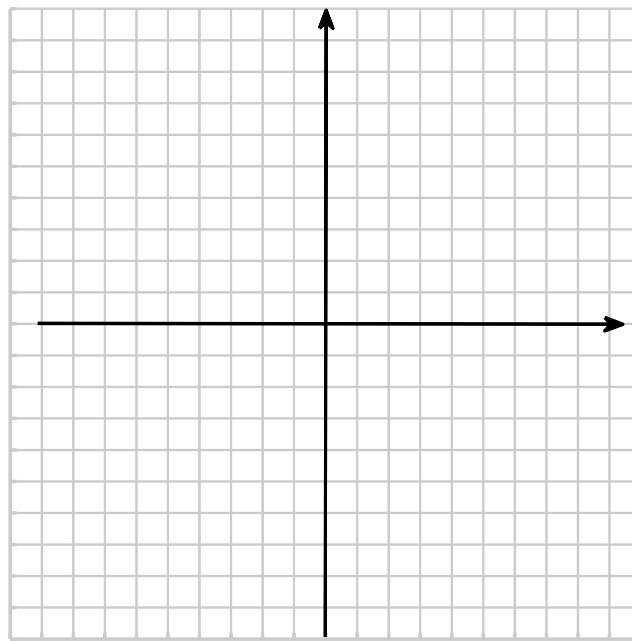
Math 176
Exam 1
Professor Busken **Name:** ______________________

Directions: You may not use a calculator. The use of any other electronic devices are strictly prohibited. Show your work on ALL of the questions. Scratch paper is not allowed. You will not be allowed to leave to use the restroom.

1. (5 points) Find an equation of the line that is tangent to the circle $x^2 + y^2 = 25$ at the point $(x, y) = (3, 4)$.



2. (5 points) Suppose $f(x) = \left\{ \begin{array}{ll} 4 - 3x & \text{if } x < 0 \\ 5x & \text{if } 0 \le x \le 2 \\ (x-5)^2 & \text{if } x > 2 \end{array} \right\}$. Evaluate the piecewise defined function at the values indicated below.

   (a) $f(-5)$
   (b) $f(0)$
   (c) $f(1)$
   (d) $f(2)$
   (e) $f(5)$

(a) ____________
(b) ____________
(c) ____________
(d) ____________
(e) ____________

3. (5 points) Let $g(x) = \dfrac{3}{1-x}$. Find a simplified form for the difference quotient $\dfrac{g(a+h)-g(a)}{h}$, where $h \neq 0$.

3. ______________

4. (3 points) Find the domain of $f(x) = \dfrac{2}{\sqrt{x+13}}$. Express the domain set using interval notation.

4. ______________

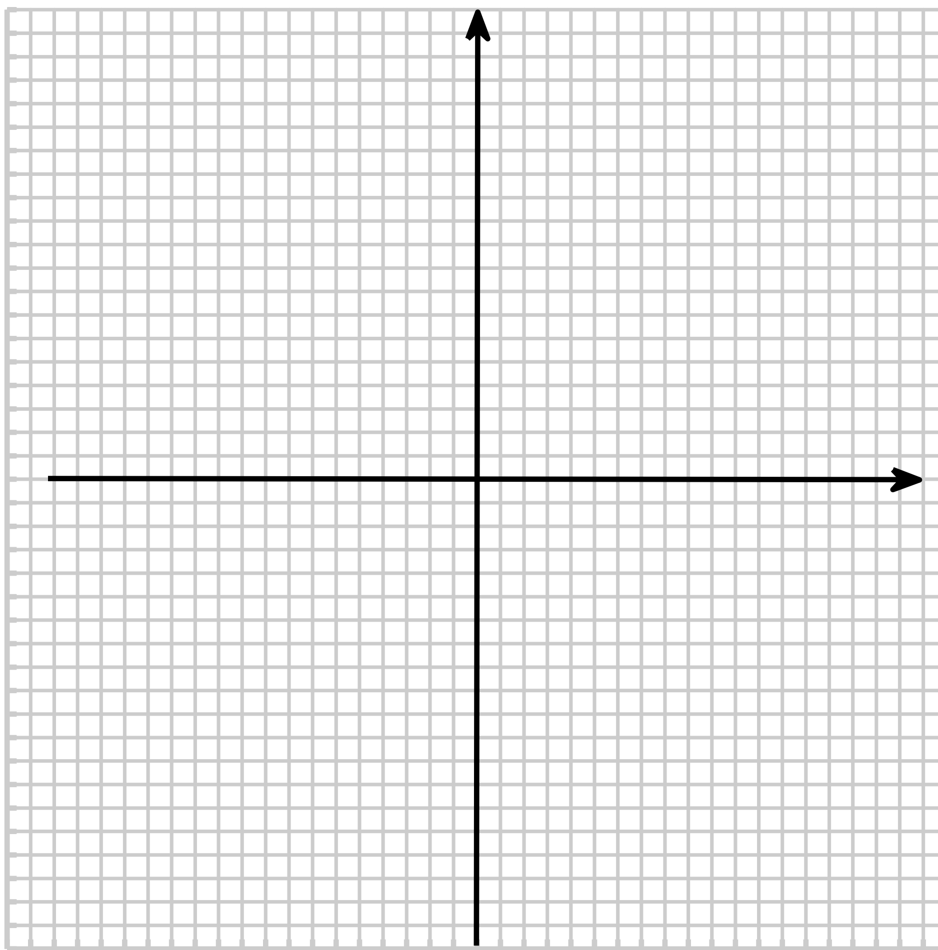
5. (3 points) Find the domain of $f(x) = \sqrt[3]{x-13}$. Express the domain set using interval notation.

5. ______________

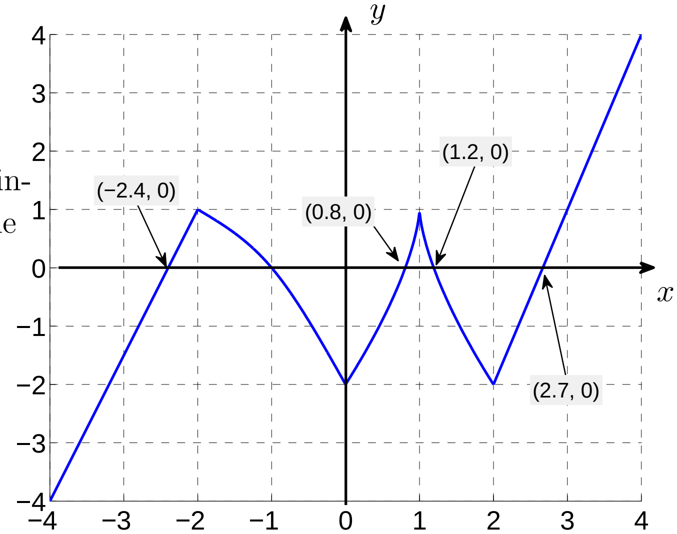
6. (4 points) Suppose $f(t) = 3t + t^2$ Find the average value of f over time interval $[5, 10]$.

6. ______________

7. (5 points) Sketch the graph of $f(x) = \left\{ \begin{array}{ll} -3x & \text{if } x < 0 \\ \sqrt{16 - x^2} & \text{if } 0 \le x < 4 \\ (x-4)^2 & \text{if } x \ge 4 \end{array} \right\}.$



8. (10 points) The graph of a function f is given. **Assume the entire graph of f is shown in the figure.**



(a) Find all ***local*** maximum and minimum values of the function and the value of x at which each occurs.

(b) State the x intervals for which $f(x) > 0$.

(c) State the x intervals for which $f(x) < 0$.

(d) Find the x intervals on which the function is ***increasing***.

(e) Find the x intervals on which the function is ***decreasing***.

(f) Find $f(4)$.

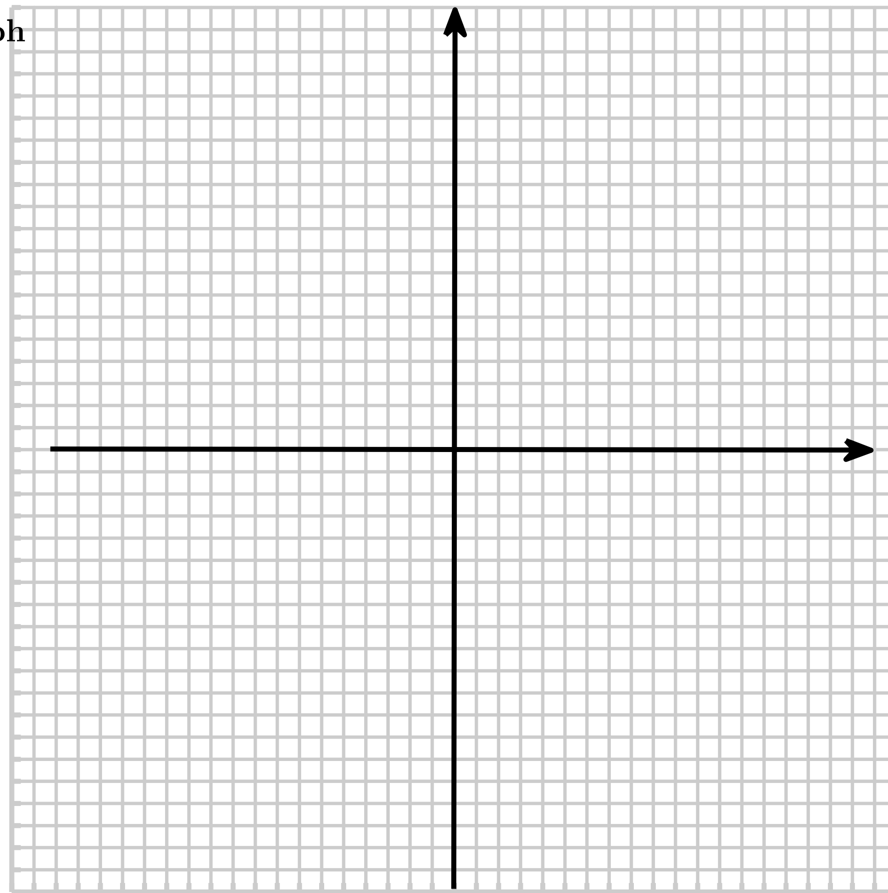
(f) ______________

(g) Find $f(-1)$.

(g) ______________

9. (5 points) **Use the graph of $f(x) = \sqrt{x}$ to sketch the graph of the given function.**

$h(x) = -3\sqrt{x-4}+1$



10. (2 points) What interval represents the domain of h? 10. ____________

11. (2 points) What interval represents the range of h? 11. ____________

12. (5 points) This is a ***Matching question*** associated with the theory on graphical translations of functions. Suppose $f(x) = x^3$. Relative to the graph of $f(x)$ the graphs of the following functions have been changed in what way?

_____ $g(x) = x^3 + 5$ a.) shifted 5 units left

_____ $g(x) = (x+5)^3$ b.) reflected about the x axis

_____ $g(x) = -2 \cdot x^3$ c.) shifted 5 units down

_____ $g(x) = (x-5)^3$ d.) shifted 5 units right

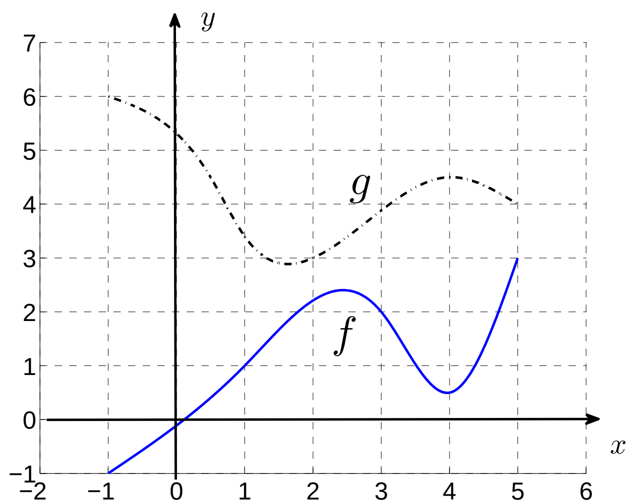
_____ $g(x) = x^3 - 5$ e.) shifted 5 units vertically up

13. (6 points) **Use the graph to find the indicated functional values.**

(a) $(f+g)(5)$

(b) $(f \circ g)(-1)$

(c) $\dfrac{f}{g}(3)$



14. (6 points) Suppose $f(x) = \sqrt{25 - x^2}$ and $g(x) = \sqrt{2 + x}$. Find $f + g$ and f/g, AND THEIR DOMAINS.

15. (2 points) Suppose f is an invertible function and suppose that $f(2) = -5$. Find $(f(2))^{-1}$

16. (2 points) Suppose f is an invertible function and suppose that $f(3) = 5$. Find $(f^{-1}(5))^{-1}$

17. (5 points) Find $f^{-1}(x)$ if $f(x) = \dfrac{1+3x}{5-2x}$.

17. ____________

18. (5 points) Suppose $f(x) = \dfrac{2x}{1-x}$ and $g(x) = 2 + 7x$. Find $f \circ g$ and its domain.

18. ____________

19. (5 points) Write $f(x) = 3x^2 + 6x + 1$ in standard (vertex) form.

19. ______________

20. (8 points) Use this equation to answer parts a through h: $y = -(x+2)^2 + 4$

(a) Determine whether the parabola has a maximum or a minimum and give the value.

(a) ____________

(b) Find the vertex (and plot it on the graph below).

(b) ____________

(c) What is the equation that represents the axis of symmetry (and draw it on the graph).

(c) ____________

(d) Find a second point (and plot it on the graph).

(d) ____________

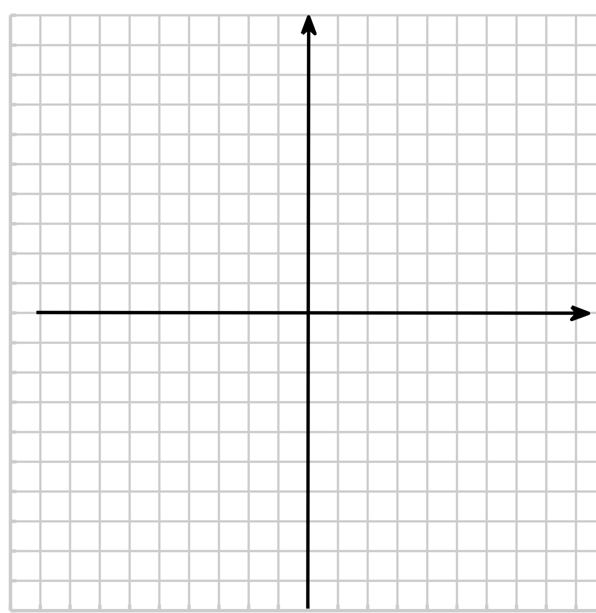
(e) Use symmetry to find a third point (and plot it on the graph)

(e) ____________

(f) Find the x-intercepts

(f) ____________

(g) Sketch the parabola



(h) What interval represents the range of the function? (h) ____________

Math 176 — Exam 2 **Name:** _________________________

Directions: You may not use a calculator. The use of any other electronic devices are strictly prohibited. Show your work on ALL 33 questions (6 pages). You will not be allowed to leave to use the restroom. Pages 24, 43 and 44 of your workbook (the graphing algorithms) have been included at the end of this document along with a scratch piece of paper. I won't grade the scratch paper.

1. (3 points) Find the quotient and the remainder for $\dfrac{x^5 - 2x^3 + 2x + 1}{x^2 + 1}$

1. ____________

2. (3 points) Divide and simplify $\dfrac{2+3i}{3-5i}$. Write your solution in the form $a + bi$.

2. ____________

3. (3 points) Solve $\ln(x+5) = \ln(x-1) - \ln(x+1)$.

3. ____________

For questions 4 through 9, use $f(x) = x^4 + 10x^3 + 35x^2 + 50x + 24$***.***

4. (3 points) Find all the zeros of $f(x)$ 4. ____________

5. (1 point) What is the domain of $f(x)$? 5. ____________

6. (1 point) Find the y-intercept of $f(x)$ 6. ____________

7. (2 points) Write an end behavior description of $f(x)$

8. (2 points) Find the solution set to $f(x) < 0$ 8. ____________

9. (2 points) Use synthetic division to find the value of $f(-5)$ 9. ____________

For questions 10 through 16, use $f(x) = \dfrac{x^2 - 5x + 6}{3x^2 - 15x - 18}$

10. (2 points) Find the vertical asymptote(s) of $f(x)$ 10. ____________

11. (1 point) Find the domain of $f(x)$ 11. ____________

12. (2 points) Find the zeros of $f(x)$ 12. ____________

13. (1 point) Find the y-intercept of $f(x)$ 13. ____________

14. (2 points) Find the horizontal asymptote of $f(x)$ 14. ____________

15. (4 points) Describe the behavior of the graph of f around its vertical asymptote(s).

16. (2 points) Describe the end behavior of the graph of f.

17. (3 points) Find a 2nd-degree polynomial function with integer coefficients that has a zero at $x = 2 + 5i$. Write the polynomial in descending order (leaving your polynomial in factored form doesn't constitute a full credit answer).

17. ____________

For questions 18 through 23, use $f(x) = 2 - 3^{x+1}$

18. (2 points) Find the range of $f(x)$ 18. ____________

19. (2 points) Find the horizontal asymptote(s) of $f(x)$ 19. ____________

20. (1 point) Find the domain of $f(x)$ 20. ____________

21. (2 points) Find the zero(s) of $f(x)$ if there are any. You can leave your answer(s) in exact form.

21. ____________

22. (1 point) Find the y-intercept of $f(x)$ if there is one. 22. ____________

23. (2 points) Describe the end behavior of the graph of f.

For questions 24 through 29, use $f(x) = 2 - \log_3(x+1)$

24. (2 points) Find the range of $f(x)$ 24. ____________

25. (2 points) Find the vertical asymptote(s) of $f(x)$ 25. ____________

26. (1 point) Find the domain of $f(x)$ 26. ____________

27. (2 points) Find the zero(s) of $f(x)$ if there are any. You can leave your answer(s) in exact form.

27. ____________

28. (1 point) Find the y-intercept of $f(x)$ if there is one. 28. ____________

29. (2 points) Describe the end behavior of the graph of f.

30. (2 points) Find the slant asymptote(s) of $f(x) = \dfrac{x^2 + 2x + 2}{x}$

30. ____________

31. (2 points) Write a statement describing the end behavior of $f(x) = \dfrac{x^2 + 2x + 2}{x}$

31. ____________

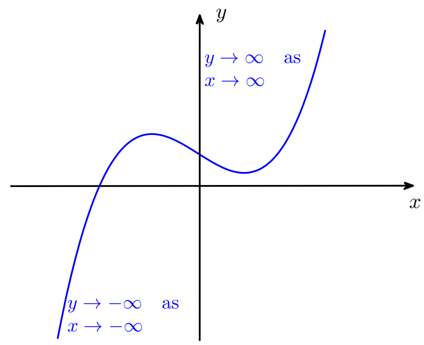
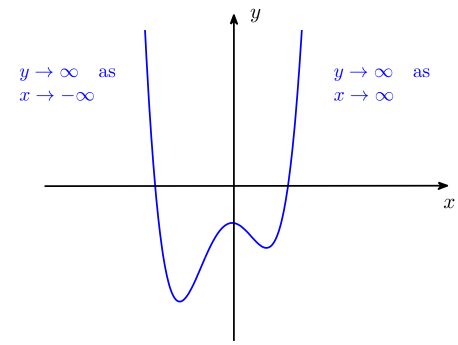
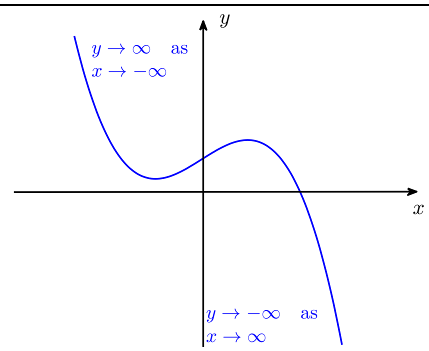
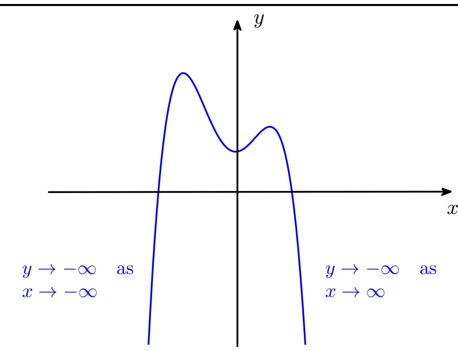
32. (2 points) Evaluate $\log_5\left(\sqrt[3]{5}\right) - \log_2\left(\dfrac{1}{8}\right)$

32. ____________

33. (3 points) The half-life of Plutonium-240 is 6537 years. If a sample has a mass of 130 kg, find a function that models the mass that remains after t years. (Hint: use the continuous growth/decay model $A = P_0 e^{rt}$.)

GRAPHING POLYNOMIAL FUNCTIONS

1. Determine if the graph has any symmetry. Locate the y intercept.
2. Factor the polynomial and find the zeros.
3. Determine the x intervals for which $f(x) > 0$ (is above the y axis) and $f(x) < 0$ (is below the y axis).
4. Plot the zeros on the real number line. Label each zero as being either odd or even.
5. Make a table of values. Mark the end behavior of the graph.

	n odd	n even
$a_n > 0$		
$a_n < 0$		

6. Begin graphing starting at the left 'end behavior point' that you marked your graph with. Proceed to the first zero on the left:
   - if the zero is odd, pass through the x axis,
   - if the zero is even 'bounce off' the x axis.
7. Continue to the next zero and repeat the process.
8. When you have finished this procedure with the last zero on the right, the graph should connect with the right end behavior arrow that you marked in step 4.
9. As a 'check point' you can determine the sign (+ or -) of the y intercept (when $x = 0$) and see if the result is consistent with your graph.

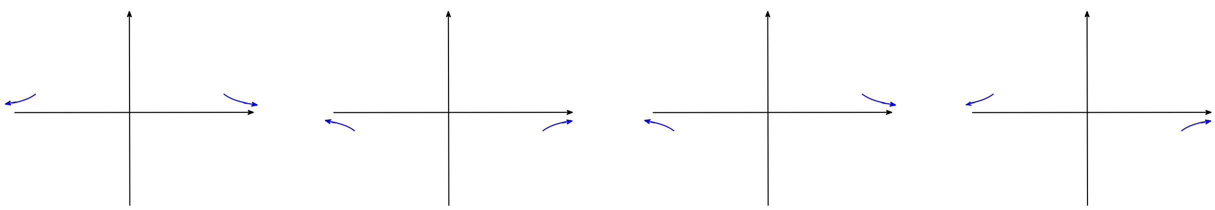
Graphing Rational Functions Algorithm

1. Write the numerator and denominator in factored form. Determine the locations of the x and y intercepts.
2. Determine the equation(s) that represent the Vertical Asymptote(s): set the denominator equal to zero and solve for x.
3. Determine the equation that represents the Horizontal or Slant Asymptote (HA or SA).
4. Write a statement describing the end behavior of the graph:

$$y \to HA \quad \text{as} \quad x \to \pm\infty \qquad \text{or} \qquad y \to SA \quad \text{as} \quad x \to \pm\infty$$

5. Determine the x intervals for which $f(x) < 0$ and $f(x) > 0$. Make a sign chart and use either the test point method or the multiplicity method to mark the sign (+ or -) of $f(x)$ for x in each interval.
6. Mark the "end behavior" of the graph, depending on whether $n < d$, $n = d$, or $n > d$; where n is the degree of the polynomial in the numerator and d is the degree of the polynomial in the denominator.

   Case 1: ($n < d$) The HA is $y = 0$. The end behavior of the graph will look like one of the following 4 pictures: Your inequality sign chart will indicate which one of the four



   you have; but you may have to compute a couple of y values be sure.

   Case 2: ($n = d$) The HA occurs at $y = \frac{a}{b}$. The end behavior will look like one of the four above graphs, but the HA is shifted up or down to $y = \frac{a}{b}$.

   Case 3: ($n > d$) The graph doesnt have a HA, it has a SA at $y = mx + b$, where $mx + b$ is the linear part of the resulting quotient, after the polynomial in the numerator has been divided by the polynomial in the denominator. The end behavior for the graph of the rational function will be the same as the end behavior of the SA. Draw the graph of $y = mx + b$ with a dotted line.
7. Identify and plot the zeros (also called roots) of the rational function.
   - Label these odd (O) or even (E) as in polynomial graphing
   - also plot the y intercept
8. Sketch the vertical asymptotes as vertical dotted lines: Label these VAs as being odd (O) or even (E) at the top of the dotted line depending on their multiplicities.
9. Determine the behavior of the graph (y value) around the VAs.

$$y \to \pm\infty \quad \text{as} \quad x \to a^- \quad \text{and} \quad y \to \pm\infty \quad \text{as} \quad x \to a^+$$

Whether or not y goes to plus or minus infinity depends on the sign of f(x) in that interval.

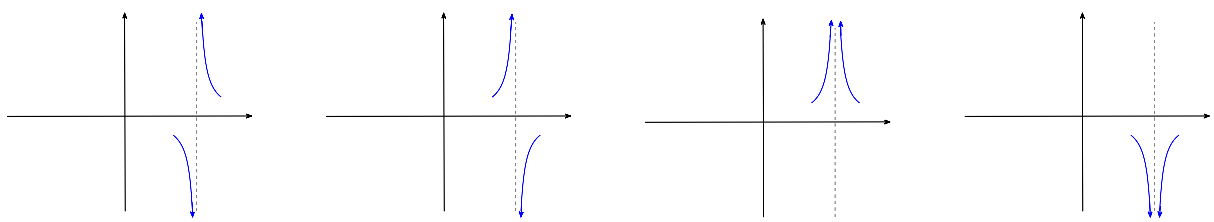
10. Begin graphing from the left end point and proceed to the first critical value
    - a critical value is a zero or a vertical asymptote
    - you may need to make a small table of values for better plotting accuracy.
11. At each zero proceed as in polynomial graphing, ie. If the zero is odd the graph goes thru the x axis; if the zero has even multiplicity, then it touches the axis and bounces off.
12. At each vertical asymptote, as you approach from the left go "north" to $+\infty$ or "south" to $-\infty$.

    At each **odd** vertical asymptote, as you step across the dotted line

    Switch signs from $+\infty$ or to $-\infty$ or from $-\infty$ or to $+\infty$

    At each **even** vertical asymptote, as you step across the dotted line

    Keep the **same signs** from $+\infty$ or to $+\infty$ or from $-\infty$ or to $-\infty$



13. Continue to the next critical value (zero or asymptote) repeat the process of step 8 or 9; ignore the vertical scale as you pass from one zero to the next; item When you have finished this procedure with the last zero on the right, the graph should connect with the right "endpoint" that you marked in step 4
14. As a "check point" you can determine the sign (+ or −) of the y intercept (when $x = 0$) and see if the result is consistent with your graph

Math 176 — Exam 3 **Name:** ____________________

<u>Directions</u>: **NO CALCULATORS OR ANY OTHER ELECTRONIC DEVICES are permitted on this section. Once you turn this section in, you may not have it back.**

1. (4 points) Sketch the graph of $f(x) = 1 - \cos\left(\pi x - \frac{\pi}{2}\right)$

2. (1 point) What is the period of f? 2. ____________

3. (1 point) What is the domain of f? 3. ____________

4. (1 point) What is the range of f? 4. ____________

5. (1 point) Given an angle θ, recite the definition of θ's reference angle, $\acute{\theta}$.

6. (2 points) Find the reference angle $\acute{\theta}$, for the angle $\theta = 540^o$.

6. ___________

7. (3 points) State the Pythagorean identities.

7. ___________

8. (3 points) Determine, without graphing, if the graph of

$$f(x) = 3x^3 - \sin(x)$$

has any symmetry. If it does state which type of symmetry it has.

9. (2 points) Find an angle in the interval $[0, 360°)$ that is coterminal to $\alpha = \dfrac{53\pi}{6}$

9. ___________

10. (2 points) Evaluate $\tan\left(-\frac{7\pi}{3}\right)$

10. ____________

11. Suppose $t = \frac{-7\pi}{2}$

(a) (1 point) Draw the angle t

(b) (1 point) What is the value of $\sin(t)$?

(b) ____________

(c) (1 point) What is the value of $\cos(t)$?

(c) ____________

12. (2 points) Evaluate $\sin^{-1}\left(-\frac{1}{2}\right)$

12. ____________

13. (2 points) Evaluate $\tan^{-1}(0)$

13. ____________

14. (2 points) Evaluate $\tan\left(\cos^{-1}\left(\frac{1}{2}\right)\right)$

14. ____________

15. (4 points) Find the domain and range of $y = 3\csc\left(\frac{2}{5}x - 1\right) - 1$

15. ____________

Math 176 **Test 4** **Name:** ____________________

No Calculators or Computing Devices. Use Algebraic Notation AND Show All of Your Work. No Assistance or Collaboration!

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1. (5 points) Verify the identity: $\dfrac{\sin(x+y)+\sin(x-y)}{\cos(x+y)+\cos(x-y)} = \tan(x)$

2. (4 points) Solve the given equation in the interval $[0, 2\pi)$

$$2\sqrt{3}\cos(\theta)+3=0$$

2. ____________

3. (5 points) Verify the identity: $\dfrac{\sin(3x) + \cos(3x)}{\cos(x) - \sin(x)} = 1 + 2\sin(2x)$

4. (5 points) Solve the given equation in the interval $[0, 2\pi)$

$$2\cos^2(\theta) = \cos(\theta) + 1$$

4. ____________

5. (4 points) Find the exact value of $\cos\left(\dfrac{7\pi}{12}\right)$

5. ____________

6. (5 points) Find the exact value of the expression $\tan\left(2\cos^{-1}\left(\frac{3}{7}\right)\right)$

6. ____________

7. (3 points) Convert the polar point $(r, \theta) = \left(-\sqrt{3}, \frac{2\pi}{3}\right)$ to its equivalent rectangular coordinate.

7. ____________

8. (5 points) Consider the complex number $1 + i\sqrt{3}$. **(a)** Graph the complex number in the complex plane. **(b)** Find the modulus and the argument. **(c)** Write the number in polar form. 8. __________

9. (5 points) Use DeMoivre's Theorem to find $(4 - 4i)^5$

9. __________

10. (5 points) Find the square roots of $4 - 4i$

10. ___________

11. (5 points) Find the length and direction of the vector $\vec{u} = \langle -3, -3 \rangle$

11. ___________

12. (4 points) Consider the parametric curves:

$$x = 1 - t^2, \qquad y = 1 + t$$

(a) Sketch the curve represented by the parametric equations. (b) Find a rectangular coordinate equation for the curve by eliminating the parameter.

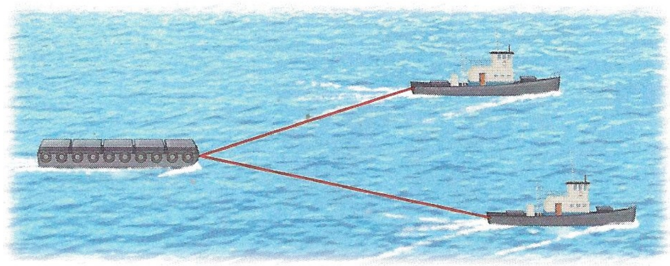
12. ____________

13. (5 points) Suppose the vector $\vec{u}$ has length $\left|\vec{u}\right| = 20$ and direction $\theta = 60^\circ$ are given. Express $\vec{u}$ in component form.

13. ____________

Calculator Section **Name:** ____________________

14. (6 points) Two tugboats are pulling a barge as shown in the figure.



One pulls with a force of 2.0 $\times 10^4$ lb. in the direction N 50° E, and the other pulls with a force of 3.4 $\times 10^4$ lb. in the direction S 75° E.

a.) Find the resultant force on the barge as a vector.

b.) Find the magnitude and direction of the resultant force.

14. __________

Math 176 Test 5 Name: ____________________

No Calculators or Computing Devices allowed! Use Algebraic Notation AND Show All of Your Work.

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1. (6 points) Use Gaussian elimination to find the complete solution of the system, or show that no solution exists.

$$\begin{cases} x - y + 2z = 0 \\ 2x - 4y + 5z = -5 \\ 2y - 3z = 5 \end{cases}$$

1. ____________

2. (a) (2 points) Write a matrix equation equivalent to the following system.

$$\begin{cases} 4x - 3y = 10 \\ 3x - 2y = 30 \end{cases}$$

(a) ____________

(b) (4 points) Find the inverse of the coefficient matrix, and use it to solve the system.

(b) ____________

3. (5 points) Solve $\left\{\begin{array}{r} x - y = 1 \\ 4x + 3y = 18 \end{array}\right\}$ using Cramer's Rule.

3. ____________

4. Let $A = \begin{bmatrix} 1 & -5 \\ -3 & 7 \end{bmatrix}$, $\quad B = \begin{bmatrix} -2 & -6 \\ 2 & 7 \\ 1 & 0 \end{bmatrix}$, $\quad C = \begin{bmatrix} 1 & 3 & 1 \\ -2 & 7 & 2 \\ 0 & 2 & 4 \end{bmatrix}$

Carry out the indicated operation, or <u>explain, using complete sentences, why</u> it cannot be performed.

(a) (2 points) $\quad A + B$

(b) (2 points) $\quad AB$

(c) (2 points) $\quad BA - 3A$

(d) (2 points) $\quad B^{-1}$

(e) (2 points) $\quad \det(B)$

5. (6 points) Find the inverse of $C = \begin{bmatrix} 1 & 0 & 4 \\ -1 & 1 & 2 \\ 0 & 1 & 3 \end{bmatrix}$ if it exists.

5. ___________

6. (6 points) Find the partial fraction decomposition of $\dfrac{2x-3}{x^3+3x}$.

6. ______________

7. Only one of the following two matrices has an inverse.

$$A = \begin{bmatrix} -2 & 5 & -2 \\ 0 & 7 & 0 \\ -2 & 1 & -2 \end{bmatrix}, \qquad B = \begin{bmatrix} 4 & 5 & 6 \\ 2 & 7 & 1 \\ -7 & 1 & -3 \end{bmatrix}$$

(a) (5 points) Find the determinant of each matrix. (a) ____________

(b) (1 point) Use the determinants from part (a) to identify which matrix has an inverse.

(b) ____________

8. (3 points) Find the first three terms of the sequence $a_n = 2n^2 - 1$

8. ____________

9. (3 points) Find the third partial sum of the sequence $a_n = 2n^2 - 1$

9. ____________

10. (3 points) A sequence is defined recursively by $a_{n+1} = 2a_n - 3n$, with $a_1 = 2$. Find the first 4 terms of the sequence.

10. ____________

11. An arithmetic sequence begins with 6, 13, 20, 27, ...

(a) (1 point) Find the common difference, d, for this sequence.

(a) ____________

(b) (2 points) Find a formula for the n^{th} term, a_n, of the sequence.

(b) ____________

(c) (2 points) Find the 36^{th} term, a_{36}, of the sequence.

(c) ____________

12. A geometric sequence begins with 12, 3, 3/4, 3/16, 3/64, ...

(a) (1 point) Find the common ratio, r, for this sequence.

(a) ____________

(b) (2 points) Find a formula for the n^{th} term, a_n, of the sequence.

(b) ____________

(c) (2 points) Find the 10^{th} term, a_{10}, of the sequence.

(c) ____________

13. (6 points) Expand $(2x-1)^4$

14. (6 points) Express the repeating decimal $0.\overline{051}$ as a fraction in lowest terms.

14. ____________

15. (4 points) Write the sum using sigma notation. Do not evaluate.

$$3 + 6 + 9 + 12 + \cdots + 99$$

16. (3 points) ***EXTRA CREDIT*** Find the sum $\sum_{k=3}^{5} (k+1)^2$