Ch. 5 Wkshts

1

a) There are 4 digits to select, __ __ __ __, or 4 slots to fill. A digit can take on 1 of ten values from 0, 1, 2, ..., 9. A digit is allowed to be selected more than once, for example 1111 is a valid 4-digit selection. Using the multiplication rule for counting, the number of different possible selections is

$$\underline{10}\cdot\underline{10}\cdot\underline{10}\cdot\underline{10} = 10^4 = 10{,}000$$

b) There are 10,000 different, equally likely sequences, so

$$P(\text{winning}) = 1/10{,}000 = 0.0001$$

c) net profit $= -50¢ + \$2000 = \$1999.50$

d) Expected Value: let x = the

event	x	P(x)	x · P(x)
win	\$1999.50 net profit	$1/10{,}000 = 0.0001$	\$0.19995
lose	−\$0.50	$9999/10{,}000 = 0.9999$	−\$0.49995

$$\mu = \sum x\cdot p(x) = -\$0.30$$

$$= \boxed{-30¢}$$

(2)

x	$P(x)$
4	0.15
5	0.30
6	0.25
7	0.30

Let X = the number of games it takes for a team to win a playoff series.

← probability distribution (given)

Using the 5% rule, it would not be unusual since $30\% \not\leq 5\%$,

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(3) a) calculator $\mu = 7.9$ years

b) calculator $\sigma^2 = 67.14 - 7.9^2 = 4.73$, so $\sigma = \sqrt{\sigma^2} = \sqrt{4.73} \approx 2.2$

c) $X_{max} = \mu + 2\sigma = 7.9 + 2(2.2) = 12.3$ yrs

$X_{min} = \mu - 2\sigma = 7.9 - 2(2.2) = 3.5$ yrs

So, the range of usual patent life lengths is between 3.5 years and 12.3 years.

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(4) Not a binomial distribution since there are more that 2 possible outcomes (namely, there are 6) on each trial (roll).

(5) Yes: there are $n=19$ trials (rolls), each independent, with 2 possible outcomes ("roll a 5" or "didn't roll a 5"), and the probability of success, $P(5) = 1/6$, is the same in all 19 trials.

(6) Not a binomial distribution, since the 5 trials are not all independent.

(7) $n=7$ trials, dependent, with 2 possible outcomes (red or not red), but the prob. of getting a red marble is not the same for all 7 trials.

(8)

$X=3$
$n=10$
$p=0.2$
$q=0.8$

Q3	Q4	Q5
1st right question	2nd Right Question	3rd Right Question

$$P(x) = \underbrace{(nC_x)}_{\text{The number of outcomes with exactly } x\text{-successes among } n \text{ trials}} \cdot \underbrace{p^x \cdot q^{n-x}}_{\text{The probability of 3 right ("successes") among } n=10 \text{ questions (trials)}}$$

(the # of ways to choose 3 right answers from 10 questions)
order of the 3 right questions does not matter

$$P(3) = ({}_{10}C_3) \cdot (0.2)^3 \cdot (0.8)^{10-3} \doteq \boxed{0.201}$$

or use binompdf(n, p, x)

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(9)

X = the number of students who recognize the brand name
$n=10$

$p=0.55$

$q=0.45$

find $P(x \neq 4)$

$P(x\neq 4) = P(x=0) + P(x=1) + P(x=2) + P(x=3) + P(x=5)$ [redacted] $+ P(x=10)$

$= 1 - P(X=4)$ by the complement rule.

$= 1 - \text{binompdf}(n, p, x) \approx \boxed{0.8404}$

$= 1 - ({}_{10}C_4)(0.55)^4(0.45)^6$

(10) $X = $ # of correct answers

$n = 10$

$p = 0.5$ Find $P(x \geq 6)$

$q = 0.5$

$P(x \geq 6) = P(x=6) + P(7) + P(8) + P(9) + P(10)$ ← Add these 5 values from the binomial tables, or

$= 1 - P(x \leq 5) = 1 - \text{binomcdf}(n, p, 5) \approx \boxed{0.3770}$

use the complement rule since the binomcdf function yields cumulative probabilities. (or just sum the probabilities in the table associated with $x = 6, 7, 8, 9$ and 10.) That is, the command "binomcdf" provides the sum of all probabilities from $x = 0$ through the specific value of x. Whereas, binompdf(n, p, x) provides the probability associated with a single value of x.

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(11) $p = 0.42$, $q = 0.58$

$n = 11$

$x = $ the # of people in excellent health

Find $P(x \leq 3)$

$P(x \leq 3) = P(3) + P(2) + P(1) + P(0)$

$= \text{binomcdf}(n, p, 3) = \boxed{0.2510}$

(12)

X is the discrete random variable: number of persons who show up (out of 24 booked)

$p = 0.0995$

$q = 1 - 0.0995 = 0.9005$

$n = 24$ bookings

Find $P(X = 23 \text{ or } X = 24)$.

$P(X = 23 \cup X = 24) = P(23) + P(24)$

$= \text{binompdf}(n, q, 23) + \text{binompdf}(n, q, 24)$

$= 0.295$ this is not low.

overbooking should be a real concern.

13. $P(\text{exactly none}) = P(x=0) = ({}_{50}C_0) \cdot (0.07)^0 \cdot (0.93)^{50}$

$$\overset{\text{or}}{=} \text{binompdf}(n, p, x)$$

$$= \text{binompdf}(50, 0.07, 0) \approx \boxed{0.2656}$$

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14. $P(\text{not } 5) = P(x \neq 5)$

$$= P(x = 0 \text{ or } 1 \text{ or } 2 \text{ or } 3 \text{ or } 4 \text{ or } 6 \text{ or } 7 \text{ or } \ldots \text{ or } 50)$$

$$= P(x=0) + P(x=1) + \ldots + P(x=4) + P(x=6) + P(x=7) + \ldots + P(x=50)$$

use the complement rule $P(A) = 1 - P(\bar{A})$

$$= 1 - P(x=5) = 1 - \left[{}_{50}C_5 \cdot (0.07)^5 \cdot (0.93)^{50-5}\right]$$

$$\overset{\text{or}}{=} 1 - \text{binompdf}(n, p, x)$$

$$= 1 - \text{binompdf}(50, 0.07, 5) \approx \boxed{0.8741}$$

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15. $P(\text{at least } 5) = P(x \geq 5) = P(x=5) + P(x=6) + \ldots + P(x=49) + P(x=50)$

$$= 1 - P(x < 5)$$

The complement of $x \geq 5$ is $x < 5$

$$= 1 - P(x \leq 4)$$

$$= 1 - [P(x=0) + P(x=1) + P(x=2) + P(x=3) + P(x=4)]$$

$$= 1 - \text{binomcdf}(n, p, 4)$$

c for cumulative probabilities

$$= 1 - \text{binomcdf}(50, 0.07, 4)$$

$$\approx 0.2710$$

(16) $P(\text{at most } 5) = P(x \leq 5)$

$= P(0) + P(1) + P(2) + P(3) + P(4) + P(5)$

$= \text{binomcdf}(n, p, x)$

$= \text{binomcdf}(50, 0.07, 5) = 0.8650$

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(17) $P(\text{will not exceed } 3) = P(x \not> 3)$ (will not be greater than or equal to)

$= P(x \leq 3)$

$= P(0) + P(1) + P(2) + P(3)$

$= \text{binomcdf}(50, 0.07, 3) = 0.5327$

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(18) $P(\text{must exceed } 3) = P(x > 3)$

$= P(4) + P(5) + P(6) + \ldots + P(49) + P(50)$

$= 1 - P(x \leq 3)$ by the complement rule. We do this because the binomial cumulative density function (ie, "binomcdf") provides the sum of all probabilities from $x = 0$ through the specific value of x; And, in this case, there are less probabilities to add up. The last expression is equal to $= 1 - [P(0) + P(1) + P(2) + P(3)]$

$= 1 - \text{binomcdf}(n, p, x)$ in terms of calculator syntax

$= 1 - \text{binomcdf}(50, 0.07, 3) = 0.4673$

(19) P(is less than 3) = P(x < 3)

= P(x ≤ 2) because x is a discrete random variable

= P(x=0) + P(x=1) + P(x=2)

= binomcdf(50, 0.07, 2) = 0.3108

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(20) P(is more than 3) = P(x > 3) = P(4) + P(5) + P(6) + ... + P(50)

= P(x ≥ 4) ← because x is a discrete random variable

= 1 − P(x ≤ 3) by the complement rule

= 1 − binomcdf(50, 0.07, 3) = 0.4673

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(21) P(between 2 and 5 inclusive)

= P(2 ≤ x ≤ 5) = P(2) + P(3) + P(4) + P(5). We can put the probability distribution table in L1 and L2, then we can get the values of each of the 4 probabilities from off of the table, then sum them. In order to install the dist. table in L1 and L2, we place the numbers 0 through 50 in L1. To do this, open your spreadsheet with [stat] [enter], then arrow over and up until your cursor highlights L1. Press:

[2nD] [stat] [▶] . [5] [X,T,Θ,n] [,] [X,T,Θ,n] [,] [0] [,] [5] [0] [enter]

Then, to get the corresponding probabilities in L2, we type:

[2nd] [VARS] [0] [5] [0] [,] [.] [0] [7] [enter]

comma ↑ ↑ decimal

(21) (continued)

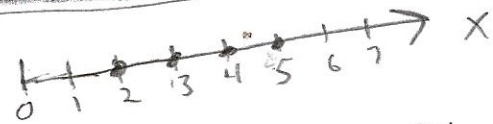
The distribution is in L1 and L2

$P(2 \le X \le 5) = P(2) + P(3) + P(4) + P(5)$

$\doteq 0.1843 + 0.22195 + 0.19629 + 0.13593 \approx 0.7385$

~~21 Alt soln: binomcdf(N,P,5) – binomcdf(N,P,1) ≈ 0.7385~~

(21) Alternate Soln.



$P(2 \le X \le 5) = P(X \le 5) - P(X \le 1)$

$= \text{binomcdf}(50, 0.07, 5) - \text{binomcdf}(50, 0.07, 1)$

$= 0.7385$

(22) $P(2 < X < 5) = P(3 \le X \le 4)$

$= P(3) + P(4)$

$= \text{binompdf}(n, p, 3) + \text{binompdf}(n, p, 4)$

≈ 0.4182

(23) P(is a minimum of 5) = P(5 or more) = $P(X \ge 5)$ = See Soln 15

(24) P(is a maximum of 5) = P(5 or less) = $P(X \le 5)$ = binomcdf(n, p, 5)

(25) P(is no more than 5) = P(5 or less) = $P(X \le 5)$

(26) P(no less than 5) = P(5 or more) = $P(X \ge 5)$ = 1 – binomcdf(n, p, 5)