

Math 160 – Professor Busken
Probability Worksheet 1

Name: Key

Experiment: Toss a single die and observe the number that appears on the upper face.
Here are some possible events:

- Event A Observe an even number
- Event B Observe a number less than 3
- Event E_1 Observe a 1
- Event E_2 Observe a 2
- Event E_3 Observe a 3
- Event E_4 Observe a 4
- Event E_5 Observe a 5
- Event E_6 Observe a 6

} each event is equally likely.

1. Are events A and B mutually exclusive? 1. No
2. Are events E_1 and E_2 mutually exclusive? 2. Yes
3. Are simple events always mutually exclusive? 3. Yes
4. What is the sample space for the experiment? 4. $S = \{E_1, E_2, E_3, E_4, E_5, E_6\}$
5. Compute the following probabilities:
 - $P(E_1) = 1/6 = 0.\overline{16}$
 - $P(E_5) = 1/6 = 0.\overline{16}$
 - $P(A) = 3/6 = 0.50$
 - $P(B) = 2/6 = 0.\overline{3}$
6. Roll a pair of dice. Record the sum of the two numbers that appear on the upper faces of the dice.
 - $P(\text{the sum is } 8) = \frac{5}{36} = 0.1\overline{38}$
 - $P(\text{rolling a double } 1) = 6/36 = 0.\overline{16}$
7. Find the probability that when a couple has three children, they will have exactly 2 girls. Assume that girls and boys are equally likely and that the gender of one child is not influenced by the gender of any other child. $3/8 = 0.375$
8. Pick a card at random from a shuffled deck of cards. Determine the following probabilities:
 - $P(\text{the card is a four of hearts}) = 13/52 = 0.25$
 - $P(\text{the card is a queen}) = 4/52 = 0.0769$
 - $P(\text{the card is not an ace}) = 48/52 = 0.923$

9. In the last 30 years, death sentence executions in the United States included 795 men and 10 women (based on data from the Associated Press). If an execution is randomly selected, find the probability that the person executed is a woman. Is it unusual for a woman to be executed?

$$\frac{10}{805} = 0.0124$$

9. 0.0124

10. Suppose A is any event, either simple or compound. Are the events A and A complement mutually exclusive?

10. yes

11. Women have a 0.25% rate of red/green color blindness. If a woman is randomly selected, what is the probability that she does not have red/green color blindness?

$$P(A) = 1 - P(\bar{A}) = 1 - 0.0025 = 0.9975$$

11. 0.9975

12. Consider again tossing a single die. Determine $P(A \cup B)$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B) = \frac{3}{6} + \frac{2}{6} - \frac{1}{6}$$

12. 0.6

13. Roll a pair of dice again. Record the sum of the two numbers that appear on the upper faces of the dice. Let A be the event the observed pair sums to 10 and let B be the event the observed pair is a double. Determine $P(A \cup B)$.

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= \frac{3}{36} + \frac{6}{36} - \frac{1}{36} = \frac{8}{36} = 0.2$$

13. 0.2

14. Roll a pair of dice again. Record the sum of the two numbers that appear on the upper faces of the dice. Let A be the event the observed pair sums to 10 and let B be the event the observed pair sums to 4. Determine $P(A \cup B)$.

$$P(A \cup B) = P(A) + P(B) - \cancel{P(A \cap B)}$$

$$= \frac{3}{36} + \frac{3}{36} = \frac{6}{36} = 0.1\bar{6}$$

14. 0.16

15. Pick a card at random from a shuffled deck of cards. Let A be the event the observed card is a 4 and let B be the event the card is a heart. Determine $P(A \cup B)$.

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= \frac{4}{52} + \frac{13}{52} - \frac{1}{52} = \frac{16}{52} = .308$$

16. Let A be the event the observed card is a 4 and let B be the event the card is a 10. Determine $P(A \cup B)$.

$$P(A \cup B) = P(A) + P(B) - \cancel{P(A \cap B)}$$

$$= \frac{4}{52} + \frac{4}{52} = \frac{8}{52} = 0.154$$

17. Pick a card at random from a shuffled deck of cards. Two cards are randomly selected without replacement. Find the probability the first card is an ace and the second card is an ten.

$$\begin{aligned}
 P(\text{1st card ace And 2nd card } 10) &= P(A \text{ And } B) \\
 &= P(A) \cdot P(B|A) \quad \text{since A and B are} \\
 &= \frac{4}{52} \cdot \frac{4}{51} = 0.00603 \quad \text{Dependent events}
 \end{aligned}$$

18. Pick a card at random from a shuffled deck of cards. Two cards are randomly selected with replacement. Find the probability the first card is an ace and the second card is an ten.

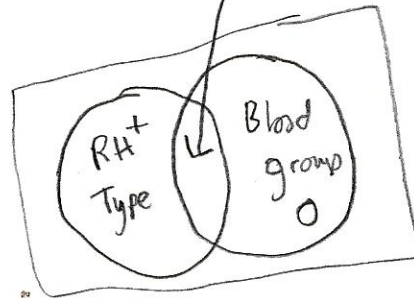
$$\begin{aligned}
 P(A \text{ And } B) &= P(A) \cdot P(B) \quad \text{since A and B are independent} \\
 &= \frac{4}{52} \cdot \frac{4}{52} = 0.00591 \quad \text{events}
 \end{aligned}$$

19. Two cards are randomly selected. Find the probability that the draw includes and ace and a ten.

$$\begin{aligned}
 &P(\underbrace{[\text{1st card } 10 \text{ and 2nd card Ace}]}_A \text{ OR } \underbrace{[\text{1st card Ace and 2nd card } 10]}_B) \\
 &= P(A) + P(B) - P(A \cap B) \\
 &= P(\text{1st card } 10 \text{ And 2nd card Ace}) + P(\text{1st card Ace And 2nd card } 10) \\
 &= \frac{4}{52} \cdot \frac{4}{51} + \frac{4}{52} \cdot \frac{4}{51} = 0.0121
 \end{aligned}$$

20. **Homework #17:** Use the data in the following table, which summarizes blood type and Rh types for 100 subjects.

		Blood Type ^{group}			
		O	A	B	AB
Rh Type	Rh ⁺	39	35	8	4
	Rh ⁻	5	6	2	1



If 2 out of the 100 subjects are randomly selected, find the probability that they are both blood group O and Rh type Rh⁺.

- Assume that the selections are made with replacement.
- Assume that the selections are made without replacement.

Let A be the event "the 1st person selected is O⁺" and

let B " " " " " the 2nd person selected is O⁺."

a) $P(A \cap B) = P(A) \cdot P(B)$ since A and B are dependent event

$$= \frac{39}{100} \cdot \frac{39}{100} = 0.1521$$

b) $P(A \cap B) = P(A) \cdot P(B|A)$ since A and B are independent events

$$= \frac{39}{100} \cdot \frac{38}{99} \approx 0.1497$$

21. With one method of a procedure called acceptance sampling, a sample of items is randomly selected without replacement and the entire batch is accepted if every item in the sample is okay. The Telektronics Company manufactured a batch of 400 back up power supply units for computers, and 8 of them are defective. If 3 of the units are randomly selected for testing, what is the probability that the entire batch will be accepted?

We need $P(\text{All 3 units okay})$ which equals

$$P(\text{1st unit okay AND 2nd unit okay AND 3rd unit okay})$$

$$= \frac{392}{400} \cdot \frac{391}{399} \cdot \frac{390}{398}$$

Since there are 3 dependent, simple events that compose the compound event "All 3 units okay."