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Chapter 6
Continuous Random Variables

## Chapter 6

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The main focus of Chapter 6 is two-fold!


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Continuous random variables (CRVs) can take on numerical values that fall in an interval where there are no gaps between the numbers.

Examples of CRVs: distance, speed, time, shelf life of foods and medicines, heights and weights, volumes, surface areas.


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Unlike discrete random variables, CRVs take on an infinite number of values in an interval. If you try to assign a probability to each of the infinite values in the interval, the sum of the probabilities is no longer 1 (or 100\%)!

So, we must take a different approach.


$\overline{\text { An example of a discrete probability distribution (left figure) and a continuous probability distribution (right) are }}$ shown above. Remember that not all probability distributions are bell-shaped!

## Probability Density Function <br> (PDF)

Suppose you have a set of measurements on a continuous random variable and you create a relative frequency histogram to describe their distribution. For a small number of measurements, you could use a small number of classes; then as more and more measurements are collected, you can use more classes, and reduce the class width.

The outline of the histogram will change slightly, for the most part becoming less and less irregular, as shown in the animation (right).

As the number of measurements becomes large and the class widths become more narrow, the relative frequency histogram appears more and more like a smooth, continuous curve.

This smooth curve describes the probability distribution of the continuous random variable, and is called a probability density function.


Not all PDFs are bell-shaped!

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## Concept

$\checkmark$ If we take an infinite number of measurements and shrink the class width to near zero, then the histogram outline takes on the shape of a smooth curve. We can show mathematically that the area under the smooth, PDF curve is 1-resulting in a correspondence between area and probability.

As class widths decrease, more rectangles are required to construct the probability histogram. Additionally, once the class widths shrink to zero, there are an infinite number of rectangles under the curve, so every real number in the interval becomes a distinct class. As a result of this construction, for any particular value of $x$, such as $x=a$,

$$
P(x=a)=0 .
$$

That is, the probability associated with any single value of $x$ is zero. This is a major difference between continuous random variables and discrete random variables. Therefore, for continuous random variables, we can only determine the probability that $x$ will be between two values.

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## How to Find Probabilities for Continuous Random Variables

(1) First identify the correct probability density function (PDF) that is associated with the continuous random variable, $x$.
(2) The probability that a continuous random variable $x$ assumes a value in the interval from $a$ to $b$ is the area under the PDF between vertical lines $x=a$ and $x=b$.


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## Properties of PDFs

Definition
A probability density function (PDF) is the graph of a continuous probability distribution. It must satisfy the following properties:
(1) The total area under the curve must equal 1.
(2) Every point on the curve must have a vertical height that is 0 or greater. (That is, the curve cannot fall below the $x$-axis.)

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## Uniform Distribution

Definition
A continuous random variable, $x$ has a uniform distribution if its values are spread evenly over the range of probabilities. The graph of a uniform distribution results in a rectangular shape.

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Example: Wait times at the bus stop are uniformly distributed between 0 and 15 minutes.

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Example: Wait times at the bus stop are uniformly distributed between 0 and 15 minutes. This means that any wait time between 0 minutes and 15 minutes is possible and all of the possible wait times are equally likely.

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Example: Wait times at the bus stop are uniformly distributed between 0 and 15 minutes. This means that any wait time between 0 minutes and 15 minutes is possible and all of the possible wait times are equally likely.

If we randomly select one of the wait times and represent its value by the random variable, $x$, then $x$ has a probability distribution described by the 1st quadrant rectangular graph above: the area under probability density function (PDF), $f(x)=\frac{1}{15}$, bounded by vertical lines $x=0$ and $x=15$.

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(1) The total area under the curve must equal 1.
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Example: Wait times at the bus stop are uniformly distributed between 0 and 15 minutes. $\overline{\text { Determine }}$ the probability that a randomly selected wait time is between 8 and 12 minutes.

Solution: Compute the area under the uniform PDF from 8 to 12 minutes:

$$
P(8<x<12)=\text { base } \cdot \text { height }=(12-8) \cdot \frac{1}{15} \doteq 0.2667
$$

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## Click this text to try a similar exercise.

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## The Normal Probability Distribution

## Definition

If a continuous random variable has a probability distribution with a graph that is symmetric and bell-shaped, and it can be described by the function equation

$$
f(x)=\frac{1}{\sigma \sqrt{2 \pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^{2}} \quad-\infty \leq x \leq \infty
$$

then we say it has a normal distribution.

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then we say it has a normal distribution.

Note that $\pi \doteq 3.1416$ and $e \doteq 2.7183$ in the formula. When the parameters $\mu$ and $\sigma$ are fixed constant, the above equation becomes a function of a single variable $x$; and a particular normal distribution is determined.

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Note that $\pi \doteq 3.1416$ and $e \doteq 2.7183$ in the formula. When the parameters $\mu$ and $\sigma$ are fixed constant, the above equation becomes a function of a single variable $x$; and a particular normal distribution is determined.

The figure (right) shows four different normal probability curves determined by different values of these parameters. We show in another class that the area underneath each of these curves, between the $x$ axis and $f(x)$, is 1 .

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$$
f(x)=\frac{1}{\sigma \sqrt{2 \pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^{2}} \quad-\infty \leq x \leq \infty
$$

then we say it has a normal distribution.

The mean, $x=\mu$, locates the center of the distribution. The vertical line, $x=\mu$ is an axis of symmetry for the PDF.

The population standard deviation, $\sigma$, affects the shape of the distribution.

Large values of $\sigma$ decrease the height of the peak and increase the spread of the distribution (along the $x$ axis; small values of $\sigma$ raise the height of the peak and decrease the spread.


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T⿴囗十丁口⿹丁口一 The probability that a continuous random variable $x$ assumes a value in the interval from $a$ to $b$ is the area under the probability density function between vertical lines $x=a$ and $x=b$ ．


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四 The probability that a continuous random variable $x$ assumes a value in the interval from $a$ to $b$ is the area under the probability density function between vertical lines $x=a$ and $x=b$.

T1 Since normal curves have different population means and standard deviations, there are infinitely many large number of normal distributions.

A separate table listing the areas for each of these curves is obviously impractical.

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## Tabulated Areas of the Normal <br> Probability Distribution

（1⿴囗⿰丨丨丁口 The probability that a continuous random variable $x$ assumes a value in the interval from $a$ to $b$ is the area under the probability density function between vertical lines $x=a$ and $x=b$ ．

四 Since normal curves have different population means and standard deviations， there are infinitely many large number of normal distributions．

四 A separate table listing the areas for each of these curves is obviously impractical．
四 Instead，we use a standardization procedure that allows us to use the same table for all normal distributions．


## Converting to the Standard Normal Distribution

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## Definition

A normal random variable $x$ is standardized by expressing its value as the number of standard deviations $(\sigma)$ it lies to the left or right of its mean $\mu$. The standardized normal random variable, $z$, is defined as

$$
\begin{aligned}
& z=\frac{x-\mu}{\sigma} \\
& x=\mu+z \sigma
\end{aligned}
$$

or equivalently,

From the formula for $z$, we can draw the following conclusions.
$\checkmark$ When $x$ is less than the mean $\mu$, the value of $z$ is negative.
$\checkmark$ When $x$ is greater than the mean $\mu$, the value of $z$ is positive.
$\checkmark$ When $x=\mu$, the value of $z=0$


The probability distribution for $z$ is shown in the figure (right) is called the standard normal distribution because its mean is 0 and its standard deviation is 1 .

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## Tabulated Areas of the Normal Probability Distribution

(四 This standardization process helps us convert normal distributions whose mean is not 0 or whose standard deviation is not 1 to the standard normal distribution.

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四 This standardization process helps us convert normal distributions whose mean is not 0 or whose standard deviation is not 1 to the standard normal distribution.
(四 We do this (because it works and) so that we may use the same table of probabilities when working with any normal distribution.

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四 This standardization process helps us convert normal distributions whose mean is not 0 or whose standard deviation is not 1 to the standard normal distribution．

四 We do this（because it works and）so that we may use the same table of probabilities when working with any normal distribution．

四 That table of probabilities is Table A2 in the back of your textbook． （click here to view a copy）

## Using Table A－2

四 It is designed only for the standard normal distribution，which has a mean of 0 and a standard deviation of 1 ．

| Standard Normal（z）Distribution： |  |  |  |  |  | Tumulative Area from the LEFT |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $z$ | ． 00 | .01 | ． 02 | ． 03 | ． 04 | ． 05 | .06 | ． 07 | ． 08 | ． 09 |
| $\begin{aligned} & -3.50 \\ & \text { and } \\ & \text { lower } \end{aligned}$ | ． 0001 |  |  |  |  | 2 |  |  |  |  |
| －3．4 | ． 0003 | ． 0003 | ． 0003 | ． 0003 | ． 0003 | ． 0003 | ． 0003 | ． 0003 | ． 0003 | ． 0002 |
| －3．3 | ． 0005 | ． 0005 | ． 0005 | ． 0004 | ． 0004 | ． 0004 | ． 0004 | ． 0004 | ． 0004 | ． 0003 |
| －3．2 | ． 0007 | ． 0007 | ． 0006 | ． 0006 | ． 0006 | ． 0006 | ． 0006 | ． 0005 | ． 0005 | ． 0005 |
| －3．1 | ． 0010 | ． 0009 | ． 0009 | ． 0009 | ． 0008 | ． 0008 | ． 0008 | ． 0008 | ． 0007 | ． 0007 |
| －3．0 | ． 0013 | ． 0013 | ． 0013 | ． 0012 | ． 0012 | ． 0011 | ． 0011 | ． 0011 | ． 0010 | ． 0010 |
| －2．9 | ． 0019 | ． 0018 | ． 0018 | ． 0017 | ． 0016 | ． 0016 | ． 0015 | ． 0015 | ． 0014 | ． 0014 |
| －2．8 | ． 0026 | ． 0025 | ． 0024 | ． 0023 | ． 0023 | ． 0022 | ． 0021 | ． 0021 | ． 0020 | ． 0019 |
| －2．7 | ． 0035 | ． 0034 | ． 0033 | ． 0032 | ． 0031 | ． 0030 | ． 0029 | ． 0028 | ． 0027 | ． 0026 |
| －2．6 | ． 0047 | ． 0045 | ． 0044 | ． 0043 | ． 0041 | ． 0040 | ． 0039 | ． 0038 | ． 0037 | ． 0036 |
| －2．5 | ． 0062 | ． 0060 | ． 0059 | ． 0057 | ． 0055 | ． 0054 | ． 0052 | ． 00051 | ＊．0049 | ． 0048 |
| －2．4 | ． 0082 | ． 0080 | ． 0078 | ． 0075 | ． 0073 | ． 0071 | ． 0069 | ． 0068 | 1．0066 | ． 0064 |
| －2．3 | ． 0107 | ． 0104 | ． 0102 | ． 0099 | ． 0096 | ． 0094 | ． 0091 | ． 0089 | ． 0087 | ． 0084 |
| －2．2 | ． 0139 | ． 0136 | ． 0132 | ． 0129 | ． 0125 | ． 0122 | ． 0119 | ． 0116 | ． 0113 | ． 0110 |
| －2．1 | ． 0179 | ． 0174 | ． 0170 | ． 0166 | ． 0162 | ． 0158 | ． 0154 | ． 0150 | ． 0146 | ． 0143 |
| －2．0 | ． 0228 | ． 0222 | ． 0217 | ． 0212 | ． 0207 | ． 0202 | ． 0197 | ． 0192 | ． 0188 | ． 0183 |
| －1．9 | ． 0287 | ． 0281 | ． 0274 | ． 0268 | ． 0262 | ． 0256 | ． 0250 | ． 0244 | ． 0239 | ． 0233 |
| －1．8 | ． 0359 | ． 0351 | ． 0344 | ． 0336 | ． 0329 | ． 0322 | ． 0314 | ． 0307 | ． 0301 | ． 0294 |
| －1．7 | ． 0446 | ． 0436 | ． 0427 | ． 0418 | ． 0409 | ． 0401 | ． 0392 | ． 0384 | ． 0375 | ． 0367 |
| －1．6 | ． 0548 | ． 0537 | ． 0526 | ． 0516 | ． 0505 | ＊ 0495 | ． 0485 | ． 0475 | ． 0465 | ． 0455 |
| －1．5 | ． 0668 | ． 0655 | ． 0643 | ． 0630 | ． 0618 | 4．0606 | ． 0594 | ． 0582 | ． 0571 | ． 0559 |
| －1．4 | ． 0808 | ． 0793 | 0778 | ． 0764 | ． 0749 | ． 0735 | ． 0721 | ． 0708 | ． 0694 | ． 0681 |
| $-1.3$ | ． 0968 | ． 0951 | 0934 | ． 0918 | ． 0901 | ． 0885 | ． 0869 | ． 0853 | ． 0838 | ． 0823 |
| －1．2 | ． 1151 | ． 1131 | ． 1112 | ． 1093 | ． 1075 | ． 1056 | ． 1038 | ． 1020 | ． 1003 | ． 0985 |
| －1．1 | ． 1357 | ， 1335 | ． 1314 | 1292 | 1271 | ． 3251 | ． 1230 | ． 1210 | 1190 | ． 1170 |
| －1．0 | ． 1587 | ． 1562 | 1539 | 1515 | 1492 | ． 1469 | ． 1446 | ． 1423 | ． 1401 | ． 1379 |
| －0．9 | ． 1841 | ． 1814 | ． 1788 | 1762 | ． 1736 | ． 1711 | ． 1685 | ． 1660 | ． 1635 | ． 1611 |
| －0．8 | ． 2119 | ． 2090 | ． 2061 | ． 2033 | ． 2005 | ． 1977 | ． 1949 | ． 1922 | ． 1894 | ． 1867 |
| －0．7 | ． 2420 | .2389 | ． 2358 | ． 2327 | ． 2296 | ． 2266 | .2236 | ． 2206 | ． 2177 | 2148 |
| －0．6 | ． 2743 | .2709 | ． 2676 | ． 2643 | ． 2611 | ． 2578 | ． 2546 | ． 2514 | 2483 | ． 2451 |
| －0．5 | ． 3085 | 3050 | 3015 | ． 2981 | ． 2946 | ． 2912 | ． 2877 | ． 2843 | ． 2810 | ． 2776 |
| －0．4 | ． 3446 | ． 3409 | ． 3372 | ． 3336 | ． 3300 | ． 3264 | ． 3228 | ． 3192 | ． 3156 | ． 3121 |
| －0．3 | ． 3821 | ． 3793 | ． 3745 | ． 3707 | ． 3669 | ． 3632 | 3594 | 3557 | ． 3520 | ． 3483 |
| －0．2 | ． 4207 | ． 4168 | ． 4129 | 4090 | ． 4052 | ． 4013 | ． 3974 | ． 3936 | ． 3897 | ． 3859 |
| －0．1 | ． 4602 | ． 4562 | ． 4522 | ． 4483 | ． 4443 | ． 4404 | ． 4364 | ． 4325 | ． 4286 | ． 4247 |
| －0．0 | ． 5000 | .4960 | ． 4920 | ． 4880 | ． 4840 | ． 4801 | ． 4761 | ． 4721 | ． 4681 | ． 4641 |
| NOTE：For values of $z$ below -3.49 ，use 0.0001 for the area． ＊Use these common values that result from interpolation： |  |  |  |  |  |  |  |  |  |  |
| $\frac{z \text { score }}{-1.645}$ | $\frac{\text { Area }}{0.0500}$ |  | $\square$ |  |  |  |  |  |  |  |
| $-2.575$ | $0.0050$ |  |  |  |  |  |  |  |  |  |

## Using Table A-2

四 It is designed only for the standard normal distribution, which has a mean of 0 and a standard deviation of 1 .
(四 Table A-2 is on two pages, with one page for negative $z$-scores and the other page for positive $z$-scores.


## Using Table A－2

卵 It is designed only for the standard normal distribution，which has a mean of 0 and a standard deviation of 1 ．

四 Table A－2 is on two pages，with one page for negative $z$－scores and the other page for positive $z$－scores．

TABLE A－2 $\quad$ Standard Normal（ $z$ ）Distribution：Cumulative Area from the LEFT



## POSITIVE z Scores

## Using Table A－2

四 It is designed only for the standard normal distribution，which has a mean of 0 and a standard deviation of 1 ．

四 Table A－2 is on two pages，with one page for negative $z$－scores and the other page for positive $z$－scores．


## Using Table A-2

四 There are two pieces of info on this table: $z$-scores and probabilities.

Table A-2 Standard Normal (z) Distribution: Cumulative Area from the LEFT


## Using Table A-2

There are two pieces of info on this table: z-scores and probabilities.

《 The leftmost column

TABLE A-2 Standard Normal (z) Distribution: Cumulative Area from the LEFT


## Using Table A-2

四 There are two pieces of info on this table: $z$-scores and probabilities.

The leftmost column and the top row are associated with $z$ values.


## NEGATIVE z Scores

## Using Table A－2

四 There are two pieces of info on this table：$z$－scores and probabilities．

The leftmost column and the top row are associated with $z$ values．

四 Each value in the＂body＂of the table is a cumulative area from the left up to a vertical boundary above a specific $z$－score．


## Using Table A-2

四 There are two pieces of info on this table: $z$-scores and probabilities.

The leftmost column and the top row are associated with $z$ values.

四 Each value in the "body" of the table is a cumulative area from the left up to a vertical boundary above a specific $z$-score. These area values are mathematically equivalent to probabilities.


## Using Table A－2

四 There are two pieces of info on this table：$z$－scores and probabilities．

The leftmost column and the top row are associated with $z$ values．

四 Each value in the＂body＂of the table is a cumulative area from the left up to a vertical boundary above a specific $z$－score．These area values are mathematically equivalent to probabilities．

四 The part of the $z$－score denoting hundredths is found across the top．

| z | ． 00 | ． 01 | 02 | ． 03 | ． 04 | ． 05 | .06 | ． 07 | ． 08 | ． 09 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $-3.50$ <br> and <br> lower | ． 0001 |  |  |  |  | $\stackrel{1}{2}$ |  |  |  |  |
| －3．4 | ． 0003 | ． 0003 | ． 0003 | ． 0003 | ． 0003 | ． 0003 | ． 0003 | ． 0003 | ． 0003 | ． 0002 |
| －3．3 | ． 0005 | ． 0005 | ． 0005 | ． 0004 | ． 0004 | ． 0004 | ． 0004 | ． 0004 | ． 0004 | ． 0003 |
| －3．2 | ． 0007 | ． 0007 | ． 0006 | ． 0006 | ． 0006 | ． 0006 | ． 0006 | ． 0005 | ． 0005 | ． 0005 |
| －3．1 | ． 0010 | ． 0009 | ． 0009 | ． 0009 | ． 0008 | ． 0008 | ． 0008 | ． 0008 | ． 0007 | ． 0007 |
| －3．0 | ． 0013 | ． 0013 | ． 0013 | ． 0012 | ． 0012 | ． 0011 | ． 0011 | ． 0011 | ． 0010 | ． 0010 |
| －2．9 | ． 0019 | ． 0018 | ． 0018 | ． 0017 | ． 0016 | ． 0016 | ． 0015 | ． 0015 | ． 0014 | ． 0014 |
| －2．8 | ． 0026 | ． 0025 | ． 0024 | ． 0023 | ． 0023 | ． 0022 | ． 0021 | ． 0021 | ． 0020 | ． 0019 |
| $-2.7$ | ． 0035 | 0034 | ． 0033 | ． 0032 | ． 0031 | ． 0030 | ． 0029 | ． 0028 | ． 0027 | ． 0026 |
| －2．6 | ． 0047 | ． 0045 | ． 0044 | ． 0043 | ． 0041 | ． 0040 | ． 0039 | ． 0038 | ． 0037 | ． 0036 |
| －2．5 | ． 0062 | ． 0060 | ． 0059 | ． 0057 | ． 0055 | ． 0054 | ． 0052 | ． 0051 | ＊．0049 | ． 0048 |
| －2．4 | ． 0082 | ． 0080 | ． 0078 | ． 0075 | ． 0073 | ． 0071 | ． 0069 | ． 0068 | A． 0066 | ． 0064 |
| －2．3 | ． 0107 | .0104 | ． 0102 | ． 0099 | ． 0096 | ． 0094 | ． 0091 | ． 0089 | ． 0087 | ． 0084 |
| －2．2 | ． 0139 | ． 0136 | ． 0132 | ． 0129 | ． 0125 | ． 0122 | ． 0119 | ． 0116 | ． 0113 | ． 0110 |
| －2．1 | ． 0179 | ． 0174 | ． 0170 | ． 0166 | ． 0162 | ． 0158 | ． 0154 | ． 0150 | ． 0146 | ． 0143 |
| $-2.0$ | ． 0228 | ． 0222 | ． 0217 | ． 0212 | ． 0207 | ． 0202 | ． 0197 | ． 0192 | ． 0188 | ． 0183 |
| －19 | ． 0287 | ． 0281 | ． 0274 | ． 0268 | ． 0262 | ． 0256 | ． 0250 | ． 0244 | ． 0239 | ． 0233 |
| $-1.8$ | ． 0359 | ． 0351 | ． 0344 | ． 0336 | ． 0329 | ． 0322 | ． 0314 | ． 0307 | ． 0301 | ． 0294 |
| $-1.7$ | ． 0446 | ． 0436 | ． 0427 | ． 0418 | ． 0409 | ． 0401 | ． 0392 | ． 0384 | ． 0375 | ． 0367 |
| －1．6 | ． 0548 | ． 0537 | ． 0526 | ． 0516 | ． 0505 | ＊． 0495 | ． 0485 | ． 0475 | ． 0465 | ． 0455 |
| －1．5 | ． 0668 | ． 0655 | ． 0643 | ． 0630 | ． 0618 | 4．0606 | ． 0594 | ． 0582 | ． 0571 | ． 0559 |
| －1．4 | ． 0808 | ． 0793 | 0778 | ． 0764 | ． 0749 | ． 0735 | ． 0721 | ． 0708 | ． 0694 | ． 0681 |
| $-13$ | ． 0968 | ． 0951 | ． 0934 | ． 0918 | ． 0901 | ． 0885 | ． 0869 | ． 0853 | ． 0838 | ． 0823 |
| －1．2 | ． 1151 | ． 1131 | ． 1112 | ． 1093 | ． 1075 | ． 1056 | .1038 | .1020 | ． 1003 | ． 0985 |
| －1．1 | ． 1357 | ， 1335 | ． 1314 | ． 1292 | 1271 | ． 7251 | ． 1230 | ． 1210 | ．190 | ． 1170 |
| －10 | ． 1587 | ． 1562 | 1539 | 1515 | 1492 | ． 1469 | ． 1446 | ． 1423 | ． 1401 | ． 1379 |
| －0．9 | ． 1841 | 1814 | ． 1788 | 1762 | ． 1736 | ． 1711 | ． 1685 | ． 1660 | ． 1635 | ． 1611 |
| －0．8 | ． 2119 | ． 2090 | 2061 | ． 2033 | ． 2005 | ． 1977 | ． 1949 | ． 1922 | ． 1894 | ． 1867 |
| －0．7 | ． 2420 | 2389 | ． 2358 | ． 2327 | ． 2296 | ． 2266 | ． 2236 | ． 2206 | ． 2177 | 2148 |
| －0．6 | ． 2743 | .2709 | .2676 | ． 2643 | ． 2611 | ． 2578 | ． 2546 | ． 2514 | ． 2483 | .2451 |
| －0．5 | ． 3085 | .3050 | 3015 | ． 2981 | ． 2946 | .2912 | ． 2877 | ． 2843 | ． 2810 | ． 2776 |
| －0．4 | 3446 | ． 3409 | ． 3372 | ． 3336 | ． 3300 | ． 3264 | ． 3228 | ． 3192 | ． 3156 | ． 3121 |
| －0．3 | ． 3821 | ． 3793 | 3745 | ． 3707 | ． 3669 | ． 3632 | 3594 | ． 3557 | ． 3520 | ． 3483 |
| －0．2 | ． 4207 | ． 4168 | ． 4129 | 4090 | ． 4052 | ． 4013 | ． 3974 | ． 3936 | ． 3897 | ． 3859 |
| －0．1 | 4602 | ． 4562 | ． 4522 | ． 4483 | ． 4443 | ． 4404 | ． 4364 | ． 4325 | ． 4286 | ． 4247 |
| －0．0 | ． 5000 | ． 4960 | ． 4920 | ． 4880 | ． 4840 | ． 4801 | ． 4761 | ． 4721 | ． 4681 | ． 4641 |
| NOTE：For values of $z$ below -3.49 ，use 0.0001 for the area． ＊Use these common values that result from interpolation： |  |  |  |  |  |  |  |  |  |  |
| $\frac{z \text { score }}{-1.645}$ | $\frac{\text { Area }}{0.0500}$ |  |  |  |  |  |  |  |  |  |

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Example: The Precision Scientific Instrument Company manufactures thermometers that are supposed to give readings of $0^{\circ} \mathrm{C}$ at the freezing point of water. Tests on a large sample of these instruments reveal that at the freezing point of water, some thermometers give readings below $0^{\circ} \mathrm{C}$ (denoted by negative numbers) and some give readings above $0^{\circ} \mathrm{C}$ (denoted by positive numbers). [2]

Assume that the mean reading is $0^{\circ} \mathrm{C}$ and the standard deviation of the readings is $1.00^{\circ} \mathrm{C}$. Also, assume that the readings are normally distributed. If one thermometer is randomly selected, find the probability that, at the freezing point of water, the reading is less than $-1.51^{\circ} \mathrm{C}$.

Solution: We are told the distribution is normal, with $\mu=0$ and $\sigma=1$. Let $x$ be the continuous random variable representing the temperature of a randomly selected thermometer.

NEGATIVE z Scores

We need to find the probability that $x$ is less than $-1.51^{\circ} \mathrm{C}$, or, symbolically, $P\left(x<-1.51^{\circ}\right)$. Then,

$$
P\left(x<-1.51^{\circ}\right)=P\left(z<\frac{x-\mu}{\sigma}\right)
$$

(standardize $x$

$$
=P\left(z<\frac{-1.51-0}{1}\right) \quad \begin{aligned}
& \text { since } x=-1.51, \\
& \mu=0 \text { and } \sigma=1
\end{aligned}
$$

$$
=P(z<-1.51)
$$



Solution: We are told the distribution is normal, with $\mu=0$ and $\sigma=1$. Let $x$ be the continuous random variable representing the temperature of a randomly selected thermometer.

We need to find the probability that $x$ is less than $-1.51^{\circ} \mathrm{C}$, or, symbolically, $P\left(x<-1.51^{\circ}\right)$. Then,
$P\left(x<-1.51^{\circ}\right)=P\left(z<\frac{x-\mu}{\sigma}\right)$
(standardize $x$
i.e., transform $x \rightarrow z$ )

$$
=P\left(z<\frac{-1.51-0}{1}\right) \quad \begin{aligned}
& \text { since } x=-1.51, \\
& \mu=0 \text { and } \sigma=1
\end{aligned}
$$

$$
=P(z<-1.51)
$$



This last probability is equal to the area under the Standard Normal Distribution just left of $z=-1.51$. We can find this value from Table A-2.

| TABLE A-2 $\quad$ Standard Normal (z) Distribution: Cumulative Area from the LEFT |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $z$ | . 00 | .01 | . 02 | . 03 | . 04 | . 05 | .06 | . 07 | . 08 | . 09 |
| $\begin{aligned} & -3.50 \\ & \text { and } \\ & \text { lower } \end{aligned}$ | . 0001 |  |  |  |  | - 2 |  |  |  |  |
| -3.4 | . 0003 | . 0003 | . 0003 | . 0003 | . 0003 | . 0003 | . 0003 | . 0003 | . 0003 | . 0002 |
| $-3.3$ | . 0005 | . 0005 | . 0005 | . 0004 | . 0004 | . 0004 | . 0004 | . 0004 | . 0004 | . 0003 |
| -3.2 | . 0007 | . 0007 | . 0006 | . 0006 | . 0006 | . 0006 | . 0006 | . 0005 | . 0005 | . 0005 |
| -3.1 | . 0010 | . 0009 | . 0009 | . 0009 | . 0008 | . 0008 | . 0008 | . 0008 | . 0007 | . 0007 |
| -3.0 | . 0013 | . 0013 | . 0013 | . 0012 | . 0012 | . 0011 | . 0011 | . 0011 | . 0010 | . 0010 |
| -2.9 | . 0019 | . 0018 | . 0018 | . 0017 | . 0016 | . 0016 | . 0015 | . 0015 | . 0014 | . 0014 |
| -2.8 | . 0026 | . 0025 | . 0024 | . 0023 | . 0023 | . 0022 | . 0021 | . 0021 | . 0020 | . 0019 |
| -2.7 | . 0035 | . 0034 | . 0033 | . 0032 | . 0031 | . 0030 | . 0029 | . 0028 | . 0027 | . 0026 |
| -2.6 | . 0047 | . 0045 | . 0044 | . 0043 | . 0041 | . 0040 | . 0039 | . 0038 | . 0037 | . 0036 |
| -2.5 | . 0062 | . 0060 | . 0059 | . 0057 | . 0055 | . 0054 | . 0052 | . 0051 | *.0049 | . 0048 |
| -2.4 | . 0082 | . 0080 | . 0078 | . 0075 | . 0073 | . 0071 | . 0069 | . 0068 | A. 0066 | . 0064 |
| -2.3 | . 0107 | .0104 | . 0102 | . 0099 | . 0096 | . 0094 | . 0091 | . 0089 | . 0087 | . 0084 |
| -2.2 | . 0139 | . 0136 | . 0132 | . 0129 | . 0125 | . 0122 | . 0119 | . 0116 | . 0113 | . 0110 |
| -2.1 | . 0179 | . 0174 | .0170 | . 0166 | . 0162 | . 0158 | . 0154 | . 0150 | . 0146 | . 0143 |
| -2.0 | . 0228 | . 0222 | . 0217 | . 0212 | . 0207 | . 0202 | . 0197 | . 0192 | . 0188 | . 0183 |
| -1.9 | . 0287 | . 0281 | . 0274 | . 0268 | . 0262 | . 0256 | . 0250 | . 0244 | . 0239 | . 0233 |
| -1.8 | . 0359 | . 0351 | . 0344 | . 0336 | . 0329 | . 0322 | . 0314 | . 0307 | . 0301 | . 0294 |
| -1.7 | . 0446 | . 0436 | . 0427 | . 0418 | . 0409 | . 0401 | . 0392 | . 0384 | . 0375 | . 0367 |
| -1.6 | . 0548 | . 0537 | . 0526 | . 0516 | . 0505 | *. 0495 | . 0485 | . 0475 | . 0465 | . 0455 |
| -1.5 | . 0668 | . 0655 | . 0643 | . 0630 | . 0618 | 4.0606 | . 0594 | . 0582 | . 0571 | . 0559 |
| -1.4 | . 0808 | . 0793 | 0778 | . 0764 | . 0749 | . 0735 | . 0721 | . 0708 | . 0694 | . 0681 |
| $-13$ | . 0968 | . 0951 | . 0934 | . 0918 | . 0901 | . 0885 | . 0869 | . 0853 | . 0838 | . 0823 |
| -1.2 | . 1151 | . 1131 | . 1112 | . 1093 | . 1075 | . 1056 | .1038 | . 1020 | . 1003 | . 0985 |
| -1.1 | . 1357 | . 1335 | . 1314 | . 1292 | . 1271 | . 7251 | . 1230 | . 1210 | 1190 | . 1170 |
| -1.0 | . 1587 | . 1562 | 1539 | 1515 | 1492 | . 1469 | 1446 | . 1423 | . 1401 | . 1379 |
| -0.9 | . 1841 | . 1814 | . 1788 | 1762 | . 1736 | . 1711 | 1685 | . 1660 | . 1635 | . 1611 |
| -0.8 | . 2119 | . 2090 | 2061 | . 2033 | . 2005 | . 1977 | . 1949 | . 1922 | . 1894 | . 1867 |
| -0.7 | . 2420 | . 2389 | . 2358 | . 2327 | . 2296 | . 2266 | .2236 | . 2206 | . 2177 | 2148 |
| -0.6 | . 2743 | . 2709 | . 2676 | . 2643 | . 2611 | . 2578 | . 2546 | . 2514 | . 2483 | . 2451 |
| -0.5 | . 3085 | 3050 | 3015 | . 2981 | . 2946 | 2912 | .2877 | . 2843 | .2810 | 2776 |
| -0.4 | . 3446 | . 3409 | . 3372 | . 3336 | . 3300 | . 3264 | . 3228 | . 3192 | . 3156 | . 3121 |
| -0.3 | . 3821 | . 3793 | 3745 | . 3707 | . 3669 | . 3632 | 3594 | . 3557 | . 3520 | . 3483 |
| -0.2 | . 4207 | . 4168 | . 4129 | . 4090 | . 4052 | . 4013 | . 3974 | . 3936 | . 3897 | . 3859 |
| -0.1 | . 4602 | . 4562 | 4522 | . 4483 | . 4443 | . 4404 | . 4364 | . 4325 | . 4286 | . 4247 |
| -0.0 | . 5000 | . 4960 | 4920 | . 4880 | . 4840 | . 4801 | . 4761 | . 4721 | . 4681 | . 4641 |

NOTE: For values of $z$ below -3.49 , use 0.0001 for the area.
"Use these common values that result from interpolation:

[^0]$\square$

Solution: We are told the distribution is normal, with $\mu=0$ and $\sigma=1$. Let $x$ be the continuous random variable representing the temperature of a randomly selected thermometer.

We need to find the probability that $x$ is less than $-1.51^{\circ} \mathrm{C}$, or, symbolically, $P\left(x<-1.51^{\circ}\right)$. Then,
$P\left(x<-1.51^{\circ}\right)=P\left(z<\frac{x-\mu}{\sigma}\right)$
(standardize $x$
i.e., transform $x \rightarrow z$ )

$$
=P\left(z<\frac{-1.51-0}{1}\right) \quad \begin{aligned}
& \text { since } x=-1.51, \\
& \mu=0 \text { and } \sigma=1
\end{aligned}
$$

$$
=P(z<-1.51)
$$



Locate the row with $z=-1.5$ then the column with .01 .
The intersection of this row and column gives the cumulative probability,

Table A-2 Standard Normal (z) Distribution: Cumulative Area from the LEFT


NOTE: For values of $z$ below -3.49 , use 0.0001 for the area
"Use these common values that result from interpolation:

[^1]Solution: We are told the distribution is normal, with $\mu=0$ and $\sigma=1$. Let $x$ be the continuous random variable representing the temperature of a randomly selected thermometer.

We need to find the probability that $x$ is less than $-1.51^{\circ} \mathrm{C}$, or, symbolically, $P\left(x<-1.51^{\circ}\right)$. Then,
$P\left(x<-1.51^{\circ}\right)=P\left(z<\frac{x-\mu}{\sigma}\right)$
(standardize $x$

$$
=P\left(z<\frac{-1.51-0}{1}\right) \quad \begin{aligned}
& \text { since } x=-1.51, \\
& \mu=0 \text { and } \sigma=1
\end{aligned}
$$

$$
=P(z<-1.51)
$$



Locate the row with $z=-1.5$ then the column with .01 .
The intersection of this row and column gives the cumulative probability,

```
0.0655.
```



NOTE: For values of $z$ below -3.49 , use 0.0001 for the area.
Use these common values that result from interpolation:

[^2]$\square$

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# click here to access the classroom worksheet 

click here to access Table A-2 (the z-table)

The negative $z$ table lists cumulative areas from the left of the center $(z=0)$. Notice that as the values of $z$ increase from -3.5 to 0 , so does the cumulative areas (probabilities).



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The negative $z$ table lists cumulative areas from the left of the center $(z=0)$. Notice that as the values of $z$ increase from -3.5 to 0 , so does the cumulative areas (probabilities).



## POSITIVE z Scores

The positive $z$ table lists cumulative areas from the left which are associated with $z$ scores right of the center ( $z=0$ ). Notice the cumulative areas also increase as $z$ increases.



## POSITIVE z Scores

The positive $z$ table lists cumulative areas from the left which are associated with $z$ scores right of the center ( $z=0$ ). Notice the cumulative areas also increase as $z$ increases.



## POSITIVE z Scores

The positive $z$ table lists cumulative areas from the left which are associated with $z$ scores right of the center ( $z=0$ ). Notice the cumulative areas also increase as $z$ increases.



## POSITIVE z Scores

The positive $z$ table lists cumulative areas from the left which are associated with $z$ scores right of the center ( $z=0$ ). Notice the cumulative areas also increase as $z$ increases.



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## POSITIVE z Scores

The positive $z$ table lists cumulative areas from the left which are associated with $z$ scores right of the center ( $z=0$ ). Notice the cumulative areas also increase as $z$ increases.



## POSITIVE z Scores

The positive $z$ table lists cumulative areas from the left which are associated with $z$ scores right of the center ( $z=0$ ). Notice the cumulative areas also increase as $z$ increases.




Try this! Find $P(z<2.37)$


TABLE A-2 (continued) Cumulative Area from the LEFT


Try this! Find $P(z<2.37)$

$$
P(z<2.37)=0.9911
$$




## POSITIVE z Scores



## POSITIVE z Scores

Try this! Find $P(z<2.37)$

$$
P(z<2.37)=0.9911
$$



Did you notice that in order to find the probability in the table we split the number $z=2.37$ into two parts: 2.3 and 0.07? $(2.37=2.3+.07)$

| TABLE | 2 (con | inued | Cumu | Ve Ar | from | the LEF |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| z | . 00 | . 01 | . 02 | . 03 | . 04 | . 05 | . 06 | . 07 | . 08 | . 09 |
| 0.0 | . 5000 | . 5040 | . 5080 | 5120 | . 5160 | . 5199 | . 5239 | . 5279 | 5319 | . 5359 |
| 0.1 | . 5398 | . 5438 | . 5478 | . 5517 | . 5557 | . 5596 | . 5636 | . 5675 | 5714 | . 5753 |
| 0.2 | . 5793 | . 5832 | . 5871 | . 5910 | . 5948 | . 5987 | . 6026 | . 6064 | . 6103 | . 6141 |
| 03 | . 6179 | . 6217 | . 6255 | . 6293 | . 6331 | . 6368 | . 6409 | . 6443 | . 6480 | . 6517 |
| 0.4 | . 6554 | . 6591 | . 6628 | . 6664 | . 6700 | . 6736 | . 6772 | . 6808 | . 6844 | . 6879 |
| 0.5 | 6915 | . 6950 | . 6985 | 7019 | 7054 | . 7088 | . 7123 | . 7157 | 7190 | . 7224 |
| 0.6 | . 7257 | . 7291 | . 7324 | . 7357 | . 7389 | . 7422 | . 7454 | . 7486 | . 7517 | . 7549 |
| 0.7 | . 7580 | . 7611 | . 7642 | . 7673 | .7704 | . 7734 | . 7764 | 7794 | . 7823 | 7852 |
| 0.8 | 7881 | . 7910 | . 7939 | . 7967 | . 7995 | . 8023 | . 805 | . 8078 | . 8106 | . 8133 |
| 0.9 | . 8159 | . 8186 | . 8212 | . 8238 | . 8264 | . 8289 | . 8315 | . 8340 | . 8365 | . 8389 |
| 1.0 | 8413 | . 8438 | . 8461 | . 8485 | . 8508 | 8531 | . 8554 | . 8577 | . 8599 | . 8621 |
| 1.1 | . 8643 | . 8665 | . 8686 | . 8708 | . 8729 | . 8749 | . 8770 | . 8790 | 8810 | . 8830 |
| 1.2 | . 8849 | . 8869 | . 8838 | . 8907 | . 8925 | . 8944 | . 8962 | . 8980 | . 8997 | . 9015 |
| 1.3 | . 9032 | . 9049 | . 9066 | . 9082 | . 9099 | . 9115 | 9131 | . 9147 | . 9162 | 9177 |
| 1.4 | . 9192 | . 9207 | . 9222 | . 9236 | . 9251 | . 9265 | . 9278 | . 9292 | . 9306 | . 9319 |
| 1.5 | 9332 | 9345 | . 9357 | . 9370 | . 9382 | . 9394 | 9406 | 9418 | . 9429 | . 9441 |
| 1.6 | . 9452 | . 9463 | . 9474 | . 9484 | . 9495 | *. 9505 | . 9515 | . 9525 | . 9535 | . 9545 |
| 1.7 | . 9554 | . 9564 | . 9573 | . 9582 | . 9591 | 4. 9599 | . 9608 | . 9616 | . 9625 | . 9633 |
| 1.8 | . 9641 | .9649 | . 9656 | . 9664 | . 9671 | . 9678 | . 9684 | . 9693 | . 9699 | . 9706 |
| 1.9 | . 9713 | . 9719 | . 9726 | . 9732 | .9738 | . 9744 | . 9750 | . 9756 | . 9761 | . 9767 |
| 2.0 | . 9772 | . 9778 | . 9783 | . 9788 | .9793 | . 9798 | . 980 | . 9808 | . 9812 | . 9817 |
| 2.1 | . 9821 | . 9826 | . 9830 | 9834 | . 9838 | . 9842 | . 9846 | . 9850 | . 9854 | . 9857 |
| 03 | 0801 | coca | -0,00 | 9071 | 0975 | 0978 | 989 | 9884 | 9887 | 9890 |
| 23 | 9893 | . 9896 | . 9898 | . 9901 | . 9904 | 9906 | . 9900 | . 9911 | . 9913 | 9916 |
| 2.4 | . 9978 | . 9980 | . 9982 | . 9525 | . 59827 | .9529 | , 297 | . 9950 | . 9004 | . 9950 |
| 25 | . 9938 | . 9940 | . 9941 | 9943 | . 9945 | . 9946 | . 9948 | 9949 | * 9951 | . 9952 |
| 2.6 | . 9953 | . 9955 | . 9956 | . 9957 | . 9959 | . 9960 | . 996 | . 9962 | 4.9963 | . 9964 |
| 2.7 | . 9965 | . 9966 | . 9967 | . 9968 | . 9969 | . 9970 | . 997 | . 9972 | . 9973 | . 9974 |
| 2.8 | . 9974 | . 9975 | . 9976 | . 9977 | . 9977 | . 9978 | . 9979 | . 9979 | . 9980 | . 9981 |
| 2.9 | . 9981 | . 9982 | . 9982 | . 9983 | . 9984 | . 9984 | . 9985 | . 9985 | . 9986 | . 9988 |
| 3.0 | . 9987 | . 9987 | . 9987 | . 9988 | . 9988 | . 9989 | . 9985 | . 9989 | . 9990 | . 9990 |
| 3.1 | . 9990 | . 9991 | . 9991 | 9991 | . 9992 | . 9992 | . 9991 | . 9992 | . 9993 | . 9993 |
| 3.2 | . 9993 | . 9993 | . 9994 | . 9994 | . 9994 | . 9994 | . 9994 | . 9995 | . 9995 | . 9995 |
| 3.3 | 9995 | .9995 | . 9995 | . 9996 | .9996 | .9996 | . 9996 | . 9996 | . 9996 | . 9997 |
| 3.4 | . 9997 | . 9997 | . 9997 | . 9997 | . 9997 | . 9997 | . 999 | . 9997 | . 9997 | . 9998 |
| $\begin{aligned} & 3.50 \\ & \text { and up } \end{aligned}$ | . 9999 |  |  |  |  |  |  |  |  |  |
| NOTE: For values of $z$ above 3.49 , use 0.9999 for the area. *Use these common values that result from interpolation: |  |  |  |  |  | , |  |  | Common Critical Values |  |
|  |  |  |  |  |  | Confidence Level |  |
|  | Area |  |  |  |  |  |  |
| 1.642.57 | 0.9500 |  |  |  |  | 0.9 | $1.9$ |
|  | 0.9950 |  |  |  |  |  |  |  |  | 0.9 0.9 |

## POSITIVE z Scores

Try this! Find $P(z<2.37)$

$$
P(z<2.37)=0.9911
$$



The first part, 2.3, tells us what row to look up in the table. The second part, .07, identifies what column to look at. The intersection of the row and column gives the probability (area).


Try this! Find the area between $z=-1.51$ and $z=2.37$.


POSITIVE $z$ Scores


Try this! Find the area between $z=-1.51$ and $z=2.37$.


POSITIVE z Scores

Hint: Because Table A-2 gives cumulative areas from the left, we must find the area left of $z=2.37$, then subtract from this the area that is left of $z=-1.51$.



Try this! Find the area between $z=-1.51$ and $z=2.37$.

Hint: Because Table A-2 gives cumulative areas from the left, we must find the area left of $z=2.37$, then subtract from this the area that is left of $z=-1.51$.


## Answer:

$P(-1.51<z<2.37)=P(z<2.37)-P(z<-1.51)$

$$
\begin{aligned}
& =0.9911-0.0655 \\
& =0.9256
\end{aligned}
$$



Try this! Find the probability, $P(z>2.37)$.


POSITIVE z Scores


Try this! Find the probability, $P(z>2.37)$.


POSITIVE $z$ Scores
We make use of the fact that the total area under the probability density curve is 1 . Because Table A-2 gives cumulative areas from the left, we must find the area left of $z=2.37$, then subtract this from 1 .


Answer:

$$
\begin{aligned}
P(z>2.37) & =1-P(z<2.37) \\
& =1-0.9911 \\
& =0.0089
\end{aligned}
$$



Try this! Find the $z$ score associated with a probability value of 0.8461 .


Try this! Find the $z$ score associated with a probability value of 0.8461 .


## POSITIVE $z$ Scores

We make use of the fact that the positive $z$ table gives probabilities that are between 0.5 and 1 , inclusive, to deduce that the given probability value is on the positive $z$ table.



Try this! Find the $z$ score associated with a probability value of 0.8461 .


POSITIVE z Scores

We make use of the fact that the positive $z$ table gives probabilities that are between 0.5 and 1 , inclusive, to deduce that the given probability value is on the positive $z$ table. We need to locate the probability closest to 0.8461 , then use the row and column intersection to determine the corresponding value of $z$.



Try this! Find the $z$ score associated with a probability value of 0.8461 .


## POSITIVE z Scores

We make use of the fact that the positive $z$ table gives probabilities that are between 0.5 and 1 , inclusive, to deduce that the given probability value is on the positive $z$ table. We need to locate the probability closest to 0.8461 , then use the row and column intersection to determine the corresponding value of $z$.


## Answer:

The row and column intersection give us two parts of the desired $z$ value, 1.0 and .02. Putting these values together gives us the answer, $z=$ 1.02


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## $z_{\alpha}$ Notation

The expression $z_{\alpha}$ denotes the $z$ score with an area of $\alpha$ to its right.


Try this! Find $z_{\alpha}$ if $\alpha=0.05$.


Table A-2 (continued) Cumulative Area from the LEFT


Try this! Find $z_{\alpha}$ if $\alpha=0.05$.
We draw a normal distribution and locate $z_{\alpha}$ along the horizontal axis, far right of center. We suppose the area right of $z_{\alpha}$ is $\alpha=0.05$.


Table A-2 (continued) Cumulative Area from the LEFT


Try this! Find $z_{\alpha}$ if $\alpha=0.05$.
We draw a normal distribution and locate $z_{\alpha}$ along the horizontal axis, far right of center. We suppose the area right of $z_{\alpha}$ is $\alpha=0.05$.

Because of the way the table is constructed, with cumulative areas left of some critical value of $z$, we must use the fact that total area under the the curve is 1 . The area left of $z_{\alpha}$ is

$$
1-\alpha=1-0.05=0.95 .
$$




Try this! Find $z_{\alpha}$ if $\alpha=0.05$.
We draw a normal distribution and locate $z_{\alpha}$ along the horizontal axis, far right of center. We suppose the area right of $z_{\alpha}$ is $\alpha=0.05$.

Because of the way the table is constructed, with cumulative areas left of some critical value of $z$, we must use the fact that total area under the the curve is 1 . The area left of $z_{\alpha}$ is

$$
1-\alpha=1-0.05=0.95
$$



## Answer:

We find two probability values equally close to 0.95 ; and these values are associated with $z=$ 1.64 and $z=1.65$. Notice the asterisk between these two numbers points us to the bottom left portion of the table which tells us to take $z=$ 1.645 (the midpoint between $z=1.64$ and $z=$ 1.65) as the answer.

| TABLE |  | nued | um | ve A | rom | LE |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| z | . 00 | . 01 | . 02 | . 03 | .04 | . 05 | . 06 | . 07 |  | . 08 | . 09 |
| 0.0 | . 5000 | . 5040 | . 5080 | 5120 | . 5160 | . 5199 | . 5239 | . 5279 |  | . 5319 | . 5359 |
| 0.1 | . 5398 | . 5438 | . 5478 | . 5517 | . 5557 | . 5596 | . 5636 | . 5675 |  | 5714 | . 5753 |
| 0.2 | . 5793 | . 5832 | . 5871 | . 5910 | . 5948 | . 5987 | . 6026 | . 6064 |  | . 6103 | . 6141 |
| 03 | . 6179 | . 6217 | . 6255 | . 6293 | . 6331 | . 6368 | . 6406 | . 6443 |  | 6480 | . 6517 |
| 0.4 | . 6554 | . 6591 | . 6628 | . 6664 | . 6700 | . 6736 | . 6772 | . 6808 |  | 6844 | . 6879 |
| 0.5 | 6915 | . 6950 | . 6985 | 7019 | . 7054 | 7088 | 7123 | . 7157 |  | 7190 | . 7224 |
| 0.6 | . 7257 | . 7291 | . 7324 | . 7357 | . 7389 | .7422 | . 7454 | . 7486 |  | . 7517 | . 7549 |
| 0.7 | . 7580 | . 7611 | . 7642 | . 7673 | . 7704 | . 7734 | . 7764 | 7794 |  | . 7823 | . 7852 |
| 0.8 | . 7881 | . 7910 | . 7939 | . 7967 | . 7995 | . 8023 | . 8051 | . 8078 |  | . 8106 | . 8133 |
| 0.9 | . 8159 | . 8186 | . 8212 | . 8238 | . 8264 | . 8289 | . 8315 | . 8340 |  | . 8365 | . 8389 |
| 1.0 | . 8413 | . 8438 | . 8461 | . 8485 | . 8508 | 8531 | . 8554 | . 8577 |  | . 8599 | . 8621 |
| 1.1 | . 8643 | . 8665 | . 8686 | 8708 | . 8729 | . 8749 | . 8770 | . 8790 |  | 8810 | . 8830 |
| 1.2 | . 8849 | . 8869 | . 8838 | . 8907 | . 8925 | . 8944 | . 8962 | . 8980 |  | . 8997 | . 9015 |
| 1.3 | . 9032 | . 9049 | . 9066 | . 9082 | . 9099 | 9115 | 9131 | . 9147 |  | . 9162 | 9177 |
| 1.4 | . 9192 | . 9207 | . 9222 | . 9236 | . 9251 | . 9265 | . 9279 | . 9292 |  | . 9306 | . 9319 |
|  |  |  | -20ic | 0720 | 0700 | 0704 | 0.406 | 0.418 |  | $0 \times 20$ | 0.41 |
| 1.6 | . 9452 | . 9463 | . 9474 | . 9484 | . 9495 | . 9505 | . 9515 | . 9525 |  | . 9535 | . 9545 |
| 1.7 | . 9554 | . 9564 | . 9575 | .9582 | . 9591 | . 95959 | 9608 | .9616 |  | . 9625 | . 9655 |
| 1.8 | . 9641 | . 9649 | . 9656 | . 9664 | . 9671 | . 9678 | . 9685 | . 9693 |  | . 9699 | . 9706 |
| 1.9 | . 9713 | . 9719 | . 9726 | . 9732 | . 9738 | . 9744 | . 9750 | . 9756 |  | . 9761 | . 9767 |
| 2.0 | . 9772 | . 9778 | . 9783 | . 9788 | . 9793 | 9798 | . 9803 | . 9808 |  | 9812 | . 9817 |
| 2.1 | . 9821 | . 9826 | . 9830 | 9834 | . 9838 | . 9842 | . 9846 | . 9850 |  | . 9854 | . 9857 |
| 2.2 | . 9861 | . 9864 | . 9868 | . 9871 | . 9875 | . 9878 | . 9881 | . 9884 |  | . 9887 | . 9890 |
| 23 | 9893 | . 9896 | . 9898 | . 9901 | . 9904 | 9906 | . 9909 | . 9911 |  | . 9913 | . 9916 |
| 2.4 | . 9918 | . 9920 | . 9922 | . 9925 | . 9927 | . 9929 | . 9931 | . 9932 |  | . 9934 | . 9936 |
| 25 | . 9938 | . 9940 | . 9941 | 9943 | . 9945 | 9946 | . 9948 | 9949 | * | 9951 | . 9952 |
| 2.6 | . 9953 | . 9955 | . 9956 | . 9957 | . 9959 | . 9960 | . 9961 | . 9962 | 4 | . 9963 | . 9964 |
| 2.7 | . 9965 | . 9966 | . 9967 | . 9968 | . 9969 | . 9970 | 9971 | . 9972 |  | . 9973 | . 9974 |
| 28 | . 9974 | . 9975 | . 9976 | . 9977 | . 9977 | . 9978 | . 9979 | . 9979 |  | . 9980 | . 9988 |
| 2.9 | . 9981 | . 9982 | . 9982 | . 9983 | . 9984 | . 9984 | . 9985 | . 9985 |  | . 9996 | . 9986 |
| 3.0 | . 9987 | . 9987 | . 9987 | . 9988 | . 9988 | . 9989 | . 9989 | . 9989 |  | . 9990 | . 9990 |
| 3.1 | . 9990 | . 9991 | . 9991 | 9991 | . 9992 | . 9992 | . 9992 | . 9992 |  | . 9993 | . 9993 |
| 3.2 | . 9993 | . 9993 | . 9994 | . 9994 | . 9994 | . 9994 | . 9994 | . 9999 |  | . 9995 | . 9995 |
| 3.3 | . 9995 | 9995 | . 9995 | . 9996 | 9996 | .9996 | . 9996 | . 9996 |  | . 9996 | . 9997 |
| 3.4 | . 9997 | . 9997 | . 9997 | . 9997 | . 9997 | . 9997 | . 9997 | . 9997 |  | . 9997 | . 9998 |
| 3.50 <br> and up | . 9999 |  |  |  |  |  |  |  |  |  |  |
| NOTE: For values of $z$ above 3.49 , use 0.9999 for the area. *Use these common values that result from interpolation:$\frac{z \text { score }}{1.645} \frac{\text { Area }}{0.9500}$Common Critical Value <br> Confidence <br> Critical <br> Level <br> 0.90 <br>  <br> 0.95 <br> 0.99 |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |

Try this! Find the $z$-score associated with $P_{90}$, the $90^{\text {th }}$ percentile.


POSITIVE $z$ Scores

| TABLE A-2 | (continued) Cumulative Area from the LEFT |
| :--- | :--- |



Try this! Find the $z$-score associated with $P_{90}$, the $90^{\text {th }}$ percentile.
 POSITIVE z Scores

Recall that $P_{90}$ separates the lower $90 \%$ from the upper $10 \%$. We locate a $z$ value along the horizontal axis, far left of center, and assign the probability or area to the left of the $z$ value to be 0.9000 . Afterwards, we look in the body of the positive $z$ table for 0.9000 , then determine from the row and column intersection the correct value of $z$.



Try this! Find the $z$-score associated with $P_{90}$, the $90^{\text {th }}$ percentile.

POSITIVE z Scores

Recall that $P_{90}$ separates the lower $90 \%$ from the upper $10 \%$. We locate a $z$ value along the horizontal axis, far left of center, and assign the probability or area to the left of the $z$ value to be 0.9000 . Afterwards, we look in the body of the positive $z$ table for 0.9000 , then determine from the row and column intersection the correct value of $z$.


## Answer:

It seems as if there are two probabilities in the table closest to 0.9 , namely 0.8997 and 0.9015 . Since 0.8997 is so much closer to 0.9 than 0.9015 is, we take $z=1.28$ to be the best approximate value of $z$ associated with $P_{90}$.


### 6.4 Sampling Distributions

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## Sampling Distributions

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The key point of Section 6.4 is to introduce the concept of a sampling distribution of a statistic, which is the distribution of all values of that statistic when all possible samples of the same size are taken from the same population.

Consider the goal of trying to find the true proportion of all Alzheimer's patients who will have a particular side effect if they take an experimental drug that a pharmaceutical company wants to test. Because it is impossible and impractical to conduct a census, the drug manufacturer, with the FDA's approval, conducts clinical trials or repeated samples of Alzheimer's patients. The drug is given to the patients and the sample proportions are calculated. That is, the proportion of patients from each sample experiencing the undesired side effect is determined. [1]

Conclusions the pharmaceutical company makes require that they understand the behavior of the sampling distribution of all such sample proportions. Though they may have only one or a few samples, meaningful conclusions can be drawn from sample results about the population of all Alzheimer's patients who would likely suffer from the side effect.

A major goal of the rest of the textbook is to learn how we can effectively use a sample to form conclusions about a population.

Recall: The probability distribution from tossing a single die.

Now, consider the following random experiment.

## Random Experiment

Roll a die 5 times, and each time record the number on the top face of the die. Calculate the mean, $\bar{x}$, of the five values.[2]

That sample mean is a number between 1 and 6 . Suppose the experiment is repeated many times, and that each time the sample mean is recorded.

In addition, suppose we sort the sample means into 26 classes. That is, a tally mark is made in one of 26 classes each time a mean is recorded.

Suppose that as we continue this experiment, from time to time we plot a histogram of the classes and their associated frequencies. The animation (right) shows what our histograms might look like.

The number of trials represents the number of times the experiment is repeated.


## Sampling Distribution of the Mean

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The Central
Limit Theorem

Another Random Experiment

## Definition

The sampling distribution of the mean is the probability distribution of sample means, with all samples having the same sample size $n$ taken from the same population.

The animation (right) simulates the repeated experiment up to 20,000 times, but the true sampling distribution of the mean involves repeating the experiment indefinitely.

The actual sampling distribution would reflect all possible samples, not just a few or several thousand.

As the experiment is repeated several times, the distribution of means takes on a bell shape; and the mean, $\mu_{\mathrm{x}}$, of the sample means tends to reflect the actual mean, $\mu=3.5$. (The animation illustrates frequencies of sample means piling up along the $x$ axis around 3.5)


The previous experiment introduces the concept of the sampling distribution of the mean.

## Properties of the Sampling Distribution of the Mean

$\checkmark$ Sample means tend to target the value of the population mean. (That is, the mean of the sample means is the population mean. The expected value of the sample mean is equal to the population mean.)
$\checkmark$ The distribution of the sample means tends to be a normal distribution.

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Chapter 6

## Continuous Random

 VariablesRecall: The probability distribution from tossing a single die.

| $x$ | Probability, $P(x)$ |
| :---: | :---: |
| 1 | 0.1667 |
| 2 | 0.1667 |
| 3 | 0.1667 |
| 4 | 0.1667 |
| 5 | 0.1667 |
| 6 | 0.1667 |



The variance, $\sigma^{2}$, of this probability distribution is

$$
\sigma^{2}=\frac{\sum(x-\mu)^{2}}{N}=2.9
$$

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## Random Experiment

Roll a die 5 times, and each time record the number on the top face of the die. Calculate the variance, $s^{2}$, of the five values.[2]

Additionally suppose while repeating the experiment the variance of each sample was recorded, and sorted into one of 16 classes.

The animation (right) shows what our histograms might look like. The number of samples represents the number of times the experiment is repeated.

Notice this time the mean of the sample variances targets the population variance, $\sigma^{2}=2.9$; and the distributions are skewed right.



## Definition

The sampling distribution of the variance is the probability distribution of sample variances, with all samples having the same sample size $n$ taken from the same population.

## Properties of the Sampling Distribution of the Variance

$\checkmark$ Sample variances tend to target the value of the population variance, $\sigma^{2}$. (That is, the mean of the sample variances is the population variance. The expected value of the sample variance is equal to the population variance.)
$\checkmark$ The distribution of the sample means tends to be a distribution skewed to the right.

Definition
The sampling distribution of a statistic is the relative frequency distribution of that statistic that is approached as the number of samples (not the sample size!) approaches infinity.

## Got Sampling Distributions?

What about the sampling distributions of sample
(1) proportions,
(2) medians,
(3) ranges and
(4) standard deviations?

## Definition

An unbiased estimator is a sample statistic that has a sampling distribution whose mean is equal to the mean of the corresponding population parameter. An unbiased estimator tends to target or be reflective of the true value of the population parameter it is estimating.

## Unbiased Estimators:

(1) Mean $\bar{X}$
(2) Variance $s^{2}$
(3) Proportion $\hat{p}$

## Definition

A biased estimator is a sample statistic that has a sampling distribution whose mean is not equal to the mean of the corresponding population parameter. A biased estimator does not target the true value of the population parameter it is estimating.

## Biased Estimators:

(1) Median
(2) Range
(3) Standard Deviation $s$

[^3]
## Sampling Distribution Facts

- All statistics, not just the mean, have sampling distributions.
- There is a different sampling distribution for each value of $n$, the sample size.
- $\mu_{\bar{x}}$ is the notation used to represent the mean of a sampling distribution.
- $\sigma_{\bar{x}}$ is the notation used to represent the standard deviation of a sampling distribution.

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### 6.5 The Central Limit Theorem (CLT)

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## The Central Limit Theorem (CLT)

If random samples of $n$ observations are drawn from a nonnormal population with mean $\mu$ and standard deviation $\sigma$, then, when $n$ is large, the sampling distribution of the sample means is approximately normally distributed, with mean and standard deviation

$$
\mu_{\bar{x}}=\mu \quad \text { and } \quad \sigma_{\bar{x}}=\frac{\sigma}{\sqrt{n}}
$$

The approximation becomes more accurate as $n$ becomes large.

The CLT can be applied to any probability distribution (continuous or discrete). We will use the following guidelines for our work:

- For a population with any distribution, if $n>30$, then the sample means have a distribution that can be approximated by a normal distribution with mean $\mu$ and standard deviation $\sigma / \sqrt{n}$.
- If $n \leq 30$ and the original population has a normal distribution, then the sample means have a normal distribution with mean $\mu$ and standard deviation $\sigma / \sqrt{n}$.
- If $n \leq 30$ and the original population does not have a normal distribution, then we do not apply the CLT.
- When the sampled population is approximately symmetric, the sampling distribution of $\bar{x}$ becomes approximately normal for relatively small values of $n$.

Consider the roughly symmetric probability distribution given in the top left figure of the adjacent box (right). The mean of the distribution, $\mu$, was found to be 20.3873. Suppose we repeatedly take random samples of different sizes, $n$, from this distribution.

A data entry in a sample would be an $x$ value between 1 and 43. Also suppose that each time a sample is taken, the sample mean $\bar{x}$ is calculated and a tally mark is placed into one of 50 classes.

The other five figures accompanying the probability distribution show the experiment being repeated for different sample sizes, $n$. In each case, it is clear that the sample means follow a normal distribution and they target the population mean. (Observe that the center of each histogram is very close to $x=20.3873$.)

## Sample means ( $\bar{x}$ ) tend to target population means ( $\mu$ ), and this is easily seen with repeated sampling.

Random Probability Distribution - with $\mu=20.3873$ and $\sigma=12.0106$


| $K$ | $<$ | $\triangleleft$ | $\nabla$ | $>$ | $>$ | - | $+*$ | + |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |

Random Probability Distribution - with $\mu=20.3873$ and $\sigma=12.0106$

You don't need to repeatedly sample the population thousands of times to see the result of the Central Limit Theorem.

The animation here simulates the repeated random sampling, except the vertical scaling is not fixed constant. The blue curve in each figure represents a rough approximation of the outline of each histogram.

Notice that larger sample sizes approximate the normal distribution better than smaller sample sizes do.

$$
n=2
$$




$n=30$
$n=50$




[^4]Notice also that after 10,000 repeated samples were taken, the sample standard deviation for each $n$ was placed in a green box inside each figure (right). Recall that the standard deviation is a measure of how spread out the data is along the $x$ axis.

Observe that as $n$ increases the standard deviation of each histogram decreases (the spread of each distribution is more narrow).

It turns out that multiplying each one of these values of $\sigma_{\bar{x}}$ by $\sqrt{n}$ (for its associated $n$ ) approximates the true population standard deviation, $\sigma=12.0106$ with $99 \%$ accuracy. This comes as a result of the Central Limit Theorem (CLT).

## Because of the CLT we can

 approximate previously unknown values of $\mu$ and $\sigma$, even if we don't know how the population is distributed.Random Probability Distribution - with $\mu=20.3873$ and $\sigma=12.0106$

$$
n=2
$$






[^5]If we constructed relative frequency histograms while conducting the repeated sampling, then we could try to approximate the actual sampling (probability) distribution of sample means associated with each different value of $n$ shown here.

This animation shows some relative frequency histograms from the experiment. The blue curve in each figure represents each actual probability density function*-which itself follows a normal probability distribution. This comes as a result of the Central Limit Theorem (CLT).

We will study applications of this powerful theorem in Chapters 6, 7, 8 and 9 and 10.

Random Probability Distribution - with $\mu=20.3873$ and $\sigma=12.0106$

$$
n=2
$$








[^6]
## Chapter 6

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I invite you to make your own probability distribution and try the repeated sampling to see the CLT in action. The online statbook has a pretty cool applet that lets you do this with ease. Check it out if you have time:
http://onlinestatbook.com/2/sampling_distributions/clt_demo.html
click here to access the classroom worksheet

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(1) M. F. Triola, Essentials of Statistics, Addison-Wesley, fourth ed., 2011.


[^0]:    $z$ score $\quad$ Area
    $-2.575 \quad 0.0050$

[^1]:    $z$ score Area
    -2.575 0.0050

[^2]:    $z$ score Area
    $-1.645 \quad 0.0500$

[^3]:    The bias of the sample standard deviation is relatively small in large samples, so $s$ is often used as an unbiased estimator.

[^4]:    * Recall that there is a separate sampling distribution for each different value of the sample size, $n$.

[^5]:    * Recall that there is a separate sampling distribution for each different value of the sample size, $n$.

[^6]:    * Recall that there is a separate sampling distribution for each different value of the sample size, $n$.

