Math 160
Professor Busken
The Central Limit Theorem **Name:** ____________________

Theorem 1 (The Central Limit Theorem)**.** *If random samples of n observations are drawn from a nonnormal population with mean μ and standard deviation σ, then, when n is large, the sampling distribution of the sample means is approximately normally distributed, with mean and standard deviation*

$$\mu_{\overline{x}} = \mu \quad \textit{and} \quad \sigma_{\overline{x}} = \frac{\sigma}{\sqrt{n}}$$

The approximation becomes more accurate as n becomes large.

The CLT can be applied to any probability distribution (continuous or discrete). We will use the following guidelines for our work:

☞ *For a population with any distribution, if $n > 30$, then the sample means have a distribution that can be approximated by a normal distribution with mean μ and standard deviation $\sigma/\sqrt{n}$.*

☞ *If $n \leq 30$ and the original population has a normal distribution, then the sample means have a normal distribution with mean μ and standard deviation $\sigma/\sqrt{n}$.*

☞ *If $n \leq 30$ and the original population does not have a normal distribution, then we do not apply the CLT.*

☞ *When the sampled population is approximately symmetric, the sampling distribution of $\overline{x}$ becomes approximately normal for relatively small values of n.*

There are two very different question types that read similarly, and it can be hard to distinguish between the two.

1. **Individual Value**: When working with an individual value from a normally distributed population, use $z = \dfrac{x - \mu}{\sigma}$

2. **Sample of Values**: When working with a mean for some *sample* (or group), use $z = \dfrac{x - \mu_x}{\sigma_x}$

1. Some passengers died when a water taxi sank in Baltimore's inner harbor. Men are typically heavier than women and children, so when loading a water taxi, let's assume a worst-case scenario in which all passengers are men. Based on data from the National Health and Nutrition Survey, assume that weights of men are normally distributed with a mean of 172 lb. and a standard deviation of 29 lb.

   (a) Find the probability that if an individual man is randomly selected, his weight will be greater than 175 lb.

   (b) Find the probability that 20 men will have a mean weight that is greater than 175 lb. (so that their total weight exceeds the safe capacity of 3500 lb.

2. The scores on a certain test are normally distributed with a mean score of 64 and a standard deviation of 15. What is the probability that a sample of 90 students will have a mean score of at least 70?

3. The weights of the fish in a certain lake are normally distributed with a mean of 11 lb and a standard deviation of 12. If 16 fish are randomly selected, what is the probability that the mean weight will be between 8.6 and 14.6 lb?

4. In one region, the September energy consumption levels for single-family homes are found to be normally distributed with a mean of 1050 kWh and a standard deviation of 218 kWh. If 50 different homes are randomly selected, find the probability that their mean energy consumption level for September is greater than 1075 kWh.

5. Assume that women's heights are normally distributed with a mean of 63.6 inches and a standard deviation of 2.5 inches. If 90 women are randomly selected, find the probability that they have a mean height between 62.9 inches and 64.0 inches.