Chapter 5

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Mathematics Department

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Definition

A ***Random Experiment*** is an experiment, trial, procedure or observation that can be repeated numerous times under the same conditions. The outcome of an individual random experiment must in no way be affected by any previous outcome and cannot be predicted with certainty.

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2. a ***probability*** (assigned to each outcome in the experiment)
3. a ***random variable***, and
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A variable x is a ***Random Variable*** if the numerical value that it assumes, corresponding to an outcome of an experiment, is a chance or random event.

Experiment: Toss a single die

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A ***Probability Distribution*** lists the probabilities associated with each possible outcome in the sample space for a procedure, trial or random experiment. A probability distribution can be written as a table, formula, or graph (called a probability histogram).

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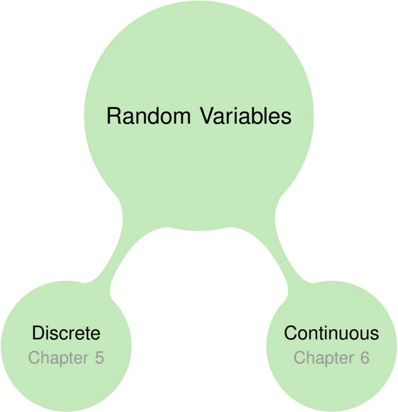
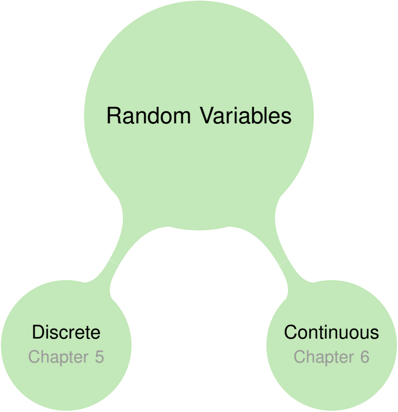

Random Variables
Discrete
Chapter 5
Continuous
Chapter 6

Random Variables can be discrete!



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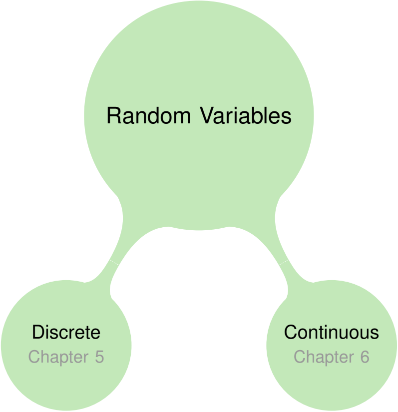

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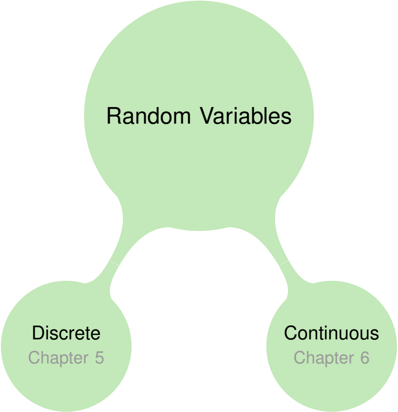

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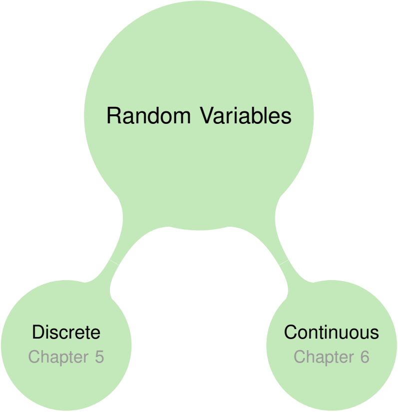

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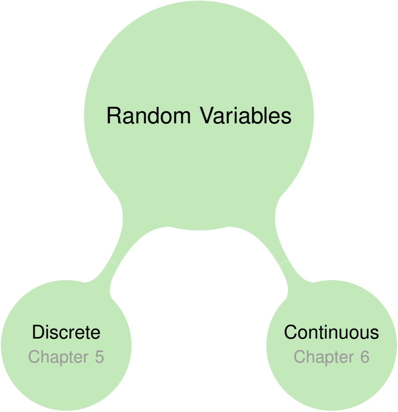

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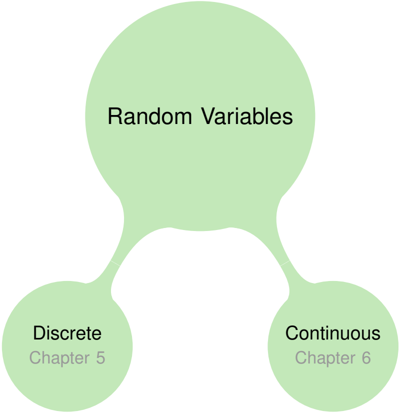

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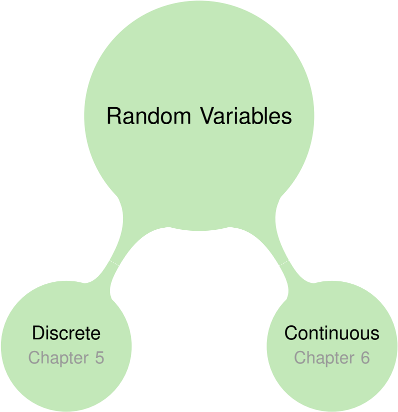

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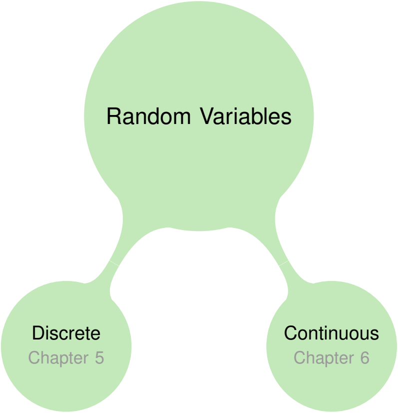

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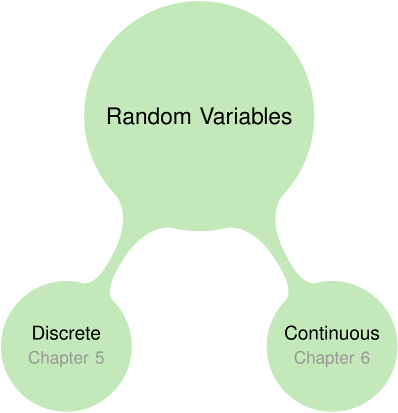

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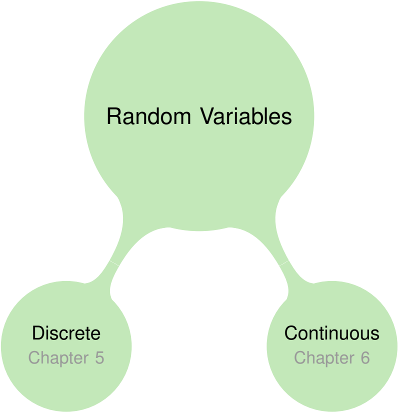

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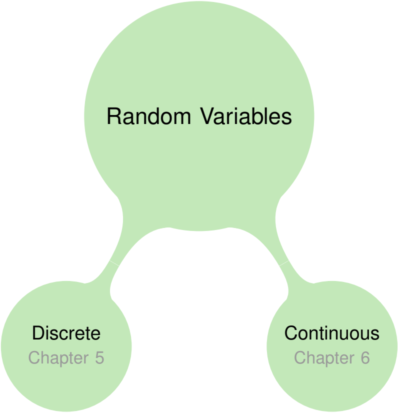

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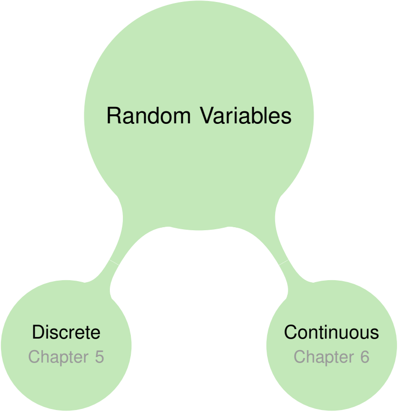

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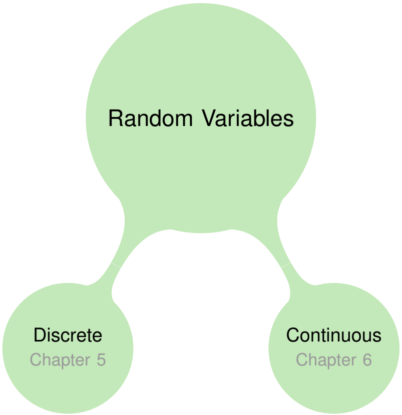

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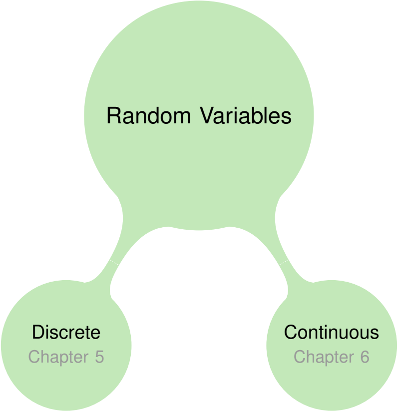

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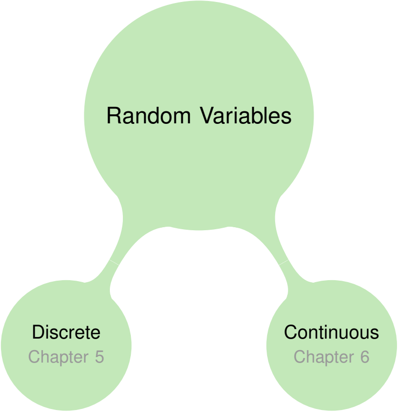

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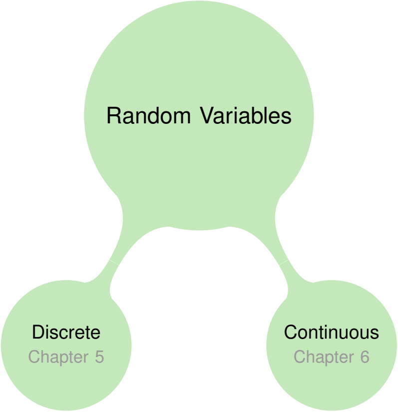

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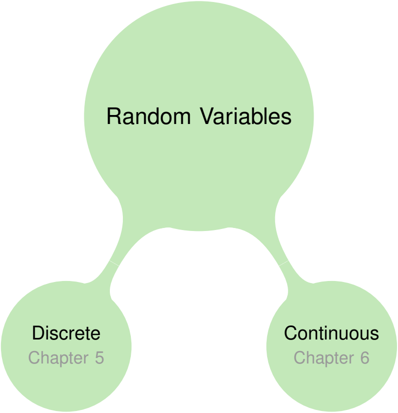

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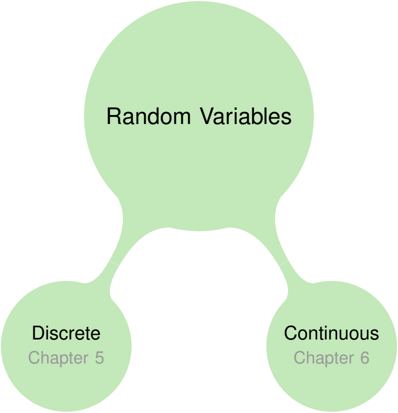

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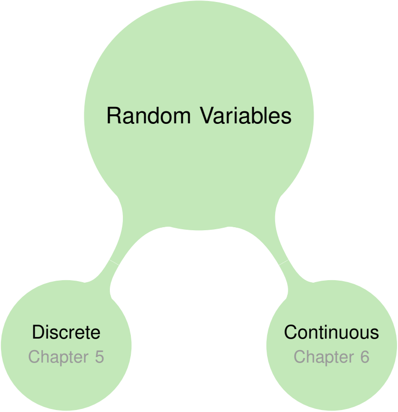

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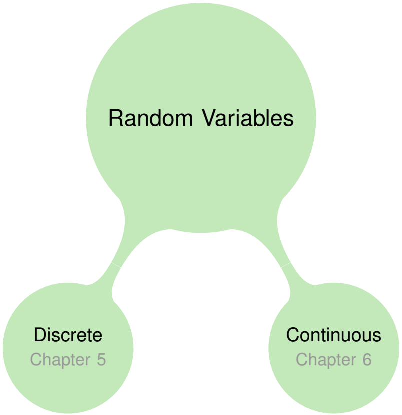

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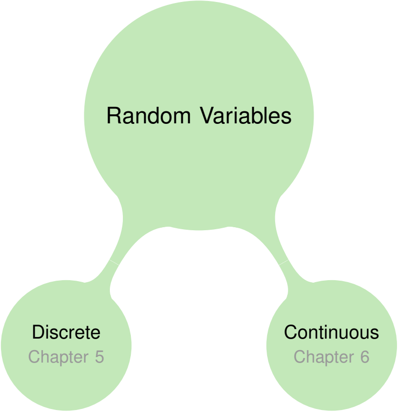

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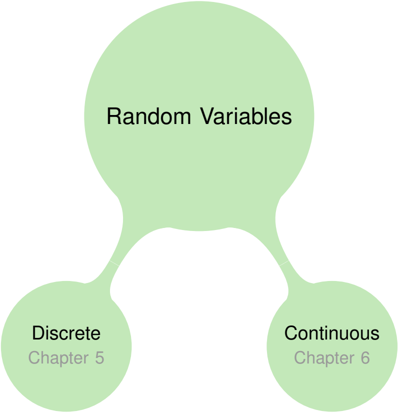

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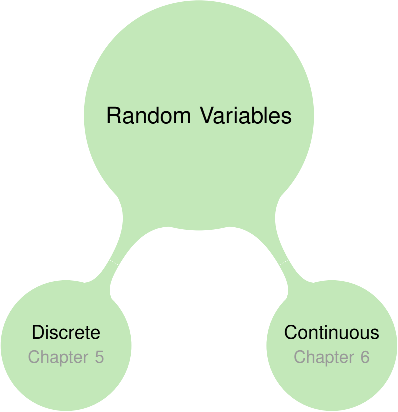

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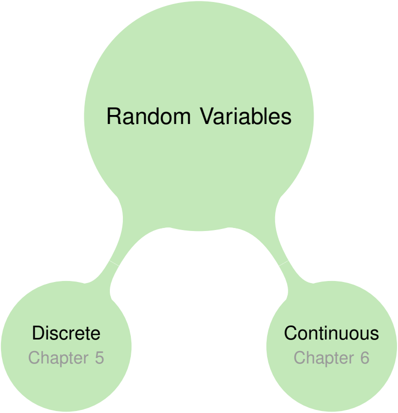

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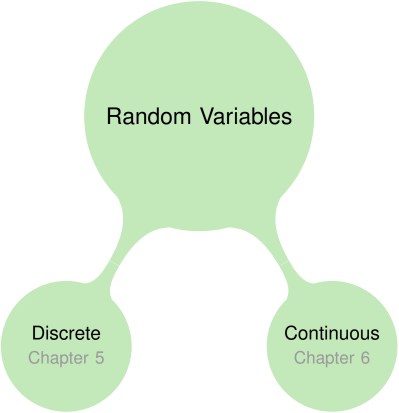

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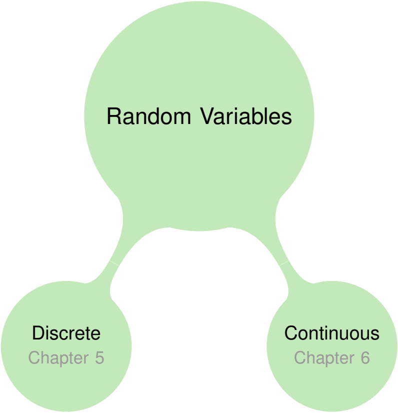

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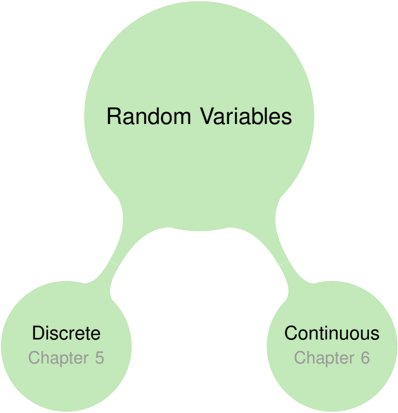

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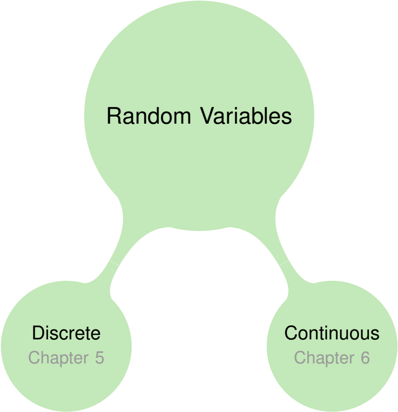

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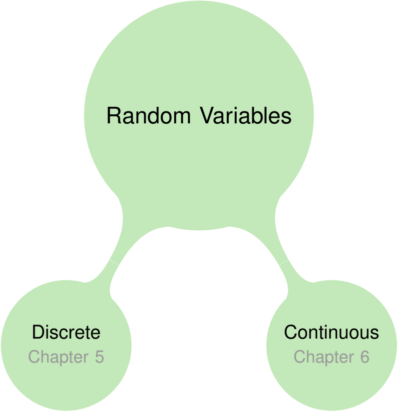

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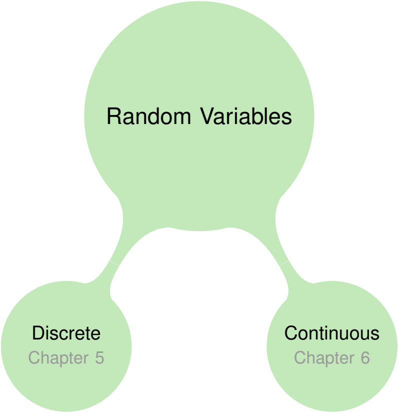


Know the difference between discrete and continuous RVs.

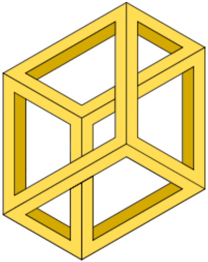
Definition (Discrete random variable)

either a finite number of values or countable number of values, where countable refers to the fact that there might be infinitely many values, but they result from a counting process

Definition (Continuous random variable)

infinitely many values, and those values can be associated with measurements on a continuous scale without gaps or interruptions

helpful hint



Random Variables

Begin Quiz **Identify the given random variable as being discrete or continuous.**

1. The number of snow storms occurring off the eastern coast of the U.S.

(a) Discrete (b) Continuous

2. The height of an ocean's tide at your favorite beach.

(a) Discrete (b) Continuous

3. The length of a king salmon

(a) Discrete (b) Continuous

4. The braking time of a car

(a) Discrete (b) Continuous

End Quiz Score: Correct

Random Variables

Begin Quiz **Identify the given random variable as being discrete or continuous.**

1. The number of phone calls made during the election on behalf of special interests.

(a) Discrete (b) Continuous

2. The number of gallons of milk produced by a single cow.

(a) Discrete (b) Continuous

3. The number of students present at graduation.

(a) Discrete (b) Continuous

4. The number of aircraft near-collisions in a year

(a) Discrete (b) Continuous

End Quiz Score: Correct

Discrete Probability Distributions

Example: As the owner of successful small business, you cannot afford to take a sick day. Suppose the random variable x represents the number of \$1000 units you sell in a day. Additionally, suppose that based on years of company records, the probability distribution is summarized in the table (right).

x	$P(x)$
0	0.191
1	0.314
2	0.363
3	0.123
4	0.009

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Requirements for Probability Distribution

1.) $\sum P(x) = 1$ The sum of all the probabilities must be 1, but values such as 0.999 or 1.001 are acceptable because they result from rounding errors.

2.) $0 \leq P(x) \leq 1$ for every each value of x. (*i.e.*, each probability value must be between 0 and 1 inclusive.)

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Begin Quiz **Identify the correct probability.**

1. P(3)

(a) 0.991 (b) 0.363 (c) 0.123 (d) 0.515

2. The probability you sell at least 2 \$1000 units.

(a) 0.363 (b) 0.132 (c) 0.495 (d) 0.505

3. The probability you sell less than 3 \$1000 units.

(a) 0.868 (b) 0.991 (c) 0.123 (d) 0.515

4. $P(x < 2)$

(a) 0.314 (b) 0.505 (c) 0.363 (d) 0.515

5. The probability you sell at least 1 unit.

(a) 0.314 (b) 0.727 (c) 0.948 (d) 0.809

End Quiz Score: Correct

Discrete Probability Distributions

Example: Air America has a policy of routinely overbooking flights. The random variable x represents the number of passengers who cannot be boarded because there are more passengers than seats.

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Quiz **Does the given table fit the requirements for a probability distribution?**

☐ yes ☐ no

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The given table does not fit the requirements for a probability distribution because the sum of the probabilities in the table is not equal to 1.

Discrete Probability Distributions

Example: Sam's Used Carpet. The random variable x represents the number of used carpets sold in a day at Sam's store

x	$P(x)$
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Quiz **Does the given table fit the requirements for a probability distribution?**

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The given table does not fit the requirements for a probability distribution because there is a probability (namely −0.231) that is not a value between 0 and 1 inclusive.

Probability Histogram

Example: As the owner of successful small business, you cannot afford to take a sick day. Suppose the random variable x represents the number of \$1000 units you sell in a day. Additionally, suppose that based on years of company records, the probability distribution is summarized in the table (right).

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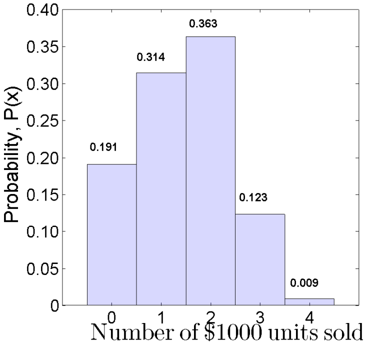

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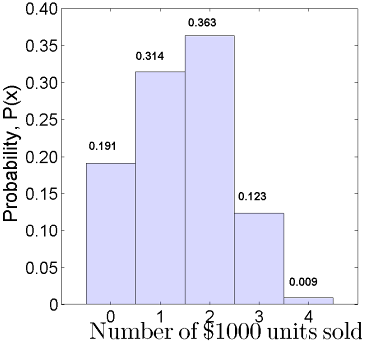
Notice that it is similar to a relative frequency histogram, but the vertical scale shows *probabilities* instead of relative frequencies based on actual sample results.



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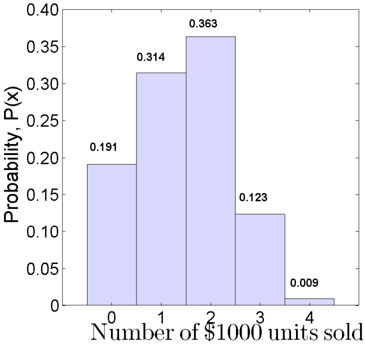
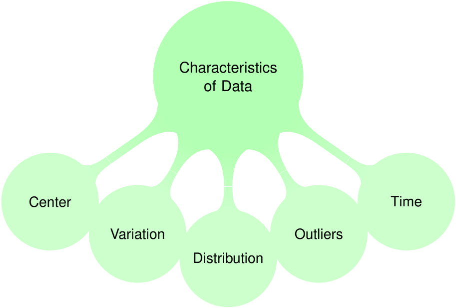
Notice that it is similar to a relative frequency histogram, but the vertical scale shows *probabilities* instead of relative frequencies based on actual sample results.

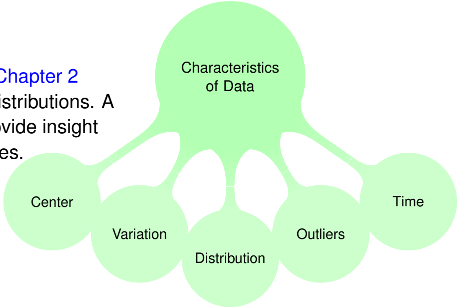
We see the values of 0, 1, 2, 3, 4 along the horizontal axis are located at the center of the rectangle. This implies that the rectangles are each 1 unit wide, so the areas of the rectangles are 0.191, 0.314, 0.363, 0.123, 0.009. The *areas* of these rectangles are the same as the probabilities in the table (above).

In later chapters, we will see that the correspondence between probabilities and area is hugely useful in statistics.




Characteristics of Data
Center
Variation
Distribution
Outliers
Time

The five characteristics of data from Chapter 2 can be used to describe probability distributions. A Probability histogram or table can provide insight into the distribution of random variables.



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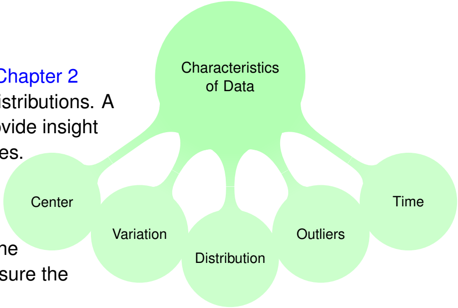

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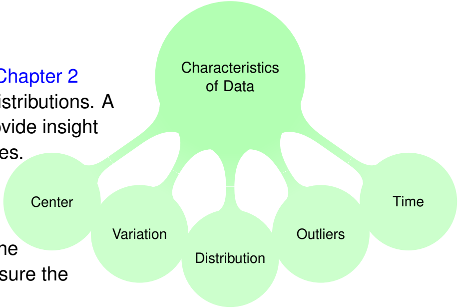

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Measures of Center and Variation for probability distributions

$$\mu = \sum [x \cdot P(x)]$$ Mean

$$\sigma^2 = \sum [(x-\mu)^2 \cdot P(x)]$$ Variance

$$\sigma^2 = \sum [x^2 \cdot P(x)] - \mu^2$$ Variance (computational shortcut formula)

$$\sigma = \sqrt{\sigma^2} = \sqrt{\sum [x^2 \cdot P(x)] - \mu^2}$$ Standard Deviation

Mean $\mu = \sum[x \cdot P(x)]$

Example: As the owner of successful small business, you cannot afford to take a sick day. Suppose the random variable x represents the number of \$1000 units you sell in a day. Additionally, suppose that based on years of company records, the probability distribution is summarized in the table (below).

Determine the mean value for the distribution.

x	$P(x)$
0	0.191
1	0.314
2	0.363
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4	0.009	$4 \cdot 0.009 = 0.036$

Multiply straight across. This is called multiplying the two columns elementwise.

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Now sum all the entries in the third column.

$$\mu = \sum[x \cdot P(x)] = 1.445$$

Mean $\mu = \sum[x \cdot P(x)]$

How can we do this with the calculator?

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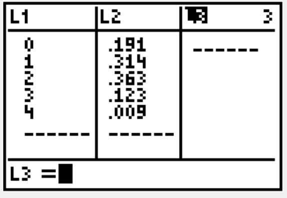

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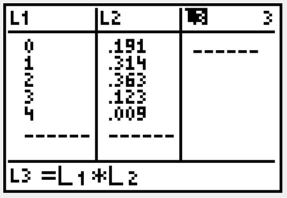

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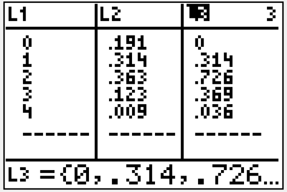

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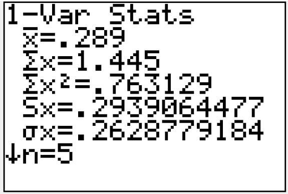

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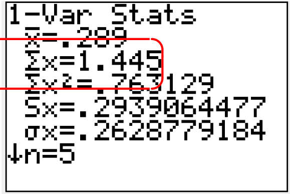

Variance $\sigma^2 = \sum[x^2 \cdot P(x)] - \mu^2$

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Square each individual value of x, then multiply it by its associated probability, $P(x)$. List these products in a separate column.

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Afterwards, subtract μ^2 from $\sum[x^2 \cdot P(x)]$.

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We take the square root of σ^2 to get the standard deviation. $\sigma = \sqrt{0.9289975} \approx 0.9638$

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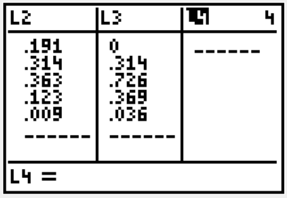

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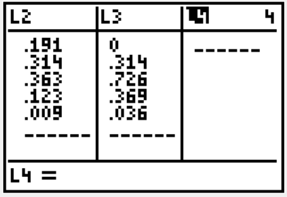

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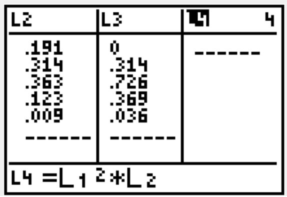

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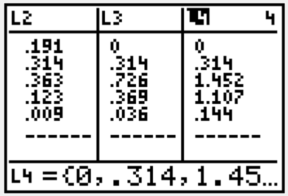

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- Calculate 1-variable statistics on L4 and take $\sum x$ to be $\sum[x^2 \cdot P(x)]$. Subtract μ^2 from this value to obtain the variance.

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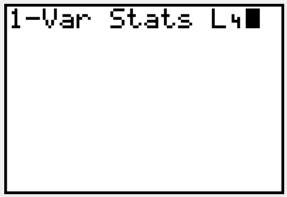

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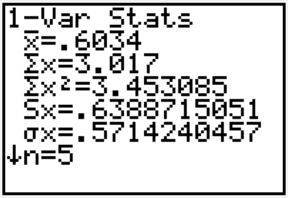

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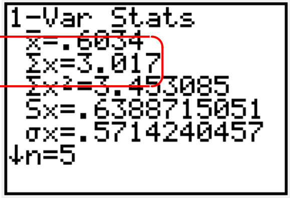


Round off Rule for μ, σ, and σ^2

Round results by carrying one more decimal place than the number of decimal places used for the random variable x. If the values of x are integers, round μ, σ and σ^2 to one decimal place. Do not round off any intermediate calculations

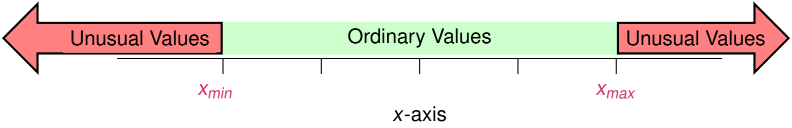
Identifying Unusual Results

Range Rule of Thumb

We can identify "unusual" values by determining if they lie outside these limits:

Maximum usual value, x_{max} $\quad x_{max} = \mu + 2\sigma$

Minimum usual value , x_{min} $\quad x_{min} = \mu - 2\sigma$



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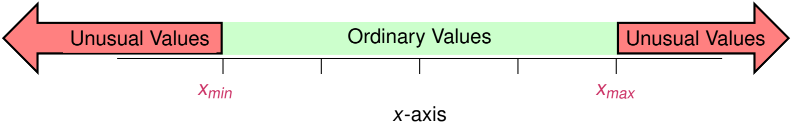

Example: Focus groups of 14 people are randomly selected to discuss products of the Yummy Company. It is determined that the mean number (per group) who recognize the Yummy brand name is 10.9, and the standard deviation is 0.98.

Quiz **Would it be unusual to randomly select 14 people and find that fewer than 7 recognize the Yummy brand name?**

☐ yes ☐ no

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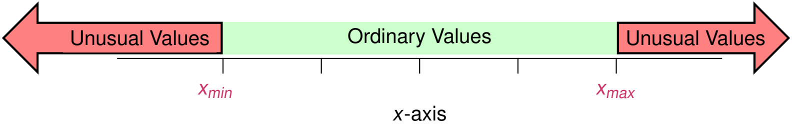

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Quiz **Would it be unusual to randomly select 14 people and find that fewer than 7 recognize the Yummy brand name?**

☐ yes ☐ no

x is the random variable representing the number of people (from a sample of 14) that recognize the Yummy brand name. $x_{min} = \mu - 2\sigma = 10.9 - 2 \cdot 0.98 = 8.94$ and $x_{max} = \mu + 2\sigma = 10.9 + 2 \cdot 0.98 = 12.86$. Since 7 people is less than x_{min}, it is considered unusual to randomly select 14 people and find that fewer than 7 recognize the Yummy brand name.

Identifying Unusual Results

Using Probabilities to Determine When Results Are Unusual

1. **Unusually high**: x successes among n trials is an unusually high number of successes if $P(x \text{ or more}) \leq 0.05$.
2. **Unusually low**: x successes among n trials is an unusually low number of successes if $P(x \text{ or fewer}) \leq 0.05$.

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Example: Suppose that weight of adolescents is being studied by a health organization and that the accompanying tables describes the probability distribution for three randomly selected adolescents, where x is the number who are considered morbidly obese.

x	$P(x)$
0	0.111
1	0.215
2	0.450
3	0.224

Quiz **Is it unusual to have no obese subjects among three randomly selected adolescents?**

☐ yes

☐ no

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Quiz **Is it unusual to have no obese subjects among three randomly selected adolescents?**

☐ yes ☐ no

It is not unusual since $0.111 \not\leq 0.05$

Expected Value

Definition

The **expected value** of a discrete random variable is denoted by E, and it represents the mean value of the outcomes. It is obtained by finding the value of $\sum[x \cdot P(x)]$.

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Example: Suppose you pay \$2.00 to roll a fair die with the understanding that you will get back \$4 for rolling a 2 or a 4, nothing otherwise.

Begin Quiz

1. What is your expected winnings from a single roll? Hint: let x be the discrete random variable representing the amount of money won or lost.

(a) $-\$0.67$ (b) \$2.00 (c) \$4.00 (d) $-\$2.00$

End Quiz Score: Correct

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Example: Suppose you pay \$2.00 to roll a fair die with the understanding that you will win \$4 for rolling a 2 or a 4, and win nothing otherwise.

Begin Quiz

1. What is your expected winnings from a single roll? Hint: let x be the discrete random variable representing the amount of money won or lost.

(a) –\$0.67 (b) \$2.00 (c) \$4.00 (d) –\$2.00

End Quiz Score: Correct

Event	x	$P(x)$	$x \cdot P(x)$
Lose	–\$2	4/6	–\$1.33
Gain (net)	\$2	2/6	\$0.67
total			–\$0.67

Now try questions 1, 2 and 3 on the worksheet that is attached to this document (or click here.)

5.3 The Binomial Distribution

The main topic of Chapter 5 is the study of ***Discrete*** Probability Distributions —which are tables of probabilities associated with random variables that take on discrete (or integer) values.

Many probability distributions are so important in theory or applications that they have been given specific names (see wikipedia topic: list of probability distributions). One specific ***Discrete*** Probability Distribution from this list is called the ***Binomial Distribution***, the topic of Section 5.3. The binomial distribution is the probability distribution that results from doing a ***"binomial experiment."***

Binomial Experiments

Definition

Binomial experiments have the following properties:

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1. The procedure has a fixed number of trials.
2. The trials must be independent. (The outcome of any individual trial doesnt affect the probabilities in the other trials.)
3. Each trial must have only two possible outcomes (commonly referred to as success and failure).
4. The probability of a success remains the same in all trials.

The word success in this context is arbitrary and does not necessarily represent something good. Either of the two possible categories may be called a success.

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Binomial Experiments

Notation for Binomial Probability Distributions

S and F (success and failure) denote the two possible categories of all outcomes; p and q will denote the probabilities of S and F, respectively.

$P(S) = p$	(p =probability of success)
$P(F) = 1 - p = q$	(q =probability of failure)
n	denotes the fixed number of trials.
x	denotes a specific number of successes in n trials, so x can be any whole number between 0 and n, inclusive.
p	denotes the probability of success in one of the n trials.
q	denotes the probability of failure in one of the n trials.
$P(x)$	denotes the probability of getting exactly x successes among the n trials.

The Binomial Probability Formula

$$P(x) = \left({}_nC_x\right) \cdot p^x \cdot q^{n-x}$$

for $x = 0, 1, 2, ..., n$, and recall that ${}_nC_x = \dfrac{n!}{(n-x)! \cdot x!}$.

Binomial Experiments

An Example of a Binomial Experiment

People with type O-negative blood are said to be "universal donors." About 7% of the U.S. population has this blood type. Suppose that 50 people show up at a blood drive. Let $x =$the number of universal donors among a random group of 50 people.

---

n	This is the number of trials. For this example, $n = 50$ (the number of blood donors).
p	This is the "success" probability. For this example, $p = 0.07$ (the probability that a randomly selected American has type O-negative blood). Note that p must be in decimal form.
x	This is the number of "successes," or type-O negative donors

Binomial Experiments

Classroom Exercise

Now try the problems on the worksheet that is attached to this document, or click here.

http://users.rowan.edu/~schultzl/TI/binomial.pdf In addition, check out the useful calculator tutorial by Dr. Laura Schultz from Rowan University in N.J. (the pdf is also attached to this pdf document.)

5.4 Mean, Variance, and Standard Deviation for the Binomial Distribution

We defined and gave the formulas for the Mean, Variance, and Standard Deviation *for Any* Discrete Probability Distribution in Section 5.2.

Measures of Center and Variation for probability distributions

$\mu = \sum[x \cdot P(x)]$	Mean
$\sigma^2 = \sum[(x-\mu)^2 \cdot P(x)]$	Variance
$\sigma^2 = \sum[x^2 \cdot P(x)] - \mu^2$	Variance (shortcut formula)
$\sigma = \sqrt{\sigma^2} = \sqrt{\sum[x^2 \cdot P(x)] - \mu^2}$	Standard Deviation

When applied to the Binomial Probability Distribution, these formulas reduce to the following.

Measures of Center and Variation for the Binomial Probability Distribution

$\mu = n \cdot p$	Mean
$\sigma^2 = n \cdot p \cdot q$	Variance
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When applied to the Binomial Probability Distribution, these formulas reduce to the following.

Measures of Center and Variation for the Binomial Probability Distribution

$\mu = n \cdot p$	Mean
$\sigma^2 = n \cdot p \cdot q$	Variance
$\sigma = \sqrt{\sigma^2} = \sqrt{npq}$	Standard Deviation

Before you attempt to complete the homework for Section 5.4, please read pages 197, 224, and 225 from the Triola[2] textbook. (A copy is also attached to this document with the name mendel.pdf.)

L. Schultz, *Using your ti-83/84 calculator: Binomial probability distributions*. http://users.rowan.edu/~schultzl/TI/binomial.pdf. Accessed: 03/16/13.

M. F. Triola, *Essentials of Statistics*, Addison-Wesley, fourth ed., 2011.