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# Chapter 4

# Professor Tim Busken

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July 5, 2015

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# Definition

**Probability** is a measure or estimation of how likely it is that something will happen or that a statement is true. Probabilities are given a value between 0 (0% chance or will not happen) and 1 (100% chance or will happen).

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# **Common Notation**

Ρ

P(A)

 $P(E_1)$ 

# denotes a probability

 $A, B, C, E_1, E_2$  notation for specific events

notation for the probability of event A occurring

notation for the probability of event  $E_1$  occurring

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# Definition

An event is an outcome of an experiment or procedure.

**Experiment**: Toss a single die and observe the number that appears on the upper face. Here are some possible events:

**Events and Simple Events** 

Event A	Observe an even number
Event B	Observe a number less than 3
Event E <sub>1</sub>	Observe a 1
Event E <sub>2</sub>	Observe a 2
Event E <sub>3</sub>	Observe a 3
Event E <sub>4</sub>	Observe a 4
Event E <sub>5</sub>	Observe a 5
Event E <sub>6</sub>	Observe a 6



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# **Events and Simple Events**

## Definition

Two events are **mutually exclusive (or called disjoint)** if, when one event occurs, the other cannot, and vice versa.

Experiment: Toss a single die



Event A	Observe an even number
Event B	Observe a number less than 3
Event E1	Observe a 1
Event E <sub>2</sub>	Observe a 2
Event E <sub>3</sub>	Observe a 3
Event E4	Observe a 4
Event E <sub>5</sub>	Observe a 5
Event E <sub>6</sub>	Observe a 6

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# **Events and Simple Events**

## Definition

Two events are **mutually exclusive (or called disjoint)** if, when one event occurs, the other cannot, and vice versa.

## Observations:

• Events A and B are *not mutually exclusive* because both events occur when the number on the upper face of the die is a 2.

Experiment: Toss a single die



Event A	Observe an even number
Event B	Observe a number less than 3
Event E1	Observe a 1
Event E2	Observe a 2
Event E <sub>3</sub>	Observe a 3
Event E4	Observe a 4
Event E <sub>5</sub>	Observe a 5
Event E <sub>6</sub>	Observe a 6

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# **Events and Simple Events**

## Definition

Two events are **mutually exclusive (or called disjoint)** if, when one event occurs, the other cannot, and vice versa.

## Observations:

- Events A and B are not mutually exclusive because both events occur when the number on the upper face of the die is a 2.
- Since event A occurs whenever the upper face is 2, 4, or 6, event A can be decomposed into a collection of simpler events—namely, E<sub>2</sub>, E<sub>4</sub>, and E<sub>6</sub>—which are themselves *mutually exclusive*.

Experiment: Toss a single die



Event A Event B Event $E_1$ Event $E_2$ Event $E_3$ Event $E_4$ Event $E_5$ Event $E_5$	Observe an even number Observe a number less than 3 Observe a 2 Observe a 3 Observe a 4 Observe a 5 Observe a 6
Event E <sub>6</sub>	Observe a 6

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# **Events and Simple Events**

Definition Two events are mutually exclusive (or called disjoint) if, when one

event occurs, the other cannot, and vice versa.

## Observations:

- Events A and B are not mutually exclusive because both events occur when the number on the upper face of the die is a 2.
- Since event A occurs whenever the upper face is 2, 4, or 6, event A can be decomposed into a collection of simpler events—namely, E<sub>2</sub>, E<sub>4</sub>, and E<sub>6</sub>—which are themselves *mutually exclusive*.
- Similarly, event B can be decomposed into the collection of simple events {*E*<sub>1</sub>, *E*<sub>2</sub>}.



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Definition

An event that cannot be decomposed is called a simple event.

Experiment: Toss a single die



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Event A Observe an even number Event B Observe a number less than 3 Event E1 Observe a 1 Event E<sub>2</sub> Observe a 2 Event E<sub>2</sub> Observe a 3 Event E Observe a 4 Event E<sub>5</sub> Observe a 5 Event E<sub>6</sub> Observe a 6

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# Definition

An event that cannot be decomposed is called a simple event.

# **Observations**:

• Events A and B are *not simple events* because both events can be decomposed into a collection of simpler events.

Experiment: Toss a single die



Event A	Observe an even number
Event B	Observe a number less than 3
Event E1	Observe a 1
Event E2	Observe a 2
Event E <sub>3</sub>	Observe a 3
Event E4	Observe a 4
Event E <sub>5</sub>	Observe a 5
Event E6	Observe a 6

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# Definition

An event that cannot be decomposed is called a simple event.

## **Observations**:

- Events A and B are *not simple events* because both events can be decomposed into a collection of simpler events.
- Events  $E_1, E_2, \ldots, E_6$  are simple events.

Experiment: Toss a single die



Event A	Observe an even number
Event B	Observe a number less than 3
Event E1	Observe a 1
Event E <sub>2</sub>	Observe a 2
Event E <sub>3</sub>	Observe a 3
Event E4	Observe a 4
Event E <sub>5</sub>	Observe a 5
Event E <sub>6</sub>	Observe a 6

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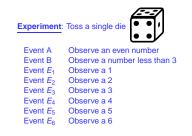
The Multiplication Rule Factorial Rule Permutations Rule Combinations Rule

# Definition

An event that cannot be decomposed is called a simple event.

## Observations:

- Events A and B are *not simple events* because both events can be decomposed into a collection of simpler events.
- Events  $E_1, E_2, \ldots, E_6$  are simple events.
- Simple events are mutually exclusive.



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Event A	Observe an even number
Event B	Observe a number less than 3
Event E1	Observe a 1
Event E2	Observe a 2
Event E <sub>3</sub>	Observe a 3
Event E4	Observe a 4
Event E <sub>5</sub>	Observe a 5
Event E <sub>6</sub>	Observe a 6

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# Definition

A sample space is the *complete* collection of simple events possible for an experiment or procedure.

Experiment: Toss a single die



3

Event A Observe an even number Event B Observe a number less than 3 Event E1 Observe a 1 Event E<sub>2</sub> Observe a 2 Event E<sub>2</sub> Observe a 3 Event E Observe a 4 Event E<sub>5</sub> Observe a 5 Event E<sub>6</sub> Observe a 6

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# Definition

A sample space is the *complete* collection of simple events possible for an experiment or procedure.

The sample space, S, for our experiment is

Experiment: Toss a single die



Event A	Observe an even number
Event B	Observe a number less than 3
Event E1	Observe a 1
Event E <sub>2</sub>	Observe a 2
Event E <sub>3</sub>	Observe a 3
Event E4	Observe a 4
Event E <sub>5</sub>	Observe a 5
Event E <sub>6</sub>	Observe a 6

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# Definition

A sample space is the *complete* collection of simple events possible for an experiment or procedure.

The sample space, S, for our experiment is

The sum of the probabilities for all simple events in any sample space, S, equals 1

Experiment: Toss a single die



Event A	Observe an even number
Event B	Observe a number less than 3
Event E1	Observe a 1
Event E <sub>2</sub>	Observe a 2
Event E <sub>3</sub>	Observe a 3
Event E4	Observe a 4
Event E <sub>5</sub>	Observe a 5
Event E <sub>6</sub>	Observe a 6

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Definition

**Event** 

 $E_1$  $E_2$ 

 $E_3$ 

Eл

 $E_5$ 

 $E_6$ 

A sample space is the *complete* collection of simple events possible for an experiment or procedure.

The sample space, S, for our experiment is

**Probability** 

1/6

1/6

1/6

1/6

1/6

1/6

 $\sum_{i=1}^{6} P(E_i) = \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} = 1$ 

The sum of the probabilities for all simple events in any sample space, S, equals 1

Experiment: Toss a single die



Event A	Observe an even number
Event B	Observe a number less than 3
Event E1	Observe a 1
Event E2	Observe a 2
Event E3	Observe a 3
Event E4	Observe a 4
Event E5	Observe a 5
Event E <sub>6</sub>	Observe a 6

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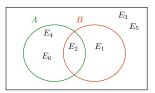
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The Multiplication Rule Factorial Rule Permutations Rule Combinations Rule It is often helpful to visualize an experiment using a Venn Diagram, (right). The outer box represents the sample space, which contains all of the mutually exclusive, simple events.



Experiment: Toss a single die

S



Event A Observe an even number Event B Observe a number less than 3 Event E1 Observe a 1 Event E<sub>2</sub> Observe a 2 Event E<sub>3</sub> Observe a 3 Event E Observe a 4 Event E<sub>5</sub> Observe a 5 Observe a 6 Event Ee

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Event A is the circled collection of simple events,  $\{E_2, E_4, E_6\}$ .

Event B is the circled collection of simple events,  $\{E_1, E_2\}$ .

# $\begin{array}{c|c} A & B & E_3 \\ \hline & E_4 \\ \hline & E_6 & E_2 \\ \hline & E_1 \\ \hline & \\ \end{array} \\ \begin{array}{c} E_5 \\ E_5 \\ \hline \\ & \\ \end{array} \\ \begin{array}{c} E_5 \\ E_5 \\ \hline \\ & \\ \end{array} \\ \begin{array}{c} E_5 \\ E_5 \\ \hline \\ & \\ \end{array} \\ \begin{array}{c} E_5 \\ E_5 \\ \hline \\ & \\ \end{array} \\ \begin{array}{c} E_6 \\ E_2 \\ E_1 \\ \hline \\ & \\ \end{array} \\ \begin{array}{c} E_5 \\ E_5 \\ \hline \\ & \\ \end{array} \\ \begin{array}{c} E_5 \\ E_5 \\ \hline \\ & \\ \end{array} \\ \begin{array}{c} E_5 \\ E_5 \\ E_5 \\ \hline \\ & \\ \end{array} \\ \begin{array}{c} E_5 \\ E_5 \\ E_5 \\ \hline \\ & \\ \end{array} \\ \begin{array}{c} E_5 \\ E_5 \\ E_5 \\ E_5 \\ \hline \\ \end{array} \\ \begin{array}{c} E_5 \\ E_5 \\ E_5 \\ E_5 \\ E_5 \\ \end{array} \\ \begin{array}{c} E_5 \\ E_5$

Experiment: Toss a single die



Event A	Observe an even number
Event B	Observe a number less than 3
Event <i>E</i> 1	Observe a 1
Event E <sub>2</sub>	Observe a 2
Event E <sub>3</sub>	Observe a 3
Event E4	Observe a 4
Event E <sub>5</sub>	Observe a 5
Event E <sub>6</sub>	Observe a 6

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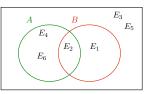
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Event A is the circled collection of simple events,  $\{E_2, E_4, E_6\}$ .

Event B is the circled collection of simple events,  $\{E_1, E_2\}$ .

Events A and B are called **compound events** because they are events combining two or more simple events. S



Experiment: Toss a single die



Event A	Observe an even number
Event B	Observe a number less than 3
Event E1	Observe a 1
Event E2	Observe a 2
Event E <sub>3</sub>	Observe a 3
Event E4	Observe a 4
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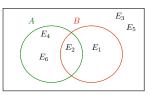
Event A is the circled collection of simple events,  $\{E_2, E_4, E_6\}$ .

Event B is the circled collection of simple events,  $\{E_1, E_2\}$ .

Events A and B are called **compound events** because they are events combining two or more simple events.

$$P(A) = P(E_2 \text{ or } E_4 \text{ or } E_6)$$
  
=  $P(E_2) + P(E_4) + P(E_6)$   
=  $\frac{1}{6} + \frac{1}{6} + \frac{1}{6}$   
=  $\frac{3}{6} = 0.5$ 

S



Experiment: Toss a single die



Event A	Observe an even number
Event B	Observe a number less than 3
Event E1	Observe a 1
Event E2	Observe a 2
Event E <sub>3</sub>	Observe a 3
Event E4	Observe a 4
Event E <sub>5</sub>	Observe a 5
Event E <sub>6</sub>	Observe a 6

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The Multiplication Rule Factorial Rule Permutations Rule Combinations Rule Suppose a couple plans to have three children. Assume that girls and boys are equally likely and that the gender of one child is not influenced by the gender of any other child. What is the sample space, or set of all possible outcomes?

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The Multiplication Rule Factorial Rule Permutations Rule Combinations Rule Suppose a couple plans to have three children. Assume that girls and boys are equally likely and that the gender of one child is not influenced by the gender of any other child. What is the sample space, or set of all possible outcomes?

3rd Child

2<sup>nd</sup> Child

1<sup>st</sup> Child

E1: GGG girl girl E<sub>2</sub>: GGB bov girl  $E_3$ : GBG girl boy E<sub>4</sub>: GBB boy girl E<sub>5</sub>: BGG girl E<sub>6</sub>: BGB bov boy E7: BBG girl boy bov E<sub>8</sub>: BBB

 $S = \{E_1, E_2, E_3, E_4, E_5, E_6, E_7, E_8\}$ 

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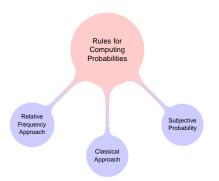
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# **Computing Probabilities**

# Definition (The Classical Approach)

Assume that a given procedure has n different simple events and that each of those simple events has an equal chance of occurring. If event A can occur in s of these n ways, then

$$P(A) = \frac{\#\text{of ways } A \text{ can occur}}{\#\text{of different simple events}} = \frac{s}{n}$$

Example: Toss a single die. Determine the following probabilities:

1	$P(E_1)$
2	$P(E_5)$
3	P(A)
4	P(B)



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# **Computing Probabilities**

# Definition (The Relative Frequency Approach)

Conduct (or observe) a procedure, and count the number of times event A actually occurs. Based on these actual results, P(A) is approximated as

 $P(A) = \frac{\#\text{of times A occurred}}{\#\text{of times procedure was repeated}}$ 

**Example**: When trying to determine the probability that an individual car crashes in a year, we must examine past results to determine the number of cars in use in a year and the number of them that crashed, then find the ratio of the two.[?]

$$P(crash) = \frac{\text{\#of times cars that crashed}}{\text{total \#of cars}} = \frac{6,511,100}{135,670,000} = 0.0480$$



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# **Computing Probabilities**

# Definition (The Relative Frequency Approach)

Conduct (or observe) a procedure, and count the number of times event A actually occurs. Based on these actual results, P(A) is approximated as

 $P(A) = \frac{\#\text{of times A occurred}}{\#\text{of times procedure was repeated}}$ 

**Example**: When trying to determine the probability that an individual car crashes in a year, we must examine past results to determine the number of cars in use in a year and the number of them that crashed, then find the ratio of the two.[?]

 $P(crash) = \frac{\text{\#of times cars that crashed}}{\text{total \#of cars}} = \frac{6,511,100}{135,670,000} = 0.0480$ 

## Theorem (Law of Large Numbers)

As a procedure is repeated again and again, the relative frequency probability of an event tends to approach the actual probability.



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# **Computing Probabilities**

## Definition (Subjective Probability)

P(A), the probability of event A, is estimated by using knowledge of the relevant circumstances.

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# **Computing Probabilities**

## Definition (Subjective Probability)

P(A), the probability of event A, is estimated by using knowledge of the relevant circumstances.

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# **Experiment**: Roll a pair of dice. Record the sum of the two numbers that appear on the upper faces of the dice.

F	Roll						Probability
	2						<u>1</u> 36
	3						<u>2</u> 36
	4	•••	•••				<u>3</u> 36
	5						$\frac{4}{36}$
	6	•					<u>5</u> 36
	7				. :		<u>6</u> 36
	8			8.			<u>5</u> 36
	9						$\frac{4}{36}$
	10			∷∷			<u>3</u> 36
	11						<u>2</u> 36
	12						<u>1</u> 36

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## Determine the following probabilities:

- P(the sum is 8)
- *P*(rolling a double 1)

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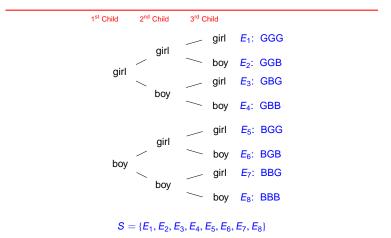
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The Multiplication Rule Factorial Rule Permutations Rule Find the probability that when a couple has three children, they will have exactly 2 girls. Assume that girls and boys are equally likely and that the gender of one child is not influenced by the gender of any other child. [?]



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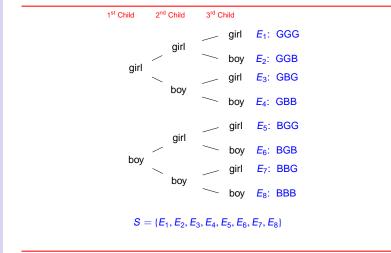
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Find the probability that when a couple has three children, they will have exactly 2 girls. Assume that girls and boys are equally likely and that the gender of one child is not influenced by the gender of any other child. [?]



**Classical Approach** 

$$P(2 \text{ boys in 3 births}) = \frac{\#\text{of ways A can occur}}{\#\text{of different simple events}} = \frac{3}{8} = 0.375$$

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# **Experiment**: Pick a card at random from a shuffled deck of cards.

A ∳	*	÷	<b>2</b> ♣	*	*7		* * *	ŧ	4**	*	5	*	* *	6. * *	*	ŧ	*	* * * †	8.4 4 4	***	9. * *	***	10	*** *** ***		<sup>₩</sup>
<b>A</b> ◆	¢	v V	2 +	¢	÷2	5	* * *	z £	4 ♠ ♥	* *	5	* *	♦	6 ♠ ♥	♠	9	**	* * *	8 4 4		9 4 4					× •
¢	٠	¢	2 •	*	•		•	<b>2</b>	<b>4</b> ♥	* *;	5	v .v	• 2	¢ •	*			÷ 2	8		9		<b>10</b>			K.
A •	٠	÷	2 •	•	ŧ	\$	• • •	ţ	4 ♦ ♦	• •;	<b>5</b>	•	• • •	€ ♦ ● ●	* * *	<b>)</b>	•	• • :	8.		9.		10			K.

# Determine the following probabilities:

- P(the card is a four of hearts)
- P(the card is a queen)
- P(the card is not an ace)

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# **Computing Probabilities**

**Example**: In the last 30 years, death sentence executions in the United States included 795 men and 10 women (based on data from the Associated Press). If an execution is randomly selected, find the probability that the person executed is a women. Is it unusual for a woman to be executed?

We use the **relative frequency** approach here, since the likelihood that a women or man is executed is not the same.

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# **Complementary Events**

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## Definition

The complement of event A, denoted by  $\overline{A}$  or  $A^{C}$ , consists of all the simple events in the sample space which are not in A.

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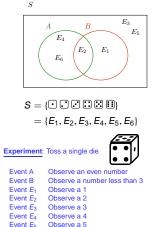
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# **Complementary Events**

## Definition

The complement of event A, denoted by  $\overline{A}$  or  $A^{C}$ , consists of all the simple events in the sample space which are not in A.



Observe a 6

3

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Event E6

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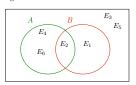
# **Complementary Events**

## Definition

The complement of event A, denoted by  $\overline{A}$  or  $A^{C}$ , consists of all the simple events in the sample space which are not in A.

## For the single die experiment, this means

- Event A observe an odd number
- Event  $\overline{B}$  observe a number greater than or equal to 3
- Event  $\overline{E_2}$  observe any number in S except 2



Experiment: Toss a single die



Event A Event B	Observe an even number Observe a number less than 3
Event E1	Observe a 1
Event E <sub>2</sub>	Observe a 2
Event E <sub>3</sub>	Observe a 3
Event E4	Observe a 4
Event E <sub>5</sub>	Observe a 5
Event E <sub>6</sub>	Observe a 6

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Definition

The complement of event A, denoted by  $\overline{A}$  or  $A^c$ , consists of all the simple events in the sample space which are not in A.

Complementary Events

## For the single die experiment, this means

- Event  $\overline{B}$  observe a number greater than or equal to 3
- Event  $\overline{E_2}$  observe any number in S except 2

A fundamental property of complementary events may now be apparent to you:

 $P(A) + P(\overline{A}) = 1$ 

the sum of the probabilities of an event and its complement is always one (regardless of whether an event is simple or compound).

# $\begin{array}{c} A \\ E_4 \\ E_6 \\ E_6 \end{array} \begin{array}{c} E_2 \\ E_1 \\ E_5 \end{array} \begin{array}{c} E_5 \\ E_5 \\ E_5 \end{array}$

Experiment: Toss a single die



Event A	Observe an even number
Event B	Observe a number less than 3
Event E1	Observe a 1
Event E2	Observe a 2
Event E3	Observe a 3
Event E4	Observe a 4
Event E <sub>5</sub>	Observe a 5
Event E6	Observe a 6



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# **Concept Check**

Question: Suppose A is any event, either simple or compound. Are the events A and A complement mutually exclusive?

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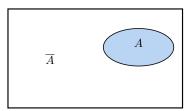
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# **Concept Check**

**Question**: Suppose A is any event, either simple or compound. Are the events A and A complement mutually exclusive?

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# **Complementary Events**

**Example**: Women have a 0.25% rate of red/green color blindness. If a women is randomly selected, what is the *probability* that she does *not* have red/green color blindness?

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# The Rare Event Rule

## Theorem (The Rare Event Rule)

If, under a given assumption, the probability of a particular <u>observed</u> event is extremely small, we conclude that the assumption is probably not correct.

**Example**: Sally thinks there is no way she can get an A on Mr. Busken's first stats exam. Then she aces the exam. By the rare event rule, her assumption must have been incorrect.

See example 12, p146 in the text for another example.

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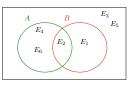
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## Experiment: Toss a single die



Event A	Observe an even number
Event B	Observe a number less than 3
Event E1	Observe a 1
Event E <sub>2</sub>	Observe a 2
Event E3	Observe a 3
Event E4	Observe a 4
Event E <sub>5</sub>	Observe a 5
Event E6	Observe a 6

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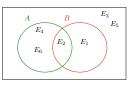
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## Experiment: Toss a single die



Event A	Observe an even number
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Event E3	Observe a 3
Event E4	Observe a 4
Event E <sub>5</sub>	Observe a 5
Event E6	Observe a 6

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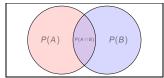
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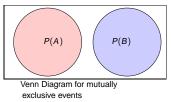
## Definition (The Addition Rule)

P(A or B) = P(A) + P(B) - P(A and B)

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$



Venn Diagram for Events that are not mutually exclusive



 $P(A \cap B) = 0$ 

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- The Rare Event Rule

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Factorial Rule

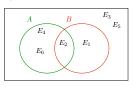
## Experiment: Toss a single die



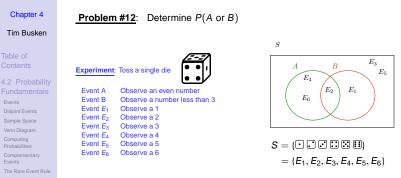
Event A	Observe an even number
Event B	Observe a number less than 3
Event E1	Observe a 1
Event E <sub>2</sub>	Observe a 2
Event E <sub>3</sub>	Observe a 3
Event E <sub>4</sub>	Observe a 4
Event E <sub>5</sub>	Observe a 5
Event E <sub>6</sub>	Observe a 6

**Problem #12**: Determine *P*(*A* or *B*)

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4.4 The Multiplication Rule

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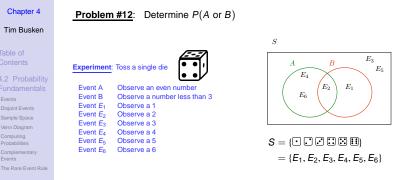
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 $P(A \cup B) = P($  observe an even number OR observe a number less than 3 )

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4.4 The Multiplication Rule

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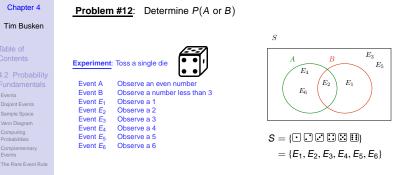
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 $P(A \cup B) = P($  observe an even number OR observe a number less than 3 )

$$= P(A) + P(B) - P(A \text{ and } B)$$



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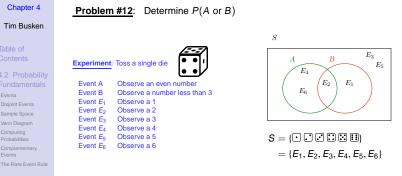
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## Works Cited

 $P(A \cup B) = P($  observe an even number OR observe a number less than 3 )

$$= P(A) + P(B) - P(A \text{ and } B)$$

$$=\frac{3}{6}+\frac{2}{6}-\frac{1}{6}$$



4.4 The Multiplication Rule

## Section 4.5

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## 4.6 Counting

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## Works Cited

 $P(A \cup B) = P($  observe an even number OR observe a number less than 3 )

= P(A) + P(B) - P(A and B)

$$= \frac{3}{6} + \frac{2}{6} - \frac{1}{6}$$
$$= \frac{4}{6} \doteq 0.67$$

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Venn Diagram

## 4.3 The Addition Rule

The Probability of "at

## **Problem #12**: Determine *P*(*A* or *B*)

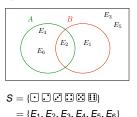
Experiment: Toss a single die



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Event A	Observe an even number
Event B	Observe a number less than 3
Event E1	Observe a 1
Event E <sub>2</sub>	Observe a 2
Event E	Observe a 3

- Event E<sub>3</sub> Observe a 3 Event E4 Observe a 4 Event E<sub>5</sub> Observe a 5 Event E6
  - Observe a 6

S



Alternatively,  $A \cup B \equiv \{E_1, E_2, E_4, E_6\}$ , so  $P(A \cup B) = \frac{4}{6} \doteq 0.67$  using the classical approach.

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## 4.6 Counting

The Multiplication Rule Factorial Rule Permutations Rule Combinations Rule **Problem #13**: Let *A* be the event the observed pair sums to 10 and let *B* be the event the observed pair is a double. Determine  $P(A \cup B)$ .

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The Multiplication Rule Factorial Rule Permutations Rule Combinations Rule **Problem #13**: Let *A* be the event the observed pair sums to 10 and let *B* be the event the observed pair is a double. Determine  $P(A \cup B)$ .

 $P(A \cup B) = P(A) + P(B) - P(A \text{ and } B)$ 

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The Multiplication Rule Factorial Rule Permutations Rule Combinations Rule **Problem #13**: Let A be the event the observed pair sums to 10 and let B be the event the observed pair is a double. Determine  $P(A \cup B)$ .

$$P(A \cup B) = P(A) + P(B) - P(A \text{ and } B)$$

$$=\frac{3}{36}+\frac{6}{36}-\frac{1}{36}$$

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The Multiplication Rule Factorial Rule Permutations Rule Combinations Rule **Problem #13**: Let *A* be the event the observed pair sums to 10 and let *B* be the event the observed pair is a double. Determine  $P(A \cup B)$ .

$$P(A \cup B) = P(A) + P(B) - P(A \text{ and } B)$$
$$= \frac{3}{36} + \frac{6}{36} - \frac{1}{36}$$
$$= \frac{8}{36} \doteq 0.22$$

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# **Problem #14**: Let *A* be the event the observed pair sums to 10 and let *B* be the event the observed pair sums to 4. Determine $P(A \cup B)$ .

 $P(A \cup B) = P(A) + P(B) - P(A \text{ and } B)$ 

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$$P(A \cup B) = P(A) + P(B) - P(A \text{ and } B)$$

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$$= \frac{3}{36} + \frac{3}{36} - \frac{0}{36}$$
$$= \frac{6}{36} \doteq 0.17$$

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**<u>Problem #15</u>**: Pick a card at random from a shuffled deck. Let *A* be the event the observed card is a 4 and let *B* be the event the card is a heart. Determine  $P(A \cup B)$ .

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**Problem #15**: Pick a card at random from a shuffled deck. Let *A* be the event the observed card is a 4 and let *B* be the event the card is a heart. Determine  $P(A \cup B)$ .

 $P(A \cup B) = P(A) + P(B) - P(A \text{ and } B)$ 

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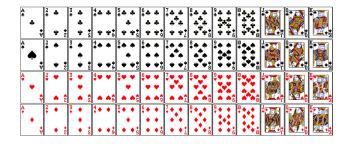
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$$P(A \cup B) = P(A) + P(B) - P(A \text{ and } B)$$
  
=  $\frac{4}{52} + \frac{3}{52} - \frac{1}{52}$ 

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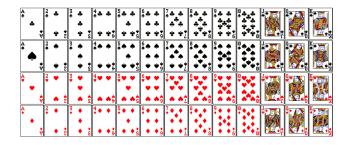
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**Problem #15**: Pick a card at random from a shuffled deck. Let *A* be the event the observed card is a 4 and let *B* be the event the card is a heart. Determine  $P(A \cup B)$ .

 $P(A \cup B) = P(A) + P(B) - P(A \text{ and } B)$  $= \frac{4}{52} + \frac{3}{52} - \frac{1}{52}$  $= \frac{16}{52} \doteq 0.31$ 

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**Problem #16**: Let *A* be the event the observed card is a 4 and let *B* be the event the card is a 10. Determine  $P(A \cup B)$ .

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**Problem #16**: Let A be the event the observed card is a 4 and let B be the event the card is a 10. Determine  $P(A \cup B)$ .

 $P(A \cup B) = P(A) + P(B) - P(A \text{ and } B)$ 

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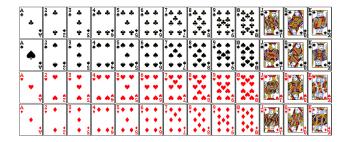
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 $P(A \cup B) = P(A) + P(B) - P(A \text{ and } B)$ =  $\frac{4}{52} + \frac{4}{52} - \frac{0}{52}$ 

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# The Multiplication Rule

## Notation

P(B|A) represents the probability of event B occurring after it is assumed that event A has already occurred (read B|A as "B given A").

## Definition

Two events A and B are independent if the occurrence of one does not affect the probability of the occurrence of the other. If A and B are not independent, they are said to be dependent.

## Definition

Two events A and B are said to be independent if and only if either

P(B|A) = P(B) or P(A|B) = P(A)

## Theorem (The Multiplication Rule)

 $P(A \text{ and } B) = P(A) \cdot P(B)$  $P(A \text{ and } B) = P(A) \cdot P(B|A)$  (if A and B are independent)

(if A and B are dependent)

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# Key Concept

The basic multiplication rule is used for finding P(A and B), the probability that event A occurs in a first trial and event B occurs in a second trial.



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# Key Concept

The basic multiplication rule is used for finding P(A and B), the probability that event A occurs in a first trial and event B occurs in a second trial.

**Example**: Suppose you are given a two-question quiz, where the first question is a true/false question and the second question is a multiple choice question with 5 possible answers. Suppose you guess on both questions. What is the probability that you correctly answered both questions?

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## Key Concept

The basic multiplication rule is used for finding P(A and B), the probability that event A occurs in a first trial and event B occurs in a second trial.

**Example:** Suppose you are given a two-question quiz, where the first question is a true/false question and the second question is a multiple choice question with 5 possible answers. Suppose you guess on both questions. What is the probability that you correctly answered both questions?

Notice that the notation P(both correct) is equivalent to P(the first answer is correct AND the second answer is correct).

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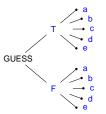
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Notice that the notation *P*(both correct) is equivalent to *P*(the first answer is correct AND the second answer is correct). The sample space,

 $S = \{Ta, Tb, Tc, Td, Te, Fa, Fb, Fc, Fd, Fe\},\$ 

has 10 simple events.

## Key Concept



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The basic multiplication rule is used for finding P(A and B), the probability that event A occurs in a first trial and event B occurs in a second trial.

**Example:** Suppose you are given a two-question guiz, where the first question is a true/false question and the second question is a multiple choice question with 5 possible answers. Suppose you guess on both guestions. What is the probability that you correctly answered both questions?

> Notice that the notation P(both correct) is equivalent to P(the first answer is correct AND the second answer is correct). The sample space.

> > $S = \{Ta, Tb, Tc, Td, Te, Fa, Fb, Fc, Fd, Fe\}$

has 10 simple events. Only one of these is a correct outcome, so

$$P(\text{both correct}) = \frac{1}{10} = 0.1$$

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## Key Concept

The basic multiplication rule is used for finding P(A and B), the probability that event A occurs in a first trial and event B occurs in a second trial.

**Example:** Suppose you are given a two-question guiz, where the first question is a true/false question and the second question is a multiple choice question with 5 possible answers. Suppose you guess on both guestions. What is the probability that you correctly answered both questions?

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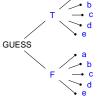
has 10 simple events. Only one of these is a correct outcome, so

 $P(\text{both correct}) = \frac{1}{10} = 0.1$ 

Suppose the correct answers are T and c. We can also obtain the correct probability by multiplying the individual probabilities:

P(both correct) = P(T and c)

$$= P(T) \cdot P(c) = \frac{1}{2} \cdot \frac{1}{5} = \frac{1}{10} = 0.1$$



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Experiment: Now pick two cards at random from a shuffled deck of playing cards.

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**Example**: Two cards are randomly selected *without replacement*. Find the probability the first card is an ace and the second card is an ten.

**Example**: Two cards are randomly selected *with replacement*. Find the probability the first card is an ace and the second card is an ten.

**Example**: Two cards are randomly selected. Find the probability that the draw includes and ace and a ten.

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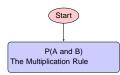
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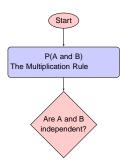
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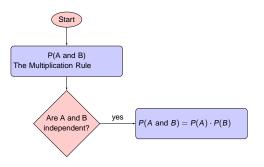
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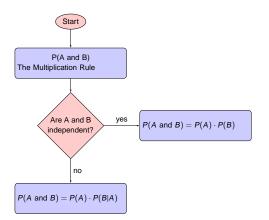
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Homework #17: Use the data in the following table, which summarizes blood type and Rh types for 100 subjects.

- -

	Blood Type								
	O A B AB								
Rh Type	$Rh^+$	39	35	8	4				
	Rh⁻	5	6	2	1				

If 2 out of the 100 subjects are randomly selected, find the probability that they are both blood group O and Rh type  $Rh^+$ .

- 1 Assume that the selections are made with replacement.
- 2 Assume that the selections are made without replacement.

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Homework #22: With one method of a procedure called acceptance sampling, a sample of items is randomly selected without rplacement and the entire batch is accepted if every item in the sample is okay. The Telektronics Company manufactured a batch of 400 back up power supply units for computers, and 8 of them are defective. If 3 of the units are randomly selected for testing, what is the probability that the entire batch will be accepted?

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## Key Concept

In this section, we extend our multiplication rule to the two special applications:

## 1 Determine the Probability of "at least one"

## **2** Conditional probability

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In this section, we extend our multiplication rule to the two special applications:

Obtermine the Probability of "at least one": Find the probability that among several trials, we get at least one of some specified event.

## **2** Conditional probability

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## Key Concept

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In this section, we extend our multiplication rule to the two special applications:

- Obtermine the Probability of "at least one": Find the probability that among several trials, we get at least one of some specified event.
- 2 Conditional probability: Find the probability of an event when we have additional information that some other event has already occurred.

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## The Probability of "at least one"

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- "at least one" is equivalent to "one or more."
- The complement of getting at least one item of a particular type is that you get no items of that type.



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## The Probability of "at least one"

Find the probability of finding at least one of some event by using these steps[?]:

**()** Use the symbol A to denote the event of getting at least one.

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# The Probability of "at least one"

Find the probability of finding at least one of some event by using these steps[?]:

- **1** Use the symbol A to denote the event of getting at least one.
- O Then A represents the event of getting none of the items being considered.

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- Subtract the result from 1. That is, evaluate

P(at least one) = 1 - P(none)

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**Example**: Find the probability of a couple having at least 1 girl among 4 children. Assume that boys and girls are equally likely and that the gender of one child is not influenced by the gender of any other child.

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**Example**: Find the probability of a couple having at least 1 girl among 4 children. Assume that boys and girls are equally likely and that the gender of one child is not influenced by the gender of any other child.

Let A = at least 1 of the 4 children is a girl.

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**Example**: Find the probability of a couple having at least 1 girl among 4 children. Assume that boys and girls are equally likely and that the gender of one child is not influenced by the gender of any other child.

- Let A = at least 1 of the 4 children is a girl.
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$$=\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}=\frac{1}{16}=0.0625$$

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$$=\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}=\frac{1}{16}=0.0625$$

 $\textbf{9} \quad \text{Finally, } P(A) = 1 - P(\overline{A})$ 

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# The Probability of "at least one"

Find the probability of finding at least one of some event by using these steps[?]:

- Use the symbol A to denote the event of getting at least one.
- **②** Then  $\overline{A}$  represents the event of getting none of the items being considered.
- Calculate the probability that none of the outcomes results in the event being considered.
- Subtract the result from 1. That is, evaluate

P(at least one) = 1 - P(none)

**Example**: Find the probability of a couple having at least 1 girl among 4 children. Assume that boys and girls are equally likely and that the gender of one child is not influenced by the gender of any other child.

- Let A = at least 1 of the 4 children is a girl.
- Then  $\overline{A}$  = none of the 4 children are girls.
  - = all 4 children are boys
  - = the 1st child is a boy AND the 2nd child is a boy AND the 3rd child is a boy AND the 4th child is a boy
- $P(\overline{A}) = P($  the 1st child is a boy AND the 2nd child is a boy AND the 3rd child is a boy AND the 4th child is a boy)
  - = P( the 1st child is a boy $) \times P($  the 2nd child is a boy $) \times P($  the 3rd child is a boy $) \times P($  the 4th child is a boy)

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$$=\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}=\frac{1}{16}=0.0625$$

• Finally,  $P(A) = 1 - P(\overline{A}) = 1 - 0.0625$ 

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  - = the 1st child is a boy AND the 2nd child is a boy AND the 3rd child is a boy AND the 4th child is a boy
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  - = P( the 1st child is a boy $) \times P($  the 2nd child is a boy $) \times P($  the 3rd child is a boy $) \times P($  the 4th child is a boy)

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$$=\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}=\frac{1}{16}=0.0625$$

**9** Finally,  $P(A) = 1 - P(\overline{A}) = 1 - 0.0625 = 0.9375$ .

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# Worksheet

Example: A study conducted at a certain college shows that 59% of the school's graduates find a job in their chosen field within a year after graduation. Find the probability that among 6 randomly selected graduates, at least one finds a job in his or her chosen field within a year of graduating.

Example: In a batch of 8,000 clock radios 6% are defective. A sample of 8 clock radios is randomly selected without replacement from the 8,000 and tested. The entire batch will be rejected if at least one of those tested is defective. What is the probability that the entire batch will be rejected?

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## **Conditional Probability**

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## Definition

A conditional probability of an event is a probability obtained with the additional information that some other event has already occurred. P(B|A) denotes the conditional probability of event B occurring, given that event A has already occurred, and it can be found by dividing the probability of events A and B both occurring by the probability of event A:

$$P(B|A) = rac{P(A ext{ and } B)}{P(A)}$$

## Table 4 - 1 Results from Experiments with Polygraph Instruments

	No (Did Not Lie)	Yes (Lied)
Positive Test Result	15	42
(The polygraph test indicated that the subject lied.)	(false positive)	(true positive)
Negative Test Result	32	9
(The polygraph test indicated that the subject did not lie.)	(true negative)	(false negative)

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# Worksheet

Example: If one of the 98 subjects is randomly selected, find the probability that the subject had a positive test result, given that the subject actually lied. That is find P(positive test result|subject lied).

Example: If one of the 98 subjects is randomly selected, find the probability that the subject actually lied, given that he or she had a positive test result.

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		Light	Heavy	
	Nonsmoker	Smoker	Smoker	Total
Men	306	74	66	446
Women	345	68	81	494
Total	651	142	147	940



# Worksheet

Consider the following events:

Event N: The person selected is a nonsmoker Event L: The person selected is a light smoker Event H: The person selected is a heavy smoker Event M: The person selected is a male Event F: The person selected is a female

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**Example**: Suppose one of the 940 subjects is chosen at random. Compute the following probabilities:

- a. P(N|F)
- b. P(F|N)
- c.  $P(H \cup M)$
- d.  $P(M \cap L)$
- e. P(the person selected is a smoker)
- f.  $P(F \cap \overline{H})$

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	Nonsmoker	Smoker	Smoker	Total
Men	306	74	66	446
Women	345	68	81	494
Total	651	142	147	940



Consider the following events:

Worksheet

Event N: The person selected is a nonsmoker Event L: The person selected is a light smoker Event H: The person selected is a heavy smoker Event M: The person selected is a male Event F: The person selected is a female

**Example:** Now suppose that two people are selected from the group, *without replacement*. Let A be the event "the first person selected is a nonsmoker," and let B be the event "the second person is a light smoker." What is  $P(A \cap B)$ ?

**Example**: Two people are selected from the group, *with replacement*. What is the probability that both people are nonsmokers?

**Example**: Two people are selected from the group. What is the probability that both people are smokers?

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# 4.6 Counting Rules

Counting the number of simple events in a sample space is one of the hardest problems to deal with when finding probabilities.

### The Multiplication Rule

For a sequence of two events in which the first event can occur *m* ways and the second event can occur *n* ways, the events together can occur a total of  $m \cdot n$  ways.

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**Example**: Suppose you are given a two-question quiz, where the first question is a true/false question and the second question is a multiple choice question with 5 possible answers. Suppose you guess on both questions. How many simple events are in the sample space?

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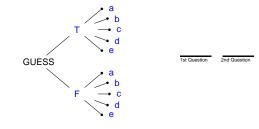
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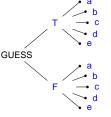
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**Example:** Suppose you are given a two-question quiz, where the first question is a true/false question and the second question is a multiple choice question with 5 possible answers. Suppose you guess on both questions. How many simple events are in the sample space?





How many ways can you guess at a true/false question?

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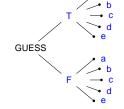
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# 4.6 Counting Rules

Counting the number of simple events in a sample space is one of the hardest problems to deal with when finding probabilities.

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For a sequence of two events in which the first event can occur *m* ways and the second event can occur *n* ways, the events together can occur a total of  $m \cdot n$  ways.

**Example:** Suppose you are given a two-question quiz, where the first question is a true/false question and the second question is a multiple choice question with 5 possible answers. Suppose you guess on both questions. How many simple events are in the sample space?

2 · 5

How many ways can you guess at the multiple choice question?

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# 4.6 Counting Rules

Counting the number of simple events in a sample space is one of the hardest problems to deal with when finding probabilities.

### The Multiplication Rule

GUESS

For a sequence of two events in which the first event can occur *m* ways and the second event can occur *n* ways, the events together can occur a total of  $m \cdot n$  ways.

**Example**: Suppose you are given a two-question quiz, where the first question is a true/false question and the second question is a multiple choice question with 5 possible answers. Suppose you guess on both questions. How many simple events are in the sample space?

 $2 \cdot 5 = 10$ 

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# 4.6 Counting Rules

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For a sequence of two events in which the first event can occur *m* ways and the second event can occur *n* ways, the events together can occur a total of  $m \cdot n$  ways.

**Example:** Suppose you are given a two-question quiz, where the first question is a true/false question and the second question is a multiple choice question with 5 possible answers. Suppose you guess on both questions. How many simple events are in the sample space?



5

The sample space,  $S = \{Ta, Tb, Tc, Td, Te, Fa, Fb, Fc, Fd, Fe\}$ , has 10 simple events.

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= 10

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Counting the number of simple events in a sample space is one of the hardest problems to deal with when finding probabilities.

### The Multiplication Rule

For a sequence of two events in which the first event can occur *m* ways and the second event can occur *n* ways, the events together can occur a total of  $m \cdot n$  ways.

**Example:** Suppose you roll a pair of dice and record the sum of the two numbers that land on the upper faces of the die. How many simple events are in the sample space?

Roll 2	۰				Probability	
3					2 36	
4					36	
5		<b>B</b> :	60		$\frac{4}{36}$	
6		80	•••		5 36	
7		∎⊙	80	80	<u>6</u> 36	
8		88	80	80	5 36	
9		802	80		4 36	
10	<b>#</b> 11	∎⊡	82		3 36	
11	80	∎⊠			2 36	
12	•••				1 36	

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Counting the number of simple events in a sample space is one of the hardest problems to deal with when finding probabilities.

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Roll 2	۰				Probability
3	٠				2 36
4					3 36
5		88⊙	60		$\frac{4}{36}$
6		80	80		<u>5</u> 36
7		∎⊙	80	80	<u>6</u> 36
8		88	82	80	<u>5</u> 36
9		802	80		4 36
10	80	88			3 36
11	80	<b>B</b> Z			2 36
12					1 36



How many ways can the first die land?

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Counting the number of simple events in a sample space is one of the hardest problems to deal with when finding probabilities.

### The Multiplication Rule

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**Example**: Suppose you roll a pair of dice and record the sum of the two numbers that land on the upper faces of the die. How many simple events are in the sample space?

Roll 2	۰				Probability
3					2 36
4					38
5		80	60		$\frac{4}{36}$
6		80	80		<u>5</u> 36
7		∎⊙	80	82	<u>6</u> 36
8		88	82	80	<u>5</u> 36
9		802	80		$\frac{4}{36}$
10		88			$\frac{3}{36}$
11	80	<b>B</b> Z			2 36
12					$\frac{1}{36}$

6	•	6
1st Die		2nd Die

How many ways can the second die land?

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Counting the number of simple events in a sample space is one of the hardest problems to deal with when finding probabilities.

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**Example:** Suppose you roll a pair of dice and record the sum of the two numbers that land on the upper faces of the die. How many simple events are in the sample space?

Roll					Probability
2					$\frac{1}{36}$
3					2 36
4					36
5		80	80		$\frac{4}{36}$
6		80	80		<u>5</u> 36
7		∎⊙	80	80	<u>6</u> 36
8		•	80	80	<u>5</u> 36
9		802	<b>8</b> ::		$\frac{4}{36}$
10	•	88⊡	82		3 36
11	80				2 36
12					1 36

 $6 \cdot 6 = 36$ 

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# 4.6 Counting Rules

Counting the number of simple events in a sample space is one of the hardest problems to deal with when finding probabilities.

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**Example:** Suppose you roll a pair of dice and record the sum of the two numbers that land on the upper faces of the die. How many simple events are in the sample space?

-					
Roll 2	۰				Probability
3	٠				2 36
4					3 36
5		88⊙	50		$\frac{4}{36}$
6		80	80		<u>5</u> 36
7		∎⊙	80	80	<u>6</u> 36
8		•	80	80	<u>5</u> 36
9		82	80		$\frac{4}{36}$
10		•••	80		3 36
11	80				$\frac{2}{36}$
12					$\frac{1}{36}$

<b>6</b> ·	6	=	36
1st Die	2nd Die		

The sample space, has 36 simple events.

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# 4.6 Counting Rules

### The Extended Multiplication Rule

For a sequence of *k* events in which the first event can occur  $n_1$  ways, the second event can occur  $n_2$  ways, ..., the *k*th event can occur  $n_k$  ways, the number of ways to carry out the the sequence of events is the product

 $n_1 \cdot n_2 \cdot n_3 \cdots n_k$ 

k factors

**Example**: Suppose a couple plans to have three children. How many simple events are in the sample space?

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# 4.6 Counting Rules

### The Extended Multiplication Rule

For a sequence of *k* events in which the first event can occur  $n_1$  ways, the second event can occur  $n_2$  ways, ..., the *k*th event can occur  $n_k$  ways, the number of ways to carry out the the sequence of events is the product

 $\underbrace{n_1 \cdot n_2 \cdot n_3 \cdots n_k}_{k}$ 

k factors

**Example**: Suppose a couple plans to have three children. How many simple events are in the sample space?

girl .	girl	_	girl	E <sub>1</sub> : GGG
		~	boy	E <sub>2</sub> : GGB
		-	girl	E <sub>3</sub> : GBG
	boy	-	boy	E <sub>4</sub> : GBB
	airl	_	girl	E <sub>5</sub> : BGG
boy	girl	/ /	girl boy	E <sub>5</sub> : BGG E <sub>6</sub> : BGB
boy	0	1 1 1	°	U U



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# 4.6 Counting Rules

**Example**: Suppose you have 3 different cars and a 3-car garage. How many different ways can you arrange (order) the way you park the cars in your garage?

garage 1 garage 2 garage 3

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# 4.6 Counting Rules

**Example**: Suppose you have 3 different cars and a 3-car garage. How many different ways can you arrange (order) the way you park the cars in your garage?

garage 1 garage 2 garage 3

How many choices of cars do you have for garage 1?

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# 4.6 Counting Rules

**Example**: Suppose you have 3 different cars and a 3-car garage. How many different ways can you arrange (order) the way you park the cars in your garage?

3 garage 1 garage 2 garage 3

You selected a car and parked it in garage 1. Now how many choices of cars do you have to park in your 2nd garage?

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# 4.6 Counting Rules

**Example**: Suppose you have 3 different cars and a 3-car garage. How many different ways can you arrange (order) the way you park the cars in your garage?

3 2 garage 1 garage 2 garage 3

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# 4.6 Counting Rules

**Example**: Suppose you have 3 different cars and a 3-car garage. How many different ways can you arrange (order) the way you park the cars in your garage?

3 · 2 garage 1 · garage 2 garage 3

You selected a car and parked it in the 2nd garage. Now how many choices of cars do you have left to park in your 3rd garage?

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#### The Multiplication Rule

# 4.6 Counting Rules

Example: Suppose you have 3 different cars and a 3-car garage. How many different ways can you arrange (order) the way you park the cars in your garage?

$$3_{\frac{2}{\text{garage 1}}} \cdot 2_{\frac{2}{\text{garage 2}}} \cdot 1_{\frac{2}{\text{garage 3}}} = 6$$

According to the Multiplication Rule, there are six different parking arrangements.

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# 4.6 Counting Rules

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**Example:** Suppose you have 3 different cars and a 3-car garage. How many different ways can you arrange (order) the way you park the cars in your garage?

 $\cdot 2 \cdot 1 = 6$ garage 1 garage 2 darade 3

Notice this was also equal to 3 factorial.

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# 4.6 Counting Rules

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3	· 2 ·	• 1	=	6
garage 1	garage 2	garage 3		

Notice this was also equal to 3 factorial.

### Definition

The factorial symbol ! denotes the product of decreasing positive whole numbers. For example,

 $4!=4\cdot 3\cdot 2\cdot 1=24$ 

By special definition, 0! = 1.

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### Definition (Factorial Rule)

A collection of *n* different items can be arranged in order *n*! different ways.

(This factorial rule reflects the fact that the first item may be selected in n different ways, the second item may be selected in n - 1 ways, and so on.)

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**Example**: Suppose you own a restaurant that has a delivery service. Suppose you need your driver to make 5 local deliveries in the next hour. How many different routes are possible?

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**Example**: Suppose you own a restaurant that has a delivery service. Suppose you need your driver to make 5 local deliveries in the next hour. How many different routes are possible?

 $5! = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 120$ 

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A collection of *n* different items can be arranged in order *n*! different ways.

Sometimes we have *n* different items to arrange, but we need to select <u>some of them</u> instead of *all* of them.

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For instance, suppose a television producer has four prizes to give away to a studio audience of 50 people. The first prize is a car, the second prize is a \$6000 TV, third prize is a \$2500 gift certificate to the mall, and fourth prize is \$500 cash. How many different ways can the producer select the four prize winners?

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 $50 \cdot 49 \cdot 48 \cdot 47 = 5,527,000$  using the Multiplication Rule

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Another way to obtain the same result is to evaluate  $\frac{50!}{46!}$ , since

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 $50 \cdot 49 \cdot 48 \cdot 47 = 5,527,000$  using the Multiplication Rule

Another way to obtain the same result is to evaluate  $\frac{50!}{46!}$ , since

$$\frac{50!}{46!} = \frac{50 \cdot 49 \cdot 48 \cdot 47 \cdot 46!}{46!} = 50 \cdot 49 \cdot 48 \cdot 47 = 5,527,000$$

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 $50 \cdot 49 \cdot 48 \cdot 47 = 5,527,000$  using the Multiplication Rule

Another way to obtain the same result is to evaluate  $\frac{50!}{46!}$ , since

$$\frac{50!}{46!} = \frac{50 \cdot 49 \cdot 48 \cdot 47 \cdot 46!}{46!} = 50 \cdot 49 \cdot 48 \cdot 47 = 5,527,000$$

This result is generalized by the *permutations rule*: if we have *n* different items available and we want to select *r* of them, then the number of different orderings is n!/(n - r)!

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# Permutations Rule (when items are all different)

### Definition (Permutations Rule)

Requirements:

- There are n different items available, with none of the items identical to any other item under consideration.
- 2 We select r of the n items (without replacement).
- 3 The ordering of the selections matter.

The number of permutations (or sequences) of *r* items selected from *n* available items (without replacement), denoted  $_nP_r$ , is

$$_{n}P_{r}=rac{n!}{(n-r)!}$$

**Example**: There are 13 members on a board of directors. How many different ways can the group select a president, vice-president and treasurer?

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The number of permutations (or sequences) of *r* items selected from *n* available items (without replacement), denoted  $_nP_r$ , is

$$_{n}P_{r}=rac{n!}{(n-r)!}$$

**Example**: There are 13 members on a board of directors. How many different ways can the group select a president, vice-president and treasurer?

$${}_{3}P_{3} = \frac{13!}{(13-3)!} = \frac{13 \cdot 12 \cdot 11 \cdot 10!}{10!} = 1716$$

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# **Combinations Rule**

### Definition (Combinations Rule)

### Requirements:

- 1 There are n different items available.
- 2 We select r of the n items (without replacement).
- 3 The ordering of the selections does not matter.

The number of combinations of *r* items selected from *n* available items (without replacement), denoted  ${}_{n}C_{r}$ , is

$${}_{n}C_{r}=\frac{n!}{(n-r)!r!}$$

**Example**: There are 13 members on a board of directors. How many different ways can the group form a subcommittee with 3 members?

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$${}_nC_r=\frac{n!}{(n-r)!r!}$$

**Example**: There are 13 members on a board of directors. How many different ways can the group form a subcommittee with 3 members?

$$_{13}C_3 = \frac{13!}{(13-3)!3!} = \frac{13 \cdot 12 \cdot 11 \cdot 10!}{10! \cdot 3!} = 286$$

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**Example**: suppose a television producer has four prizes to give away to a studio audience of 50 people. The four prizes are all the same, a \$500 gift certificate to the mall. How many different ways can the producer select the four prize winners?

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The number of combinations of *r* items selected from *n* available items (without replacement), denoted  ${}_{n}C_{r}$ , is

$${}_nC_r=\frac{n!}{(n-r)!\ r!}$$

**Example**: suppose a television producer has four prizes to give away to a studio audience of 50 people. The four prizes are all the same, a \$500 gift certificate to the mall. How many different ways can the producer select the four prize winners?

$${}_{50}C_4 = \frac{50!}{(50-4)!\,4!} = \frac{50!}{46!\cdot4!} = \frac{50\cdot49\cdot48\cdot47\cdot46!}{46!\cdot4!} = \frac{5,527,000}{24} = 230,300$$

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