Chapter 4

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Definition

Probability is a measure or estimation of how likely it is that something will happen or that a statement is true. Probabilities are given a value between 0 (0% chance or will not happen) and 1 (100% chance or will happen).

Common Notation

P	denotes a probability
A, B, C, E_1, E_2	notation for specific events
$P(A)$	notation for the probability of event A occurring
$P(E_1)$	notation for the probability of event E_1 occurring

Events and Simple Events

Definition

An **event** is an outcome of an experiment or procedure.

Experiment: Toss a single die and observe the number that appears on the upper face. Here are some possible events:

Event A	Observe an even number
Event B	Observe a number less than 3
Event E_1	Observe a 1
Event E_2	Observe a 2
Event E_3	Observe a 3
Event E_4	Observe a 4
Event E_5	Observe a 5
Event E_6	Observe a 6

Events and Simple Events

Definition

Two events are **mutually exclusive (or called disjoint)** if, when one event occurs, the other cannot, and vice versa.

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Observations:

- Events A and B are *not mutually exclusive* because both events occur when the number on the upper face of the die is a 2.

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- Events A and B are *not mutually exclusive* because both events occur when the number on the upper face of the die is a 2.
- Since event A occurs whenever the upper face is 2, 4, or 6, event A can be decomposed into a collection of simpler events—namely, E_2, E_4, and E_6—which are themselves *mutually exclusive*.

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- Similarly, event B can be decomposed into the collection of simple events $\{E_1, E_2\}$.

Experiment: Toss a single die

Event A	Observe an even number
Event B	Observe a number less than 3
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An event that cannot be decomposed is called a **simple event**.

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- Simple events are mutually exclusive.

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The sample space, S, for our experiment is

$$S = \{⚀\ ⚁\ ⚂\ ⚃\ ⚄\ ⚅\}$$
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Experiment: Toss a single die

Event	Probability
E_1	1/6
E_2	1/6
E_3	1/6
E_4	1/6
E_5	1/6
E_6	1/6

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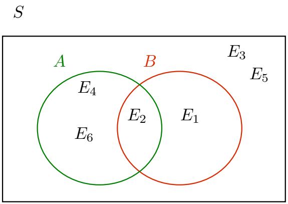
$$\sum_{i=1}^{6} P(E_i) = \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} = 1$$

It is often helpful to visualize an experiment using a **Venn Diagram**, (right). The outer box represents the sample space, which contains all of the mutually exclusive, simple events.



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It is often helpful to visualize an experiment using a **Venn Diagram**, (right). The outer box represents the sample space, which contains all of the mutually exclusive, simple events.

Event A is the circled collection of simple events, $\{E_2, E_4, E_6\}$.

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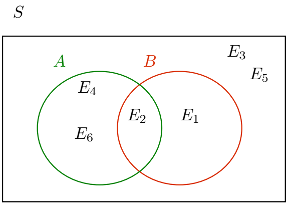

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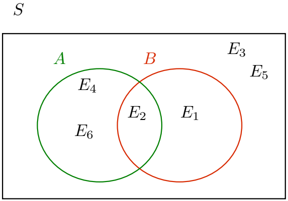

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$S = \{⚀ ⚁ ⚂ ⚃ ⚄ ⚅\}$
$= \{E_1, E_2, E_3, E_4, E_5, E_6\}$

Events A and B are called **compound events** because they are events combining two or more simple events.

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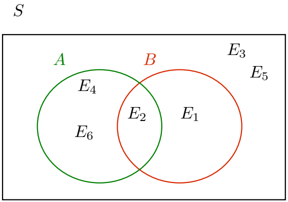

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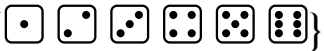
$S = \{$  $\}$

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$$
\begin{aligned}
P(A) &= P(E_2 \text{ or } E_4 \text{ or } E_6) \\
&= P(E_2) + P(E_4) + P(E_6) \\
&= \frac{1}{6} + \frac{1}{6} + \frac{1}{6} \\
&= \frac{3}{6} = 0.5
\end{aligned}
$$

Suppose a couple plans to have three children. Assume that girls and boys are equally likely and that the gender of one child is not influenced by the gender of any other child. What is the sample space, or set of all possible outcomes?

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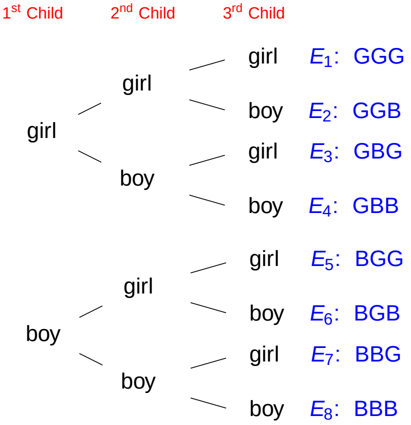

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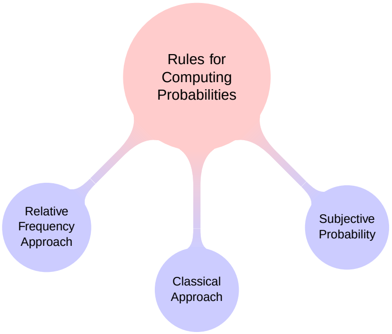
Computing Probabilities



Computing Probabilities

Definition (**The Classical Approach**)

Assume that a given procedure has n different simple events and that each of those simple events has an equal chance of occurring. If event A can occur in s of these n ways, then

$$P(A) = \frac{\#\text{of ways A can occur}}{\#\text{of different simple events}} = \frac{s}{n}$$

Example: Toss a single die. Determine the following probabilities:

1. $P(E_1)$
2. $P(E_5)$
3. $P(A)$
4. $P(B)$

Computing Probabilities

Definition (The Relative Frequency Approach)

Conduct (or observe) a procedure, and count the number of times event A actually occurs. Based on these actual results, P(A) is approximated as

$$P(A) = \frac{\#\text{of times A occurred}}{\#\text{of times procedure was repeated}}$$

Example: When trying to determine the probability that an individual car crashes in a year, we must examine past results to determine the number of cars in use in a year and the number of them that crashed, then find the ratio of the two.[**?**]

$$P(crash) = \frac{\#\text{of times cars that crashed}}{\text{total }\#\text{of cars}} = \frac{6,511,100}{135,670,000} = 0.0480$$

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Theorem (**Law of Large Numbers**)

As a procedure is repeated again and again, the relative frequency probability of an event tends to approach the actual probability.

Computing Probabilities

Definition (**Subjective Probability**)

$P(A)$, the probability of event A, is estimated by using knowledge of the relevant circumstances.

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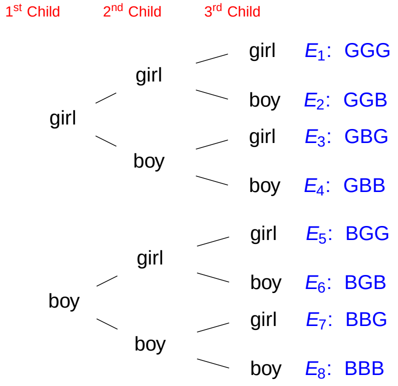
Experiment: Roll a pair of dice. Record the sum of the two numbers that appear on the upper faces of the dice.

Roll							Probability
2	⚀⚀						$\frac{1}{36}$
3	⚀⚁	⚁⚀					$\frac{2}{36}$
4	⚀⚂	⚂⚀	⚁⚁				$\frac{3}{36}$
5	⚀⚃	⚃⚀	⚂⚁	⚁⚂			$\frac{4}{36}$
6	⚀⚄	⚄⚀	⚃⚁	⚁⚃	⚂⚂		$\frac{5}{36}$
7	⚀⚅	⚅⚀	⚄⚁	⚁⚄	⚃⚂	⚂⚃	$\frac{6}{36}$
8	⚁⚅	⚅⚁	⚄⚂	⚂⚄	⚃⚃		$\frac{5}{36}$
9	⚂⚅	⚅⚂	⚄⚃	⚃⚄			$\frac{4}{36}$
10	⚃⚅	⚅⚃	⚄⚄				$\frac{3}{36}$
11	⚄⚅	⚅⚄					$\frac{2}{36}$
12	⚅⚅						$\frac{1}{36}$

Determine the following probabilities:

- P(the sum is 8)
- P(rolling a double 1)

Find the probability that when a couple has three children, they will have exactly 2 girls. Assume that girls and boys are equally likely and that the gender of one child is not influenced by the gender of any other child. [**?**]



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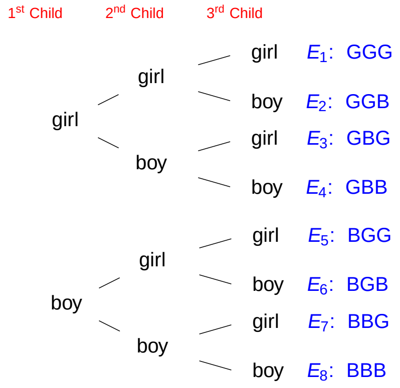

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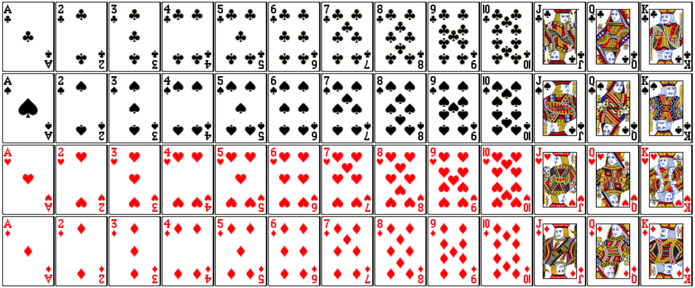
Classical Approach

$$P(2 \text{ boys in 3 births}) = \frac{\#\text{of ways A can occur}}{\#\text{of different simple events}} = \frac{3}{8} = 0.375$$

Experiment: Pick a card at random from a shuffled deck of cards.



Determine the following probabilities:

- P(the card is a four of hearts)
- P(the card is a queen)
- P(the card is not an ace)

Computing Probabilities

Example: In the last 30 years, death sentence executions in the United States included 795 men and 10 women (based on data from the Associated Press). If an execution is randomly selected, find the probability that the person executed is a women. Is it unusual for a woman to be executed?

We use the **relative frequency** approach here, since the likelihood that a women or man is executed is not the same.

Complementary Events

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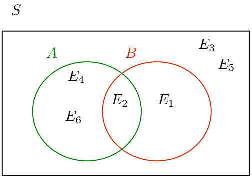

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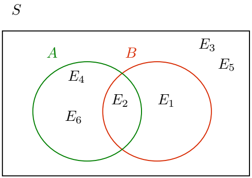
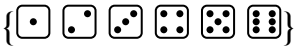


For the single die experiment, this means

Event $\overline{A}$	observe an odd number
Event $\overline{B}$	observe a number greater than or equal to 3
Event $\overline{E_2}$	observe any number in S except 2

$S = \{$  $\}$

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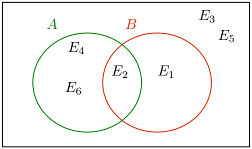

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A fundamental property of complementary events may now be apparent to you:

$$P(A) + P(\overline{A}) = 1$$

the sum of the probabilities of an event and its complement is always one (regardless of whether an event is simple or compound).

Experiment: Toss a single die

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Concept Check

Question: Suppose A is any event, either simple or compound. Are the events A and A complement mutually exclusive?

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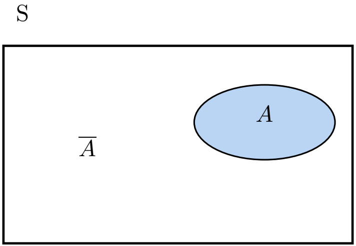

Yes they are!!!

Complementary Events

Example: Women have a 0.25% rate of red/green color blindness. If a women is randomly selected, what is the *probability* that she does *not* have red/green color blindness?

The Rare Event Rule

Theorem (The Rare Event Rule)

If, under a given assumption, the probability of a particular observed event is extremely small, we conclude that the assumption is probably not correct.

Example: Sally thinks there is no way she can get an A on Mr. Busken's first stats exam. Then she aces the exam. By the rare event rule, her assumption must have been incorrect.

See example 12, p146 in the text for another example.

Experiment: Toss a single die

Event A	Observe an even number
Event B	Observe a number less than 3
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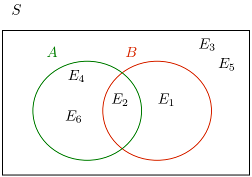
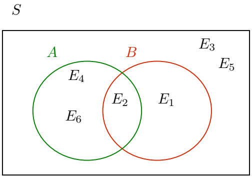


S = {⚀ ⚁ ⚂ ⚃ ⚄ ⚅}

$= \{E_1, E_2, E_3, E_4, E_5, E_6\}$

Experiment: Toss a single die

Event A	Observe an even number
Event B	Observe a number less than 3
Event E_1	Observe a 1
Event E_2	Observe a 2
Event E_3	Observe a 3
Event E_4	Observe a 4
Event E_5	Observe a 5
Event E_6	Observe a 6



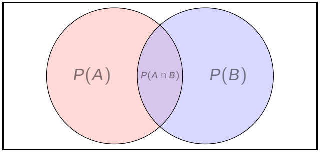
$S = \{$⚀ ⚁ ⚂ ⚃ ⚄ ⚅$\}$

$= \{E_1, E_2, E_3, E_4, E_5, E_6\}$

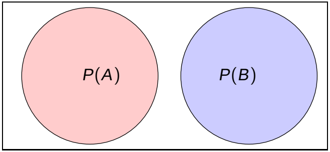
Definition (The Addition Rule)

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$



Venn Diagram for Events that are not mutually exclusive

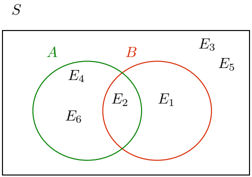


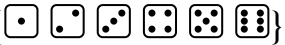
Venn Diagram for mutually exclusive events $P(A \cap B) = 0$

Problem #12: Determine $P(A \text{ or } B)$

Experiment: Toss a single die

Event A	Observe an even number
Event B	Observe a number less than 3
Event E_1	Observe a 1
Event E_2	Observe a 2
Event E_3	Observe a 3
Event E_4	Observe a 4
Event E_5	Observe a 5
Event E_6	Observe a 6



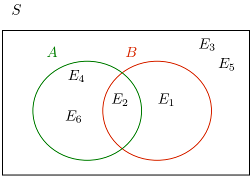
$S = \{$  $\}$

$= \{E_1, E_2, E_3, E_4, E_5, E_6\}$

Problem #12: Determine $P(A \text{ or } B)$

Experiment: Toss a single die

Event A	Observe an even number
Event B	Observe a number less than 3
Event E_1	Observe a 1
Event E_2	Observe a 2
Event E_3	Observe a 3
Event E_4	Observe a 4
Event E_5	Observe a 5
Event E_6	Observe a 6



$S = \{$⚀ ⚁ ⚂ ⚃ ⚄ ⚅$\}$

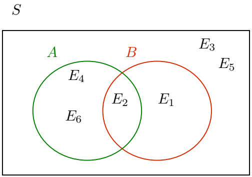
$= \{E_1, E_2, E_3, E_4, E_5, E_6\}$

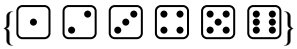
$P(A \cup B) = P($ observe an even number OR observe a number less than 3 $)$

Problem #12: Determine $P(A \text{ or } B)$

Experiment: Toss a single die

Event A	Observe an even number
Event B	Observe a number less than 3
Event E_1	Observe a 1
Event E_2	Observe a 2
Event E_3	Observe a 3
Event E_4	Observe a 4
Event E_5	Observe a 5
Event E_6	Observe a 6



$S =$ 

$= \{E_1, E_2, E_3, E_4, E_5, E_6\}$

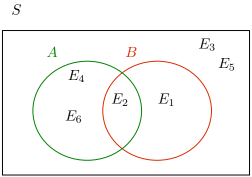
$$P(A \cup B) = P(\text{ observe an even number OR observe a number less than 3 })$$

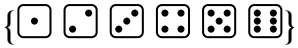
$$= P(A) + P(B) - P(A \text{ and } B)$$

Problem #12: Determine $P(A \text{ or } B)$

Experiment: Toss a single die

Event A	Observe an even number
Event B	Observe a number less than 3
Event E_1	Observe a 1
Event E_2	Observe a 2
Event E_3	Observe a 3
Event E_4	Observe a 4
Event E_5	Observe a 5
Event E_6	Observe a 6



$S =$ 

$$= \{E_1, E_2, E_3, E_4, E_5, E_6\}$$

$$P(A \cup B) = P(\text{ observe an even number OR observe a number less than 3 })$$

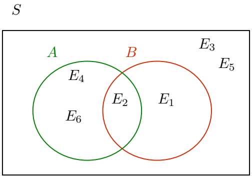
$$= P(A) + P(B) - P(A \text{ and } B)$$

$$= \frac{3}{6} + \frac{2}{6} - \frac{1}{6}$$

Problem #12: Determine $P(A \text{ or } B)$

Experiment: Toss a single die

Event A	Observe an even number
Event B	Observe a number less than 3
Event E_1	Observe a 1
Event E_2	Observe a 2
Event E_3	Observe a 3
Event E_4	Observe a 4
Event E_5	Observe a 5
Event E_6	Observe a 6



$S = \{$⚀ ⚁ ⚂ ⚃ ⚄ ⚅$\}$

$= \{E_1, E_2, E_3, E_4, E_5, E_6\}$

$$P(A \cup B) = P(\text{ observe an even number OR observe a number less than 3 })$$

$$= P(A) + P(B) - P(A \text{ and } B)$$

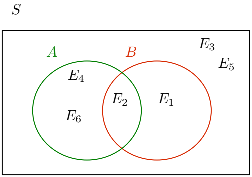
$$= \frac{3}{6} + \frac{2}{6} - \frac{1}{6}$$

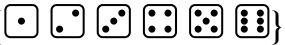
$$= \frac{4}{6} \doteq 0.67$$

Problem #12: Determine $P(A \text{ or } B)$

Experiment: Toss a single die

Event A	Observe an even number
Event B	Observe a number less than 3
Event E_1	Observe a 1
Event E_2	Observe a 2
Event E_3	Observe a 3
Event E_4	Observe a 4
Event E_5	Observe a 5
Event E_6	Observe a 6



$S =$ {  }

$= \{E_1, E_2, E_3, E_4, E_5, E_6\}$

Alternatively, $A \cup B \equiv \{E_1, E_2, E_4, E_6\}$, so $P(A \cup B) = \dfrac{4}{6} \doteq 0.67$ using the classical approach.

Roll							Probability
2	⚀⚀						$\frac{1}{36}$
3	⚀⚁	⚁⚀					$\frac{2}{36}$
4	⚀⚂	⚂⚀	⚁⚁				$\frac{3}{36}$
5	⚀⚃	⚃⚀	⚂⚁	⚁⚂			$\frac{4}{36}$
6	⚀⚄	⚄⚀	⚃⚁	⚁⚃	⚂⚂		$\frac{5}{36}$
7	⚀⚅	⚅⚀	⚄⚁	⚁⚄	⚃⚂	⚂⚃	$\frac{6}{36}$
8	⚁⚅	⚅⚁	⚄⚂	⚂⚄	⚃⚃		$\frac{5}{36}$
9	⚂⚅	⚅⚂	⚄⚃	⚃⚄			$\frac{4}{36}$
10	⚃⚅	⚅⚃	⚄⚄				$\frac{3}{36}$
11	⚄⚅	⚅⚄					$\frac{2}{36}$
12	⚅⚅						$\frac{1}{36}$

Problem #13: Let A be the event the observed pair sums to 10 and let B be the event the observed pair is a double. Determine $P(A \cup B)$.

Roll							Probability
2	⚀⚀						$\frac{1}{36}$
3	⚀⚁	⚁⚀					$\frac{2}{36}$
4	⚀⚂	⚂⚀	⚁⚁				$\frac{3}{36}$
5	⚀⚃	⚃⚀	⚂⚁	⚁⚂			$\frac{4}{36}$
6	⚀⚄	⚄⚀	⚃⚁	⚁⚃	⚂⚂		$\frac{5}{36}$
7	⚀⚅	⚅⚀	⚄⚁	⚁⚄	⚃⚂	⚂⚃	$\frac{6}{36}$
8	⚁⚅	⚅⚁	⚄⚂	⚂⚄	⚃⚃		$\frac{5}{36}$
9	⚂⚅	⚅⚂	⚄⚃	⚃⚄			$\frac{4}{36}$
10	⚃⚅	⚅⚃	⚄⚄				$\frac{3}{36}$
11	⚄⚅	⚅⚄					$\frac{2}{36}$
12	⚅⚅						$\frac{1}{36}$

Problem #13: Let A be the event the observed pair sums to 10 and let B be the event the observed pair is a double. Determine $P(A \cup B)$.

$$P(A \cup B) = P(A) + P(B) - P(A \text{ and } B)$$

Roll		Probability
2	⚀⚀	$\frac{1}{36}$
3	⚀⚁ ⚁⚀	$\frac{2}{36}$
4	⚀⚂ ⚂⚀ ⚁⚁	$\frac{3}{36}$
5	⚀⚃ ⚃⚀ ⚂⚁ ⚁⚂	$\frac{4}{36}$
6	⚀⚄ ⚄⚀ ⚃⚁ ⚁⚃ ⚂⚂	$\frac{5}{36}$
7	⚀⚅ ⚅⚀ ⚄⚁ ⚁⚄ ⚃⚂ ⚂⚃	$\frac{6}{36}$
8	⚁⚅ ⚅⚁ ⚄⚂ ⚂⚄ ⚃⚃	$\frac{5}{36}$
9	⚂⚅ ⚅⚂ ⚄⚃ ⚃⚄	$\frac{4}{36}$
10	⚃⚅ ⚅⚃ ⚄⚄	$\frac{3}{36}$
11	⚄⚅ ⚅⚄	$\frac{2}{36}$
12	⚅⚅	$\frac{1}{36}$

Problem #13: Let A be the event the observed pair sums to 10 and let B be the event the observed pair is a double. Determine $P(A \cup B)$.

$$P(A \cup B) = P(A) + P(B) - P(A \text{ and } B)$$

$$= \frac{3}{36} + \frac{6}{36} - \frac{1}{36}$$

Roll							Probability
2	⚀⚀						$\frac{1}{36}$
3	⚀⚁	⚁⚀					$\frac{2}{36}$
4	⚀⚂	⚂⚀	⚁⚁				$\frac{3}{36}$
5	⚀⚃	⚃⚀	⚂⚁	⚁⚂			$\frac{4}{36}$
6	⚀⚄	⚄⚀	⚃⚁	⚁⚃	⚂⚂		$\frac{5}{36}$
7	⚀⚅	⚅⚀	⚄⚁	⚁⚄	⚃⚂	⚂⚃	$\frac{6}{36}$
8	⚁⚅	⚅⚁	⚄⚂	⚂⚄	⚃⚃		$\frac{5}{36}$
9	⚂⚅	⚅⚂	⚄⚃	⚃⚄			$\frac{4}{36}$
10	⚃⚅	⚅⚃	⚄⚄				$\frac{3}{36}$
11	⚄⚅	⚅⚄					$\frac{2}{36}$
12	⚅⚅						$\frac{1}{36}$

Problem #13: Let A be the event the observed pair sums to 10 and let B be the event the observed pair is a double. Determine $P(A \cup B)$.

$$P(A \cup B) = P(A) + P(B) - P(A \text{ and } B)$$

$$= \frac{3}{36} + \frac{6}{36} - \frac{1}{36}$$

$$= \frac{8}{36} \doteq 0.22$$

Roll							Probability
2	⚀⚀						$\frac{1}{36}$
3	⚀⚁	⚁⚀					$\frac{2}{36}$
4	⚀⚂	⚂⚀	⚁⚁				$\frac{3}{36}$
5	⚀⚃	⚃⚀	⚂⚁	⚁⚂			$\frac{4}{36}$
6	⚀⚄	⚄⚀	⚃⚁	⚁⚃	⚂⚂		$\frac{5}{36}$
7	⚀⚅	⚅⚀	⚄⚁	⚁⚄	⚃⚂	⚂⚃	$\frac{6}{36}$
8	⚁⚅	⚅⚁	⚄⚂	⚂⚄	⚃⚃		$\frac{5}{36}$
9	⚂⚅	⚅⚂	⚄⚃	⚃⚄			$\frac{4}{36}$
10	⚃⚅	⚅⚃	⚄⚄				$\frac{3}{36}$
11	⚄⚅	⚅⚄					$\frac{2}{36}$
12	⚅⚅						$\frac{1}{36}$

Problem #14: Let A be the event the observed pair sums to 10 and let B be the event the observed pair sums to 4. Determine $P(A \cup B)$.

Roll							Probability
2	⚀⚀						$\frac{1}{36}$
3	⚀⚁	⚁⚀					$\frac{2}{36}$
4	⚀⚂	⚂⚀	⚁⚁				$\frac{3}{36}$
5	⚀⚃	⚃⚀	⚂⚁	⚁⚂			$\frac{4}{36}$
6	⚀⚄	⚄⚀	⚃⚁	⚁⚃	⚂⚂		$\frac{5}{36}$
7	⚀⚅	⚅⚀	⚄⚁	⚁⚄	⚃⚂	⚂⚃	$\frac{6}{36}$
8	⚁⚅	⚅⚁	⚄⚂	⚂⚄	⚃⚃		$\frac{5}{36}$
9	⚂⚅	⚅⚂	⚄⚃	⚃⚄			$\frac{4}{36}$
10	⚃⚅	⚅⚃	⚄⚄				$\frac{3}{36}$
11	⚄⚅	⚅⚄					$\frac{2}{36}$
12	⚅⚅						$\frac{1}{36}$

Problem #14: Let A be the event the observed pair sums to 10 and let B be the event the observed pair sums to 4. Determine $P(A \cup B)$.

$$P(A \cup B) = P(A) + P(B) - P(A \text{ and } B)$$

Roll							Probability
2	⚀⚀						$\frac{1}{36}$
3	⚀⚁	⚁⚀					$\frac{2}{36}$
4	⚀⚂	⚂⚀	⚁⚁				$\frac{3}{36}$
5	⚀⚃	⚃⚀	⚂⚁	⚁⚂			$\frac{4}{36}$
6	⚀⚄	⚄⚀	⚃⚁	⚁⚃	⚂⚂		$\frac{5}{36}$
7	⚀⚅	⚅⚀	⚄⚁	⚁⚄	⚃⚂	⚂⚃	$\frac{6}{36}$
8	⚁⚅	⚅⚁	⚄⚂	⚂⚄	⚃⚃		$\frac{5}{36}$
9	⚂⚅	⚅⚂	⚄⚃	⚃⚄			$\frac{4}{36}$
10	⚃⚅	⚅⚃	⚄⚄				$\frac{3}{36}$
11	⚄⚅	⚅⚄					$\frac{2}{36}$
12	⚅⚅						$\frac{1}{36}$

Problem #14: Let A be the event the observed pair sums to 10 and let B be the event the observed pair sums to 4. Determine $P(A \cup B)$.

$$P(A \cup B) = P(A) + P(B) - P(A \text{ and } B)$$

$$= \frac{3}{36} + \frac{3}{36} - \frac{0}{36}$$

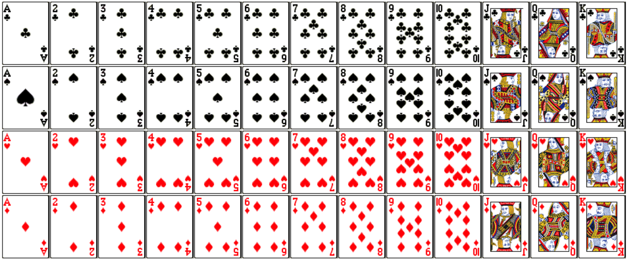
Roll		Probability
2	⚀⚀	$\frac{1}{36}$
3	⚀⚁ ⚁⚀	$\frac{2}{36}$
4	⚀⚂ ⚁⚁ ⚂⚀	$\frac{3}{36}$
5	⚀⚃ ⚃⚀ ⚂⚁ ⚁⚂	$\frac{4}{36}$
6	⚀⚄ ⚄⚀ ⚃⚁ ⚁⚃ ⚂⚂	$\frac{5}{36}$
7	⚀⚅ ⚅⚀ ⚄⚁ ⚁⚄ ⚃⚂ ⚂⚃	$\frac{6}{36}$
8	⚁⚅ ⚅⚁ ⚄⚂ ⚂⚄ ⚃⚃	$\frac{5}{36}$
9	⚂⚅ ⚅⚂ ⚄⚃ ⚃⚄	$\frac{4}{36}$
10	⚃⚅ ⚅⚃ ⚄⚄	$\frac{3}{36}$
11	⚄⚅ ⚅⚄	$\frac{2}{36}$
12	⚅⚅	$\frac{1}{36}$

Problem #14: Let A be the event the observed pair sums to 10 and let B be the event the observed pair sums to 4. Determine $P(A \cup B)$.

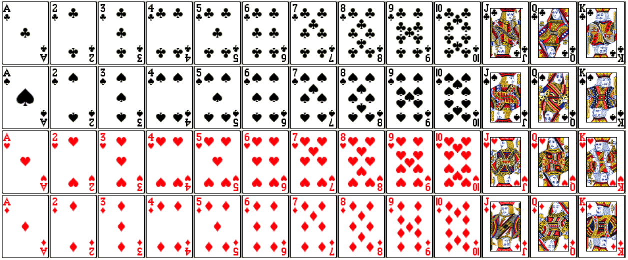
$$P(A \cup B) = P(A) + P(B) - P(A \text{ and } B)$$

$$= \frac{3}{36} + \frac{3}{36} - \frac{0}{36}$$

$$= \frac{6}{36} \doteq 0.17$$

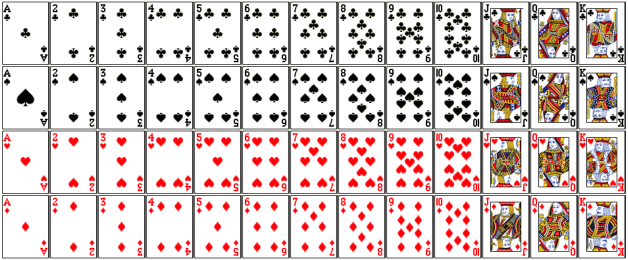


Problem #15: Pick a card at random from a shuffled deck. Let A be the event the observed card is a 4 and let B be the event the card is a heart. Determine $P(A \cup B)$.



Problem #15: Pick a card at random from a shuffled deck. Let A be the event the observed card is a 4 and let B be the event the card is a heart. Determine $P(A \cup B)$.

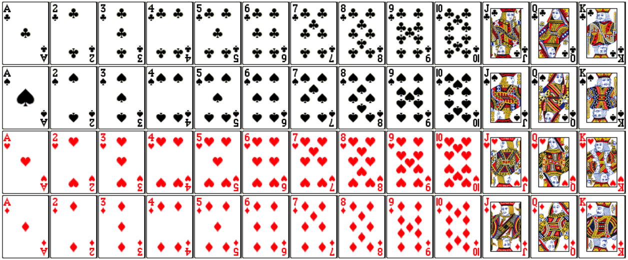
$$P(A \cup B) = P(A) + P(B) - P(A \text{ and } B)$$



Problem #15: Pick a card at random from a shuffled deck. Let A be the event the observed card is a 4 and let B be the event the card is a heart. Determine $P(A \cup B)$.

$$P(A \cup B) = P(A) + P(B) - P(A \text{ and } B)$$

$$= \frac{4}{52} + \frac{3}{52} - \frac{1}{52}$$

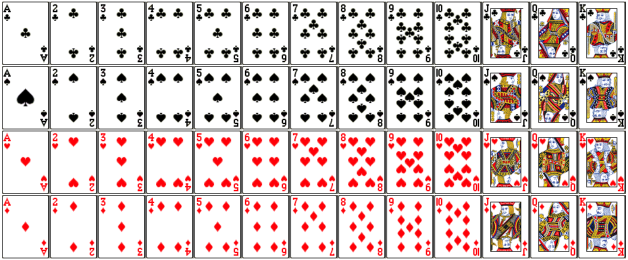


Problem #15: Pick a card at random from a shuffled deck. Let A be the event the observed card is a 4 and let B be the event the card is a heart. Determine $P(A \cup B)$.

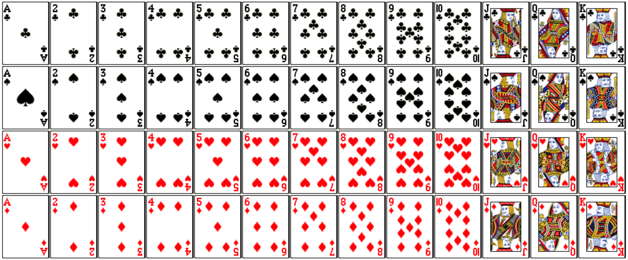
$$P(A \cup B) = P(A) + P(B) - P(A \text{ and } B)$$

$$= \frac{4}{52} + \frac{3}{52} - \frac{1}{52}$$

$$= \frac{16}{52} \doteq 0.31$$

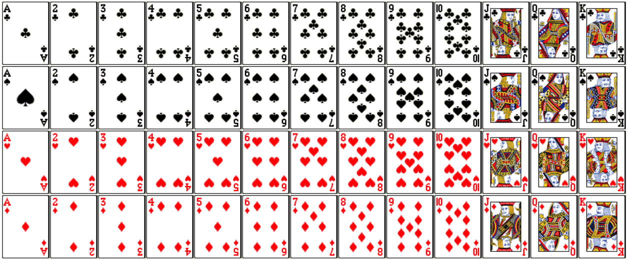


<u>Problem #16</u>: Let A be the event the observed card is a 4 and let B be the event the card is a 10. Determine $P(A \cup B)$.



Problem #16: Let A be the event the observed card is a 4 and let B be the event the card is a 10. Determine $P(A \cup B)$.

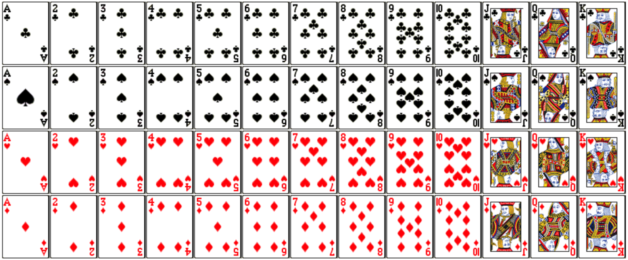
$$P(A \cup B) = P(A) + P(B) - P(A \text{ and } B)$$



Problem #16: Let A be the event the observed card is a 4 and let B be the event the card is a 10. Determine $P(A \cup B)$.

$$P(A \cup B) = P(A) + P(B) - P(A \text{ and } B)$$

$$= \frac{4}{52} + \frac{4}{52} - \frac{0}{52}$$



Problem #16: Let A be the event the observed card is a 4 and let B be the event the card is a 10. Determine $P(A \cup B)$.

$$
\begin{aligned}
P(A \cup B) &= P(A) + P(B) - P(A \text{ and } B) \\
&= \frac{4}{52} + \frac{4}{52} - \frac{0}{52} \\
&= \frac{8}{52} \doteq 0.15
\end{aligned}
$$

The Multiplication Rule

Notation

$P(B|A)$ represents the probability of event B occurring after it is assumed that event A has already occurred (read $B|A$ as "B given A").

Definition

Two events A and B are **independent** if the occurrence of one does not affect the probability of the occurrence of the other. If A and B are not independent, they are said to be **dependent** .

Definition

Two events A and B are said to be **independent** if and only if either

$$P(B|A) = P(B) \qquad \text{or} \qquad P(A|B) = P(A)$$

Theorem (**The Multiplication Rule**)

$$P(A \text{ and } B) = P(A) \cdot P(B) \qquad \textit{(if A and B are independent)}$$

$$P(A \text{ and } B) = P(A) \cdot P(B|A) \qquad \textit{(if A and B are dependent)}$$

Key Concept

The basic multiplication rule is used for finding P(A and B), the probability that event A occurs in a first trial and event B occurs in a second trial.

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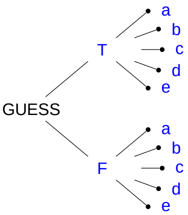

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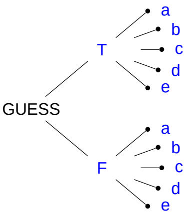

Notice that the notation P(both correct) is equivalent to P(the first answer is correct AND the second answer is correct). The sample space,

$$S = \{Ta, Tb, Tc, Td, Te, Fa, Fb, Fc, Fd, Fe\},$$

has 10 simple events.

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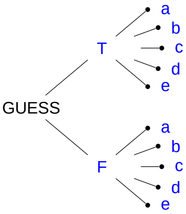

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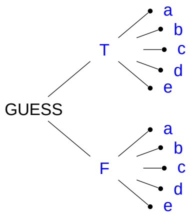

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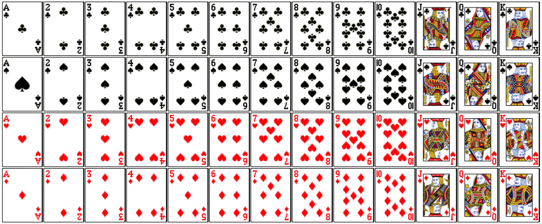
Suppose the correct answers are T and c. We can also obtain the correct probability by multiplying the individual probabilities:

$$P(\text{both correct}) = P(T \text{ and } c)$$
$$= P(T) \cdot P(c) = \frac{1}{2} \cdot \frac{1}{5} = \frac{1}{10} = 0.1$$

Experiment: Now pick two cards at random from a shuffled deck of playing cards.



Example: Two cards are randomly selected ***without replacement***. Find the probability the first card is an ace and the second card is an ten.

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Example: Two cards are randomly selected. Find the probability that the draw includes and ace and a ten.

Applying the Multiplication Rule

Start

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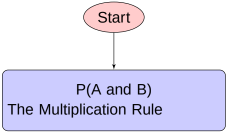

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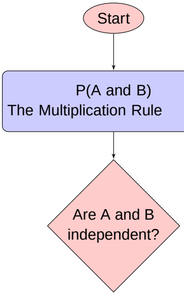

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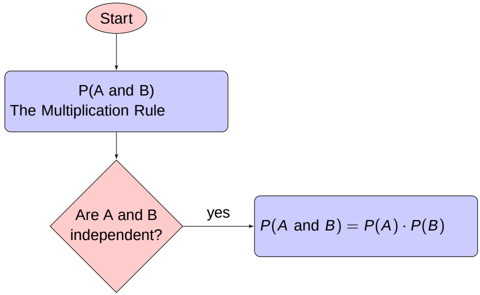

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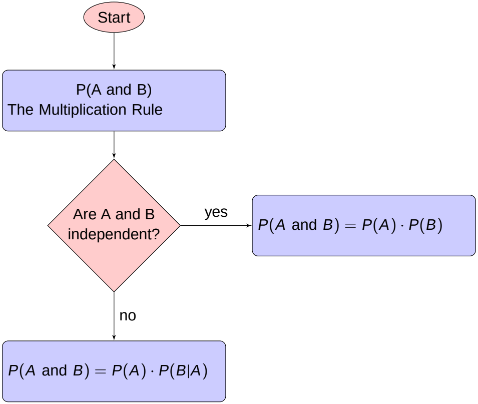


Homework #17: Use the data in the following table, which summarizes blood type and Rh types for 100 subjects.

		Blood Type			
		O	A	B	AB
Rh Type	Rh^+	39	35	8	4
	Rh^-	5	6	2	1

If 2 out of the 100 subjects are randomly selected, find the probability that they are both blood group O and Rh type Rh^+.

1. Assume that the selections are made with replacement.
2. Assume that the selections are made without replacement.

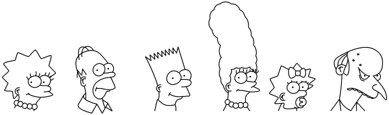
Homework #22: With one method of a procedure called acceptance sampling, a sample of items is randomly selected without rplacement and the entire batch is accepted if every item in the sample is okay. The Telektronics Company manufactured a batch of 400 back up power supply units for computers, and 8 of them are defective. If 3 of the units are randomly selected for testing, what is the probability that the entire batch will be accepted?

Key Concept

In this section, we extend our multiplication rule to the two special applications:

1. **Determine the Probability of "at least one"**

2. **Conditional probability**



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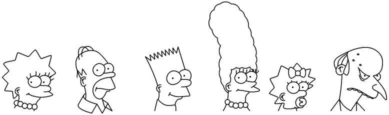

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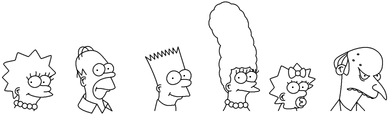
1. **Determine the Probability of "at least one"**: Find the probability that among several trials, we get at least one of some specified event.

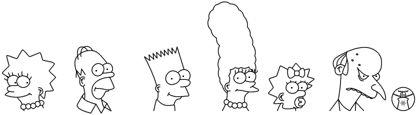
2. **Conditional probability**: Find the probability of an event when we have additional information that some other event has already occurred.



The Probability of "at least one"

- "at least one" is equivalent to "one or more."

- The **complement** of getting at least one item of a particular type is that you get **no** items of that type.



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❹ Finally, $P(A) = 1 - P(\overline{A}) = 1 - 0.0625 = 0.9375$.

Worksheet

☞ **Example**: A study conducted at a certain college shows that 59% of the school's graduates find a job in their chosen field within a year after graduation. Find the probability that among 6 randomly selected graduates, at least one finds a job in his or her chosen field within a year of graduating.

☞ **Example**: In a batch of 8,000 clock radios 6% are defective. A sample of 8 clock radios is randomly selected without replacement from the 8,000 and tested. The entire batch will be rejected if at least one of those tested is defective. What is the probability that the entire batch will be rejected?

Conditional Probability

Definition

A **conditional probability** of an event is a probability obtained with the additional information that some other event has already occurred. $P(B|A)$ denotes the conditional probability of event B occurring, given that event A has already occurred, and it can be found by dividing the probability of events A and B both occurring by the probability of event A:

$$P(B|A) = \frac{P(A \text{ and } B)}{P(A)}$$

Table 4 - 1 Results from Experiments with Polygraph Instruments

	No (Did Not Lie)	Yes (Lied)
Positive Test Result (The polygraph test indicated that the subject *lied*.)	15 (false positive)	42 (true positive)
Negative Test Result (The polygraph test indicated that the subject did not *lie*.)	32 (true negative)	9 (false negative)

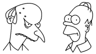
Worksheet

- **Example**: If one of the 98 subjects is randomly selected, find the probability that the subject had a positive test result, given that the subject actually lied. That is find P(positive test result|subject lied).

- **Example**: If one of the 98 subjects is randomly selected, find the probability that the subject actually lied, given that he or she had a positive test result.

Worksheet

	Nonsmoker	Light Smoker	Heavy Smoker	Total
Men	306	74	66	446
Women	345	68	81	494
Total	651	142	147	940



Consider the following events:

Event N: The person selected is a nonsmoker
Event L: The person selected is a light smoker
Event H: The person selected is a heavy smoker
Event M: The person selected is a male
Event F: The person selected is a female

Example: Suppose one of the 940 subjects is chosen at random. Compute the following probabilities:

a. $P(N|F)$

b. $P(F|N)$

c. $P(H \cup M)$

d. $P(M \cap L)$

e. P(the person selected is a smoker)

f. $P(F \cap \overline{H})$

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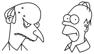

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Event F: The person selected is a female

Example: Now suppose that two people are selected from the group, ***without replacement***. Let A be the event "the first person selected is a nonsmoker," and let B be the event "the second person is a light smoker." What is $P(A \cap B)$?

Example: Two people are selected from the group, ***with replacement***. What is the probability that both people are nonsmokers?

Example: Two people are selected from the group. What is the probability that both people are smokers?

4.6 Counting Rules

Counting the number of simple events in a sample space is one of the hardest problems to deal with when finding probabilities.

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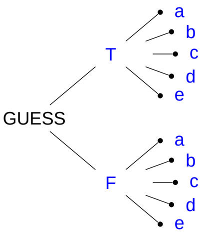

______ 1st Question   ______ 2nd Question

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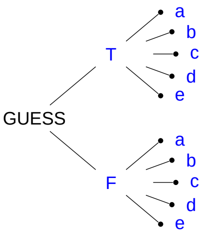

2

1st Question ____ 2nd Question ____

How many ways can you guess at a true/false question?

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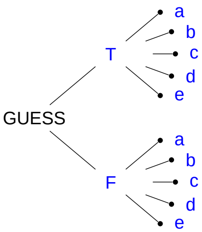

$2 \cdot 5$

1st Question   2nd Question

How many ways can you guess at the multiple choice question?

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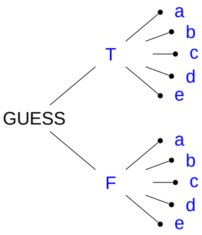

$$\underset{\text{1st Question}}{\underline{2}} \cdot \underset{\text{2nd Question}}{\underline{5}} = 10$$

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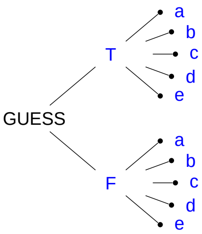

$$\underset{\text{1st Question}}{\underline{2}} \cdot \underset{\text{2nd Question}}{\underline{5}} = 10$$

The sample space, $S = \{Ta, Tb, Tc, Td, Te, Fa, Fb, Fc, Fd, Fe\}$, has 10 simple events.

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The Multiplication Rule

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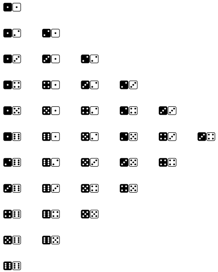
Example: Suppose you roll a pair of dice and record the sum of the two numbers that land on the upper faces of the die. How many simple events are in the sample space?

Roll	Probability
2	$\frac{1}{36}$
3	$\frac{2}{36}$
4	$\frac{3}{36}$
5	$\frac{4}{36}$
6	$\frac{5}{36}$
7	$\frac{6}{36}$
8	$\frac{5}{36}$
9	$\frac{4}{36}$
10	$\frac{3}{36}$
11	$\frac{2}{36}$
12	$\frac{1}{36}$



_____ 1st Die _____ 2nd Die

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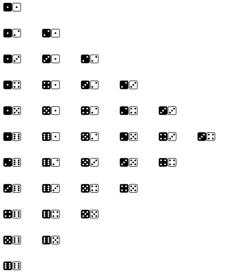

Roll	Probability
2	$\frac{1}{36}$
3	$\frac{2}{36}$
4	$\frac{3}{36}$
5	$\frac{4}{36}$
6	$\frac{5}{36}$
7	$\frac{6}{36}$
8	$\frac{5}{36}$
9	$\frac{4}{36}$
10	$\frac{3}{36}$
11	$\frac{2}{36}$
12	$\frac{1}{36}$

6

1st Die   2nd Die

How many ways can the first die land?

4.6 Counting Rules

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The Multiplication Rule

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Example: Suppose you roll a pair of dice and record the sum of the two numbers that land on the upper faces of the die. How many simple events are in the sample space?

Roll							Probability
2	⚀⚀						$\frac{1}{36}$
3	⚀⚁	⚁⚀					$\frac{2}{36}$
4	⚀⚂	⚂⚀	⚁⚁				$\frac{3}{36}$
5	⚀⚃	⚃⚀	⚂⚁	⚁⚂			$\frac{4}{36}$
6	⚀⚄	⚄⚀	⚃⚁	⚁⚃	⚂⚂		$\frac{5}{36}$
7	⚀⚅	⚅⚀	⚄⚁	⚁⚄	⚃⚂	⚂⚃	$\frac{6}{36}$
8	⚁⚅	⚅⚁	⚄⚂	⚂⚄	⚃⚃		$\frac{5}{36}$
9	⚂⚅	⚅⚂	⚄⚃	⚃⚄			$\frac{4}{36}$
10	⚃⚅	⚅⚃	⚄⚄				$\frac{3}{36}$
11	⚄⚅	⚅⚄					$\frac{2}{36}$
12	⚅⚅						$\frac{1}{36}$

$$6 \cdot 6$$

1st Die   2nd Die

How many ways can the second die land?

4.6 Counting Rules

Counting the number of simple events in a sample space is one of the hardest problems to deal with when finding probabilities.

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For a sequence of two events in which the first event can occur m ways and the second event can occur n ways, the events together can occur a total of $m \cdot n$ ways.

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Roll							Probability
2	⚀⚀						$\frac{1}{36}$
3	⚀⚁	⚁⚀					$\frac{2}{36}$
4	⚀⚂	⚂⚀	⚁⚁				$\frac{3}{36}$
5	⚀⚃	⚃⚀	⚂⚁	⚁⚂			$\frac{4}{36}$
6	⚀⚄	⚄⚀	⚃⚁	⚁⚃	⚂⚂		$\frac{5}{36}$
7	⚀⚅	⚅⚀	⚄⚁	⚁⚄	⚃⚂	⚂⚃	$\frac{6}{36}$
8	⚁⚅	⚅⚁	⚄⚂	⚂⚄	⚃⚃		$\frac{5}{36}$
9	⚂⚅	⚅⚂	⚄⚃	⚃⚄			$\frac{4}{36}$
10	⚃⚅	⚅⚃	⚄⚄				$\frac{3}{36}$
11	⚄⚅	⚅⚄					$\frac{2}{36}$
12	⚅⚅						$\frac{1}{36}$

$$\underset{\text{1st Die}}{\underline{6}} \cdot \underset{\text{2nd Die}}{\underline{6}} = 36$$

4.6 Counting Rules

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2	⚀⚀						$\frac{1}{36}$
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4	⚀⚂	⚂⚀	⚁⚁				$\frac{3}{36}$
5	⚀⚃	⚃⚀	⚂⚁	⚁⚂			$\frac{4}{36}$
6	⚀⚄	⚄⚀	⚃⚁	⚁⚃	⚂⚂		$\frac{5}{36}$
7	⚀⚅	⚅⚀	⚄⚁	⚁⚄	⚃⚂	⚂⚃	$\frac{6}{36}$
8	⚁⚅	⚅⚁	⚄⚂	⚂⚄	⚃⚃		$\frac{5}{36}$
9	⚂⚅	⚅⚂	⚄⚃	⚃⚄			$\frac{4}{36}$
10	⚃⚅	⚅⚃	⚄⚄				$\frac{3}{36}$
11	⚄⚅	⚅⚄					$\frac{2}{36}$
12	⚅⚅						$\frac{1}{36}$

$$\underset{\text{1st Die}}{\underline{6}} \cdot \underset{\text{2nd Die}}{\underline{6}} = 36$$

The sample space, has 36 simple events.

4.6 Counting Rules

The Extended Multiplication Rule

For a sequence of k events in which the first event can occur n_1 ways, the second event can occur n_2 ways, ..., the kth event can occur n_k ways, the number of ways to carry out the the sequence of events is the product

$$\underbrace{n_1 \cdot n_2 \cdot n_3 \cdots n_k}_{k \text{ factors}}$$

Example: Suppose a couple plans to have three children. How many simple events are in the sample space?

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$$\underbrace{n_1 \cdot n_2 \cdot n_3 \cdots n_k}_{k \text{ factors}}$$

Example: Suppose a couple plans to have three children. How many simple events are in the sample space?

girl
- girl
  - girl E_1: GGG
  - boy E_2: GGB
- boy
  - girl E_3: GBG
  - boy E_4: GBB

boy
- girl
  - girl E_5: BGG
  - boy E_6: BGB
- boy
  - girl E_7: BBG
  - boy E_8: BBB

$$\underset{\text{1st child}}{\underline{2}} \cdot \underset{\text{2nd child}}{\underline{2}} \cdot \underset{\text{3rd child}}{\underline{2}} = 8$$

4.6 Counting Rules

Example: Suppose you have 3 different cars and a 3-car garage. How many different ways can you arrange (order) the way you park the cars in your garage?

_____ garage 1   _____ garage 2   _____ garage 3

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______ garage 1   ______ garage 2   ______ garage 3

How many choices of cars do you have for garage 1?

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Example: Suppose you have 3 different cars and a 3-car garage. How many different ways can you arrange (order) the way you park the cars in your garage?

3		
garage 1	garage 2	garage 3

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3		
garage 1	garage 2	garage 3

You selected a car and parked it in garage 1. Now how many choices of cars do you have to park in your 2nd garage?

4.6 Counting Rules

Example: Suppose you have 3 different cars and a 3-car garage. How many different ways can you arrange (order) the way you park the cars in your garage?

$$\underset{\text{garage 1}}{\underline{\;3\;}} \cdot \underset{\text{garage 2}}{\underline{\;2\;}} \; \underset{\text{garage 3}}{\underline{\quad}}$$

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Example: Suppose you have 3 different cars and a 3-car garage. How many different ways can you arrange (order) the way you park the cars in your garage?

$$\underset{\text{garage 1}}{\underline{\;3\;}} \cdot \underset{\text{garage 2}}{\underline{\;2\;}} \quad \underset{\text{garage 3}}{\underline{\quad\;}}$$

You selected a car and parked it in the 2nd garage. Now how many choices of cars do you have left to park in your 3rd garage?

4.6 Counting Rules

Example: Suppose you have 3 different cars and a 3-car garage. How many different ways can you arrange (order) the way you park the cars in your garage?

$$\underset{\text{garage 1}}{\underline{\;3\;}} \cdot \underset{\text{garage 2}}{\underline{\;2\;}} \cdot \underset{\text{garage 3}}{\underline{\;1\;}}$$

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Example: Suppose you have 3 different cars and a 3-car garage. How many different ways can you arrange (order) the way you park the cars in your garage?

$$\underset{\text{garage 1}}{\underline{3}} \cdot \underset{\text{garage 2}}{\underline{2}} \cdot \underset{\text{garage 3}}{\underline{1}} = 6$$

According to the Multiplication Rule, there are six different parking arrangements.

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Example: Suppose you have 3 different cars and a 3-car garage. How many different ways can you arrange (order) the way you park the cars in your garage?

$$\underset{\text{garage 1}}{\underline{3}} \cdot \underset{\text{garage 2}}{\underline{2}} \cdot \underset{\text{garage 3}}{\underline{1}} = 6$$

Notice this was also equal to 3 factorial.

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Notice this was also equal to 3 factorial.

Definition

The **factorial symbol !** denotes the product of decreasing positive whole numbers. For example,

$$4! = 4 \cdot 3 \cdot 2 \cdot 1 = 24$$

By special definition, 0! = 1.

Definition (Factorial Rule)

A collection of n different items can be arranged in order $n!$ different ways.

(This factorial rule reflects the fact that the first item may be selected in n different ways, the second item may be selected in $n - 1$ ways, and so on.)

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$$5! = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 120$$

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For instance, suppose a television producer has four prizes to give away to a studio audience of 50 people. The first prize is a car, the second prize is a $6000 TV, third prize is a $2500 gift certificate to the mall, and fourth prize is $500 cash. How many different ways can the producer select the four prize winners?

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$$50 \cdot 49 \cdot 48 \cdot 47 = 5,527,000 \qquad \text{using the Multiplication Rule}$$

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This result is generalized by the *permutations rule*: if we have n different items available and we want to select r of them, then the number of different orderings is $n!/(n-r)!$

Permutations Rule (when items are all different)

Definition (Permutations Rule)

Requirements:

1. There are n different items available, with none of the items identical to any other item under consideration.
2. We select r of the n items (without replacement).
3. The ordering of the selections matter.

The number of permutations (or sequences) of *r* items selected from *n* available items (without replacement), denoted ${}_nP_r$, is

$$ {}_nP_r = \frac{n!}{(n-r)!} $$

Example: There are 13 members on a board of directors. How many different ways can the group select a president, vice-president and treasurer?

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$$ {}_{13}P_3 = \frac{13!}{(13-3)!} = \frac{13 \cdot 12 \cdot 11 \cdot \cancel{10!}}{\cancel{10!}} = 1716 $$

Combinations Rule

Definition (**Combinations Rule**)

Requirements:

1. There are n different items available.
2. We select r of the n items (without replacement).
3. The ordering of the selections does not matter.

The number of combinations of r items selected from n available items (without replacement), denoted ${}_nC_r$, is

$$ {}_nC_r = \frac{n!}{(n-r)!r!} $$

Example: There are 13 members on a board of directors. How many different ways can the group form a subcommittee with 3 members?

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Example: There are 13 members on a board of directors. How many different ways can the group form a subcommittee with 3 members?

$$ {}_{13}C_3 = \frac{13!}{(13-3)!3!} = \frac{13 \cdot 12 \cdot 11 \cdot \cancel{10!}}{\cancel{10!} \cdot 3!} = 286 $$

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Example: suppose a television producer has four prizes to give away to a studio audience of 50 people. The four prizes are all the same, a $500 gift certificate to the mall. How many different ways can the producer select the four prize winners?

$$ {}_{50}C_4 = \frac{50!}{(50-4)!\, 4!} = \frac{50!}{46! \cdot 4!} = \frac{50 \cdot 49 \cdot 48 \cdot 47 \cdot \cancel{46!}}{\cancel{46!} \cdot 4!} = \frac{5,527,000}{24} = 230,300 $$