Math 160 – Professor Busken
Probability Worksheet 2 Name: Key

1. A study conducted at a certain college shows that 59% of the school's graduates find a job in their chosen field within a year after graduation. Find the probability that among 6 randomly selected graduates, at least one finds a job in his or her chosen

① Let A = the event "at least one of the 6 have found a job in their chosen field within a year of graduation."

② Then $\bar{A}$ = the event "none of the 6 found a job..."

= 1st person selected no job AND 2nd person no job AND 3rd person NO job AND 4th person no job AND...

③ The probability that a person didn't find a job in their chosen field within a year is $1 - 59\% = 1 - 0.59 = 0.41$. Then

$P(\bar{A}) = P(\text{1st person no job}) \cdot P(\text{2nd person no job}) \cdot P(\text{3rd person no job}) \cdots$

~~2. In a batch of 8,000 clock radios 6sample of 8 clock radios is randomly selected without replacement from the 8,000 and tested. The entire batch will be rejected if at least one of those tested is defective. What is the probability that the entire batch will be rejected?~~

$\cdot P(\text{4th person no job}) \cdot P(\text{5th person no job}) \cdot P(\text{6th person no job})$

$= 0.41 \cdot 0.41 \cdot 0.41 \cdot 0.41 \cdot 0.41 \cdot 0.41$

$= (0.41)^6 = 0.00475$ to 3 significant figures.

Then $P(\text{At least}) = 1 - P(\text{none})$, or

$P(A) = 1 - P(\bar{A}) = 1 - 0.00475 = \boxed{0.9952}$

Conclusion

There is a 99.52% chance that at least one of the 6 selected has found a job in their chosen field within a year of graduation.

(2) In a batch of 8000 clock radios, 6% are defective. A sample of 8 clock radios is randomly selected without replacement from the 8,000 and tested. The entire batch is rejected if at least one of those tested is defective. What is the probability that the entire batch will be rejected?

Soln: (1) Let A = the event at least 1 of the 8 is defective

= 1 or more of the clocks are defective

(2) Then $\bar{A}$ = the event none of the 8 are defective

= 1st radio not defective AND 2nd radio not defective AND 3rd radio not defective AND 4th radio not defective AND the 5th radio is not defective AND... the 8th radio is not defective

Since the 8 are selected "without replacement," whether or not the 2nd clock is defective depends on whether or not the 1st clock is defective; And whether or not the third clock is defective depends on whether or not the 2nd clock was defective, and so on and so forth. Thus, when finding $P(\bar{A})$ we use the multiplication rule for dependent events.

(3) $P(\bar{A}) = P(\text{1st not defective}) \cdot P(\text{2nd not defective}) \cdots P(\text{8th not defective})$

$$= \frac{7520}{8000} \cdot \frac{7519}{7999} \cdot \frac{7518}{7998} \cdot \frac{7517}{7997} \cdot \frac{7516}{7996} \cdot \frac{7515}{7995} \cdot \frac{7514}{7994} \cdot \frac{7513}{7993} \approx 0.6094$$

④ Then $P(\text{at least } 1) = 1 - P(\text{none})$,

or $P(A) = 1 - P(\bar{A}) = 1 - 0.6094 \approx 0.3906$

There is a 39% chance the entire lot will be rejected.

Alternate Soln The calculation of $P(\bar{A})$ was cumbersome. We could have used the 5% guideline from page 164, which allows us to treat the dependent events as being independent (since the sample size, 8, is no more that 5% of the population of 8,000 clock radios). In that case, we could have said

$$P(\bar{A}) = (0.94)\cdot(0.94)\cdot(0.94)\cdot(0.94)\cdot(0.94)\cdot(0.94)\cdot(0.94)(0.94)$$

$$= (0.94)^8 \doteq 0.6095,$$

and then

$$P(A) = 1 - P(\bar{A}) \approx 1 - 0.6095 = 0.3904$$

Table 4 - 1 Results from Experiments with Polygraph Instruments

	No (Did Not Lie)	Yes (Lied)	totals
Positive Test Result (The polygraph test indicated that the subject *lied.*)	15 (false positive)	42 (true positive)	57
Negative Test Result (The polygraph test indicated that the subject did not *lie.*)	32 (true negative)	9 (false negative)	41
totals	47	51	98

3. If one of the 98 subjects is randomly selected, find the probability that the subject had a positive test result, given that the subject actually lied. That is find P(positive test result|subject lied).

$$P\left(\text{subject had a positive test result} \mid \text{the subject lied}\right) = \frac{42}{51} \approx 0.8235$$

Alternate soln $\quad P(B|A) = \dfrac{P(A \text{ And } B)}{P(A)} = \dfrac{\frac{42}{98}}{\frac{51}{98}} = \dfrac{42}{\cancel{98}} \cdot \dfrac{\cancel{98}}{51} = \dfrac{42}{51}$

4. If one of the 98 subjects is randomly selected, find the probability that the subject actually lied, given that he or she had a positive test result.

$$P\left(\text{subject lied} \mid \text{positive result}\right) = \frac{42}{57}$$

Alt. Soln

$$P(B|A) = \frac{P(A \text{ and } B)}{P(A)} = \frac{\frac{42}{98}}{\frac{57}{98}} = \frac{42}{\cancel{98}} \cdot \frac{\cancel{98}}{57} = \frac{42}{57} \doteq 0.7368$$

	Nonsmoker	Light Smoker	Heavy Smoker	Total
Men	306	74	66	446
Women	345	68	81	494
Total	651	142	147	940

Consider the following events:

Event N: The person selected is a nonsmoker
Event L: The person selected is a light smoker
Event H: The person selected is a heavy smoker
Event M: The person selected is a male
Event F: The person selected is a female

5. Suppose one of the 940 subjects is chosen at random. Determine the following probabilities:

a. $P(N|F)$ $= \frac{P(N \cap F)}{P(F)} = \frac{\frac{345}{940}}{\frac{494}{940}} = \frac{345}{940} \cdot \frac{940}{494} = \frac{345}{494} \doteq 0.6984$

b. $P(F|N)$ $= \frac{P(F \cap N)}{P(N)} = \frac{\frac{345}{940}}{\frac{651}{940}} = \frac{345}{940} \cdot \frac{940}{651} \doteq 0.53$

c. $P(H \cup M)$ $= P(H) + P(M) - P(H \cap M) = \frac{147}{940} + \frac{446}{940} - \frac{66}{940} \doteq 0.5606$

d. $P(M \cap L)$ $= \frac{74}{940}$

e. P(the person selected is a smoker)

$= P(L \cup H) = P(L) + P(H) - P(L \cap H) = \frac{142}{940} + \frac{147}{940} - 0 \doteq 0.3074$

f. $P(F \cap \overline{H})$

$= P($Female and not a heavy smoker$) = \frac{345+68}{940} = \frac{413}{940} = 0.4394$

6. Now suppose that two people are selected from the group, ***without replacement***. Let A be the event "the first person selected is a nonsmoker," and let B be the event "the second person is a light smoker." What is $P(A \cap B)$?

$P(A \cap B) = P(A) \cdot P(B|A) = \frac{651}{940} \cdot \frac{142}{939} = \frac{9114}{882660} = 0.1047$

7. Two people are selected from the group, ***with replacement***. What is the probability that both people are nonsmokers?

$= P($1st person selected is a non-smoker AND the 2nd person is a non smoker$)$

$= P($1st person non-smoker$) \cdot P($2nd person non smoker$) = \frac{651}{940} \cdot \frac{651}{940} = 0.4796$

8. Two people are selected from the group. What is the probability that both people are smokers?

$P(\underline{L}\ \underline{L}$ or $\underline{L}\ \underline{H}$ or $\underline{H}\ \underline{L}$ or $\underline{H}\ \underline{H})$

We can use the 5% guideline and treat events independent

$= P(LL) + P(LH) + P(HL) + P(HH) = \left(\frac{142}{940}\right)^2 + \frac{142}{940} \cdot \frac{147}{940} + \frac{147}{940} \cdot \frac{142}{940} + \left(\frac{147}{940}\right)^2$

$\doteq 0.0945$