Math 160 Professor Busken
Chapter 5 Worksheets **Name:** _____

1. **Find the expected value.** Suppose you play a "Pick 4 Lotto" where you pay 50¢ to select a sequence of four digits, such as 2118. If you select the same sequence of digits that are drawn, you win and collect $2000.
   (a) How many different selections are possible?
   (b) What is the probability of winning?
   (c) If you win, what is your net profit?
   (d) Find the expected value.

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2. Assume that there is a 0.15 probability that a basketball playoff series will last four games, a 0.30 probability that it will last five games, a 0.25 probability that it will last six games, and a 0.30 probability that it will last seven games. Is it ***unusual*** for a team to win a series in 5 games?

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3. The maximum patent life for a new drug is 17 years. Subtracting the length of time required by the FDA for testing and approval of the drug provides the actual patent life of the drug—that is, the length of time that a company has to recover research and development costs and make a profit. Suppose the distribution of the lengths of patent life for new drugs is as shown here:

Years, x	3	4	5	6	7	8	9	10	11	12	13
$p(x)$	.03	.05	.07	.10	.14	.20	.18	.12	.07	.03	.01

   (a) Find the expected number of years of patent life for a new drug.
   (b) Find the standard deviation of the probability distribution.
   (c) Use the Range Rule of Thumb to identify the range of values associated with ***usual*** patent life lengths.

Definition 1. *A **Random Experiment** is an experiment, trial, procedure or observation that can be repeated numerous times under the same conditions. The outcome of an individual random experiment must in no way be affected by any previous outcome and cannot be predicted with certainty.*

Accompanying this experiment is

1. a ***sample space*** (all possible outcomes of the experiment),
2. a ***probability*** (assigned to each outcome in the experiment)
3. a ***random variable***, and
4. a ***probability distribution***.

Definition 2. *A variable x is a **Random Variable** if the numerical value that it assumes, corresponding to an outcome of an experiment, is a chance or random event.*

Definition 3. *A **Probability Distribution** lists the probabilities associated with each possible outcome in the sample space for a procedure, trial or random experiment. A probability distribution can be written as a table, formula, or graph (called a probability histogram).*

The main topic of Chapter 5 is the study of ***Discrete*** Probability Distributions —which are tables of probabilities associated with random variables that take on discrete (or integer) values.

Many probability distributions are so important in theory or applications that they have been given specific names (see wikipedia topic: list of probability distributions). One specific ***Discrete*** Probability Distribution from this list is called the ***Binomial Distribution***, the topic of Section 5.3. The binomial distribution is the probability distribution that results from doing a ***"binomial experiment."***

Definition 4. ***Binomial experiments*** *have the following properties:*

1. *The procedure has a fixed number of trials.*
2. *The trials must be independent. (The outcome of any individual trial doesnt affect the probabilities in the other trials.)*
3. *Each trial must have only two possible outcomes (commonly referred to as success*[1] *and failure).*
4. *The probability of a success remains the same in all trials.*

Notation for Binomial Probability Distributions:

S and F (success and failure) denote the two possible categories of all outcomes; p and q will denote the probabilities of S and F, respectively.

$P(S) = p$	(p =probability of success)
$P(F) = 1 - p = q$	(q =probability of failure)
n	denotes the fixed number of trials.
x	denotes a specific number of successes in n trials, so x can be any whole number between 0 and n, inclusive.
p	denotes the probability of success in one of the n trials.
q	denotes the probability of failure in one of the n trials.
$P(x)$	denotes the probability of getting exactly x successes among the n trials.

The Binomial Probability Formula

$$P(x) = \left({}_nC_x\right) \cdot p^x \cdot q^{n-x}$$

for $x = 0, 1, 2, ..., n$, and recall that ${}_nC_x = \dfrac{n!}{(n-x)! \cdot x!}$.

Example: Assume that a procedure yields a binomial distribution with a trial repeated $n = 4$ times. Use the binomial probability formula to find the probability of $x = 3$ successes given the probability of success on a single trial is $p = \frac{1}{5}$. Round to three decimal places.

For questions, 4—7, determine whether the given procedure results in a binomial distribution. If not, state the reason why.

4. Rolling a single die 37 times, keeping track of the numbers that are rolled.

5. Rolling a single die 19 times, keeping track of the "fives" rolled.

6. Choosing 5 people (without replacement) from a group of 64 people, of which 15 are women, keeping track of the number of men chosen.

7. Choosing 7 marbles from a box of 40 marbles (20 purple, 12 red, and 8 green) one at a time without replacement, keeping track of the number of red marbles chosen.

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8. A multiple choice test has 10 questions each of which has 5 possible answers, only one of which is correct. If Judy, who forgot to study for the test, guesses on all questions, what is the probability that she will answer exactly 3 questions correctly?

9. The brand name of a certain chain of coffee shops has a 55% recognition rate at Grossmont College. An executive from the company wants to verify the recognition rate as the company is interested in opening a coffee shop on the college. She selects a random sample of 10 Grossmont College students. Find the probability that the number that recognize the brand name is not 4.

10. A test consists of 10 true/false questions. To pass the test a student must answer at least 6 questions correctly. If a student guesses on each question, what is the probability that the student will pass the test?

11. In a study, 42% of adults questioned reported that their health was excellent. A researcher wishes to study the health of people living close to a nuclear power plant. Among 11 adults randomly selected from this area, only 3 reported that their health was excellent. Find the probability that when 11 adults are randomly selected, 3 or fewer are in excellent health.

12. An airline estimates that 9.95% of people who book a flight *do not* actually show up. If the airline books 24 people for a flight on which there is 22 seats, what is the probability that the number of people who show up will exceed the capacity of the plane? Is this probability low enough so that overbooking is not a real concern?

Binomial Experiment

People with type O-negative blood are said to be "universal donors." About 7% of the U.S. population has this blood type. Suppose that 50 people show up at a blood drive. Let x =the number of universal donors among a random group of 50 people.

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n	This is the number of trials. For this example, $n = 50$ (the number of blood donors).
p	This is the "success" probability. For this example, $p = 0.07$ (the probability that a randomly selected American has type O-negative blood). Note that p must be in decimal form.
x	This is the number of "successes," or type-O negative donors

13. Find the probability that ***exactly*** none are type-O negative donors.

14. Find the probability that ***not*** 5 are type-O negative donors.

15. Find the probability that ***at least*** 5 people are type-O negative.

16. Find the probability that ***at most*** 5 people are type-O negative.

Find the probability that the number of type-O negative donors who show up:

17. ***will not exceed*** 3.

18. ***must exceed*** 3.

19. ***is less than*** 3.

20. ***is more than*** 3.

21. ***is between 2 and 5, inclusive***.

22. ***is between 2 and 5***.

23. ***is a minimum of*** 5.

24. ***is a maximum of*** 5.

25. ***is no more than*** 5.

26. ***is no less than*** 5.

TABLE A-1 Binomial Probabilities

		p												
n	x	.01	.05	.10	.20	.30	.40	.50	.60	.70	.80	.90	.95	.99	x
2	0	.980	.902	.810	.640	.490	.360	.250	.160	.090	.040	.010	.002	0+	0
	1	.020	.095	.180	.320	.420	.480	.500	.480	.420	.320	.180	.095	.020	1
	2	0+	.002	.010	.040	.090	.160	.250	.360	.490	.640	.810	.902	.980	2
3	0	.970	.857	.729	.512	.343	.216	.125	.064	.027	.008	.001	0+	0+	0
	1	.029	.135	.243	.384	.441	.432	.375	.288	.189	.096	.027	.007	0+	1
	2	0+	.007	.027	.096	.189	.288	.375	.432	.441	.384	.243	.135	.029	2
	3	0+	0+	.001	.008	.027	.064	.125	.216	.343	.512	.729	.857	.970	3
4	0	.961	.815	.656	.410	.240	.130	.062	.026	.008	.002	0+	0+	0+	0
	1	.039	.171	.292	.410	.412	.346	.250	.154	.076	.026	.004	0+	0+	1
	2	.001	.014	.049	.154	.265	.346	.375	.346	.265	.154	.049	.014	.001	2
	3	0+	0+	.004	.026	.076	.154	.250	.346	.412	.410	.292	.171	.039	3
	4	0+	0+	0+	.002	.008	.026	.062	.130	.240	.410	.656	.815	.961	4
5	0	.951	.774	.590	.328	.168	.078	.031	.010	.002	0+	0+	0+	0+	0
	1	.048	.204	.328	.410	.360	.259	.156	.077	.028	.006	0+	0+	0+	1
	2	.001	.021	.073	.205	.309	.346	.312	.230	.132	.051	.008	.001	0+	2
	3	0+	.001	.008	.051	.132	.230	.312	.346	.309	.205	.073	.021	.001	3
	4	0+	0+	0+	.006	.028	.077	.156	.259	.360	.410	.328	.204	.048	4
	5	0+	0+	0+	0+	.002	.010	.031	.078	.168	.328	.590	.774	.951	5
6	0	.941	.735	.531	.262	.118	.047	.016	.004	.001	0+	0+	0+	0+	0
	1	.057	.232	.354	.393	.303	.187	.094	.037	.010	.002	0+	0+	0+	1
	2	.001	.031	.098	.246	.324	.311	.234	.138	.060	.015	.001	0+	0+	2
	3	0+	.002	.015	.082	.185	.276	.312	.276	.185	.082	.015	.002	0+	3
	4	0+	0+	.001	.015	.060	.138	.234	.311	.324	.246	.098	.031	.001	4
	5	0+	0+	0+	.002	.010	.037	.094	.187	.303	.393	.354	.232	.057	5
	6	0+	0+	0+	0+	.001	.004	.016	.047	.118	.262	.531	.735	.941	6
7	0	.932	.698	.478	.210	.082	.028	.008	.002	0+	0+	0+	0+	0+	0
	1	.066	.257	.372	.367	.247	.131	.055	.017	.004	0+	0+	0+	0+	1
	2	.002	.041	.124	.275	.318	.261	.164	.077	.025	.004	0+	0+	0+	2
	3	0+	.004	.023	.115	.227	.290	.273	.194	.097	.029	.003	0+	0+	3
	4	0+	0+	.003	.029	.097	.194	.273	.290	.227	.115	.023	.004	0+	4
	5	0+	0+	0+	.004	.025	.077	.164	.261	.318	.275	.124	.041	.002	5
	6	0+	0+	0+	0+	.004	.017	.055	.131	.247	.367	.372	.257	.066	6
	7	0+	0+	0+	0+	0+	.002	.008	.028	.082	.210	.478	.698	.932	7
8	0	.923	.663	.430	.168	.058	.017	.004	.001	0+	0+	0+	0+	0+	0
	1	.075	.279	.383	.336	.198	.090	.031	.008	.001	0+	0+	0+	0+	1
	2	.003	.051	.149	.294	.296	.209	.109	.041	.010	.001	0+	0+	0+	2
	3	0+	.005	.033	.147	.254	.279	.219	.124	.047	.009	0+	0+	0+	3
	4	0+	0+	.005	.046	.136	.232	.273	.232	.136	.046	.005	0+	0+	4
	5	0+	0+	0+	.009	.047	.124	.219	.279	.254	.147	.033	.005	0+	5
	6	0+	0+	0+	.001	.010	.041	.109	.209	.296	.294	.149	.051	.003	6
	7	0+	0+	0+	0+	.001	.008	.031	.090	.198	.336	.383	.279	.075	7
	8	0+	0+	0+	0+	0+	.001	.004	.017	.058	.168	.430	.663	.923	8

NOTE: 0+ represents a positive probability less than 0.0005.

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