) = \dfrac{x^2+2x+2}{x}$

30. ____________

31. (2 points) Write a statement describing the end behavior of $f(x) = \dfrac{x^2+2x+2}{x}$

31. ____________

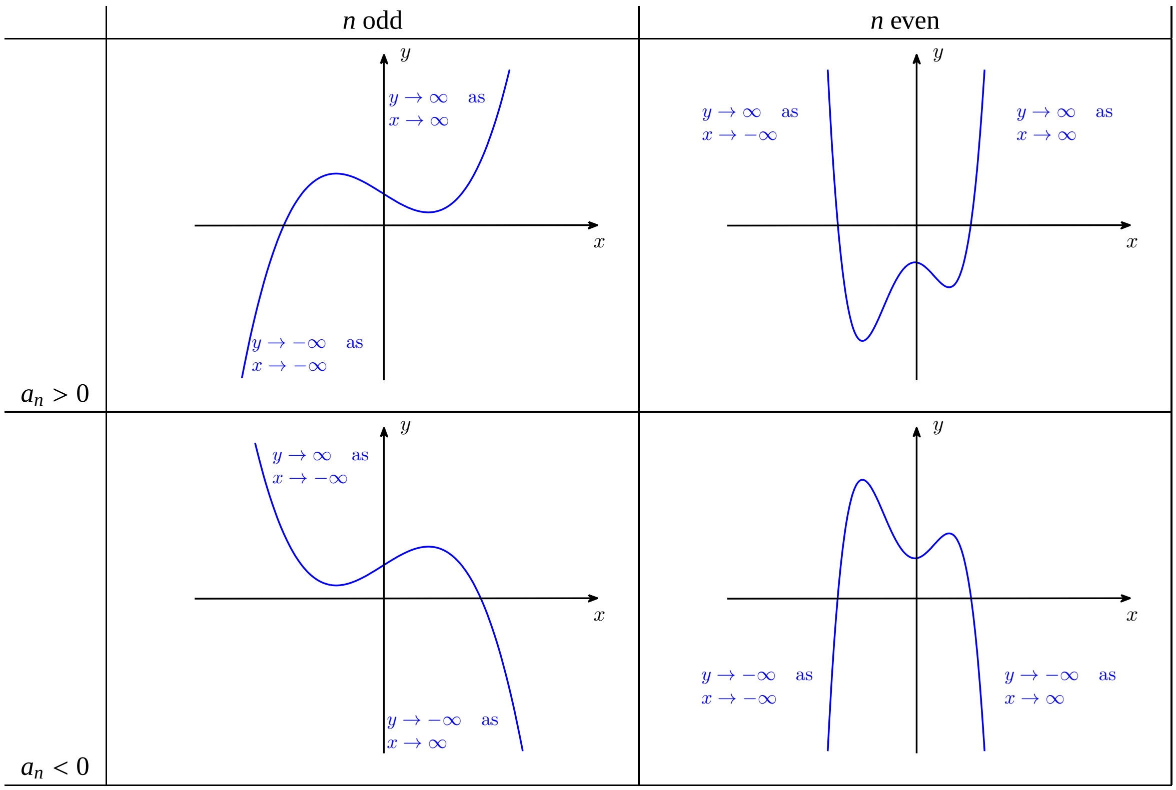
32. (2 points) Evaluate $\log_5\left(\sqrt[3]{5}\right) - \log_2\left(\dfrac{1}{8}\right)$

32. ____________

33. (3 points) The half-life of Plutonium-240 is 6537 years. If a sample has a mass of 130 kg, find a function that models the mass that remains after t years. (Hint: use the continuous growth/decay model $A = P_0 e^{rt}$.)

GRAPHING POLYNOMIAL FUNCTIONS

1. Determine if the graph has any symmetry. Locate the y intercept.
2. Factor the polynomial and find the zeros.
3. Determine the x intervals for which $f(x) > 0$ (is above the y axis) and $f(x) < 0$ (is below the y axis).
4. Plot the zeros on the real number line. Label each zero as being either odd or even.
5. Make a table of values. Mark the end behavior of the graph.



6. Begin graphing starting at the left 'end behavior point' that you marked your graph with. Proceed to the first zero on the left:
   - if the zero is odd, pass through the x axis,
   - if the zero is even 'bounce off' the x axis.
7. Continue to the next zero and repeat the process.
8. When you have finished this procedure with the last zero on the right, the graph should connect with the right end behavior arrow that you marked in step 4.
9. As a 'check point' you can determine the sign (+ or -) of the y intercept (when $x = 0$) and see if the result is consistent with your graph.

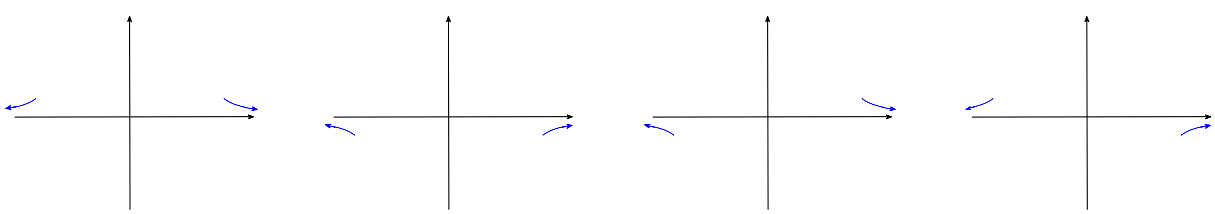
Graphing Rational Functions Algorithm

1. Write the numerator and denominator in factored form. Determine the locations of the x and y intercepts.
2. Determine the equation(s) that represent the Vertical Asymptote(s): set the denominator equal to zero and solve for x.
3. Determine the equation that represents the Horizontal or Slant Asymptote (HA or SA).
4. Write a statement describing the end behavior of the graph:

$$y \to HA \quad \text{as} \quad x \to \pm\infty \qquad \text{or} \qquad y \to SA \quad \text{as} \quad x \to \pm\infty$$

5. Determine the x intervals for which $f(x) < 0$ and $f(x) > 0$. Make a sign chart and use either the test point method or the multiplicity method to mark the sign (+ or -) of $f(x)$ for x in each interval.
6. Mark the "end behavior" of the graph, depending on whether $n < d$, $n = d$, or $n > d$; where n is the degree of the polynomial in the numerator and d is the degree of the polynomial in the denominator.

   Case 1: ($n < d$) The HA is $y = 0$. The end behavior of the graph will look like one of the following 4 pictures: Your inequality sign chart will indicate which one of the four



   you have; but you may have to compute a couple of y values be sure.

   Case 2: ($n = d$) The HA occurs at $y = \frac{a}{b}$. The end behavior will look like one of the four above graphs, but the HA is shifted up or down to $y = \frac{a}{b}$.

   Case 3: ($n > d$) The graph doesnt have a HA, it has a SA at $y = mx + b$, where $mx + b$ is the linear part of the resulting quotient, after the polynomial in the numerator has been divided by the polynomial in the denominator. The end behavior for the graph of the rational function will be the same as the end behavior of the SA. Draw the graph of $y = mx + b$ with a dotted line.
7. Identify and plot the zeros (also called roots) of the rational function.
   - Label these odd (O) or even (E) as in polynomial graphing
   - also plot the y intercept
8. Sketch the vertical asymptotes as vertical dotted lines: Label these VAs as being odd (O) or even (E) at the top of the dotted line depending on their multiplicities.
9. Determine the behavior of the graph (y value) around the VAs.

$$y \to \pm\infty \quad \text{as} \quad x \to a^- \quad \text{and} \quad y \to \pm\infty \quad \text{as} \quad x \to a^+$$

Whether or not y goes to plus or minus infinity depends on the sign of f(x) in that interval.

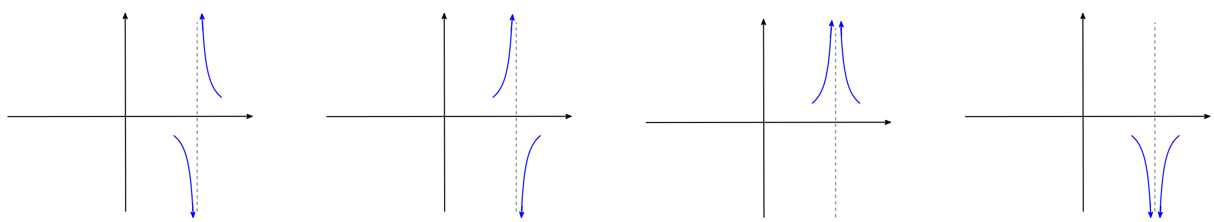
10. Begin graphing from the left end point and proceed to the first critical value
    - a critical value is a zero or a vertical asymptote
    - you may need to make a small table of values for better plotting accuracy.
11. At each zero proceed as in polynomial graphing, ie. If the zero is odd the graph goes thru the x axis; if the zero has even multiplicity, then it touches the axis and bounces off.
12. At each vertical asymptote, as you approach from the left go "north" to $+\infty$ or "south" to $-\infty$.

    At each **odd** vertical asymptote, as you step across the dotted line

    Switch signs from $+\infty$ or to $-\infty$ or from $-\infty$ or to $+\infty$

    At each **even** vertical asymptote, as you step across the dotted line

    Keep the **same signs** from $+\infty$ or to $+\infty$ or from $-\infty$ or to $-\infty$



13. Continue to the next critical value (zero or asymptote) repeat the process of step 8 or 9; ignore the vertical scale as you pass from one zero to the next; item When you have finished this procedure with the last zero on the right, the graph should connect with the right "endpoint" that you marked in step 4
14. As a "check point" you can determine the sign (+ or –) of the y intercept (when $x = 0$) and see if the result is consistent with your graph