8.2 Basics of Hypothesis Testing

Type I and II Errors

Level of Significance

P-Value Method

Identifying H_0 and H_1

Works Cited

Chapter 8
Hypothesis Testing

Professor Tim Busken

April 20, 2014

In Chapter 8, we continue our study of inferential statistics.

Concept: Inferential Statistics

The two major activities of inferential statistics are

1. to use sample data to estimate values of population parameters (proportions p, means μ, and variances σ^2), and
2. to test hypotheses or claims made about these population parameters.

Chapter 8 Contents: Hypothesis Testing

8.1 Review and Preview
8.2 Basics of Hypothesis Testing
8.3 Testing a Claim about a Proportion
8.4 Testing a Claim about a Mean
8.5 Testing a Claim about a Standard Deviation or Variance

Learning Objectives

1. Understand the definitions used in hypothesis testing.
2. State the null and alternative hypotheses.
3. Find critical values for the z test.
4. State the five steps used in hypothesis testing.
5. Test means when σ is known, using the z test.
6. Test means when σ is unknown, using the t test.
7. Test proportions, using the z test.
8. Test variances or standard deviations, using the chi-square test.
9. Test hypotheses, using confidence intervals.

Hypothesis testing

Researchers are interested in answering many types of questions. For example,

- Is the earth warming up?
- Does a new medication lower blood pressure?
- Do the overwhelming majority of Americans think it would be a change for the worse if personal and commercial drones are given permission to fly through most U.S. airspace?
- Is a new teaching technique better than a traditional one?
- What percentage of Americans think it would be a change for the worse if most people wear implants or other devices that constantly show them information about the world around them?

These types of questions can be addressed through statistical hypothesis testing, which is a decision-making process for evaluating claims about a population. [1]

Hypothesis testing

Three methods used to test hypotheses:

1. The Critical Value Method
2. The P-value Method
3. The Confidence Interval Method

Basics of Hypothesis Testing – The Critical Value Method

Every hypothesis-testing situation begins with the statement of a hypothesis.

Definition

A **statistical hypothesis** is a conjecture about a population parameter (for example $\mu, p, \sigma or \sigma^2$). This conjecture may or may not be true.

There are two types of statistical hypotheses for each situation: the null hypothesis and the alternative hypothesis.

Definition

The **null hypothesis** (denoted by H_0) is a statement that the value of a population parameter (such as proportion, mean, or standard deviation) is equal ($=, \geq$ or $\leq$) to some specific value.

Definition

The **alternative hypothesis** (denoted by H_a or H_1) is a statement that the value of a population parameter has a value that somehow differs from the null hypothesis. The symbolic form of the alternative hypothesis must use one of these symbols: $\neq$ (the "not equals" symbol), $>$ or $<$).

Hypothesis Testing

Definition

A **statistical test** uses the data obtained from a sample to make a decision about whether the null hypothesis should be rejected.

Definition

The numerical value obtained from a statistical test is called the **test value** or **test statistic**. The test statistic has the form

$$Test\ Statistic = \frac{(observed\ value) - (expected\ value)}{standard\ error}$$

where the observed value is the value of the sample statistic, the expected value is the hypothesized value of the parameter (stated in H_0), and the denominator is the standard error of the (sampling distribution of the) statistic being tested.

Example 1 – The Critical Value Method

A manufacturer of ubiquitous wearable or implanted computing devices is interested in finding out the percentage of Americans who think it would be a change for the worse if most people wear implants or other devices that constantly show them information about the world around them. Pew Research Center recently estimated that percentage to be 53%. The manufacturer claims that is not an accurate estimate, and is interested in finding out whether the true percentage is higher or lower than 53%.

The hypotheses for this situation are

$$H_0: \quad p = 0.53 \qquad\qquad H_1: \quad p \neq 0.53$$

This is called a **two-tailed** hypothesis test about a population proportion (percentage).

Example 2 – The Critical Value Method

A medical researcher is interested in finding out whether a new medication will have any undesirable side effects. The researcher is particularly concerned with the pulse rate of the patients who take the medication. Will the pulse rate increase, decrease, or remain unchanged after a patient takes the medication? The researcher knows that the mean pulse rate for the population under study is 82 beats per minute.

The hypotheses for this situation are

$$H_0: \quad \mu = 82 \; bpm \qquad\qquad H_1: \quad \mu \neq 82 \; bpm$$

This is an example of a **two-tailed** hypothesis test about a population mean.

Example 3 – The Critical Value Method

A medical researcher is interested in finding out whether her company's new fertility medication will increase the percentage likelihood that a newborn baby is a female. The researcher knows that if no treatment is applied, the percentage likelihood that a newborn baby is a female is 50%.

The hypotheses for this situation are

$$H_0: \quad p = 0.50 \qquad\qquad H_1: \quad p > 0.50$$

This is called a **right-tailed** hypothesis test.

Example 4 – The Critical Value Method

Consider an airline flight to be late if it arrives later than 15 minute past it's scheduled arrival time. A well known airline company wants to lower the percentage of flights that arrive late, which is currently 10%, by making some changes in company policy.

The hypotheses for this situation are

$$H_0: \quad p = 0.10 \qquad\qquad H_1: \quad p < 0.10$$

This is called a **left-tailed** hypothesis test.

Hypothesis Testing Algorithm Summary

After stating the hypotheses, the researcher's next step is to design the study. The researcher selects the correct statistical test, chooses an appropriate level of significance, and formulates a plan for conducting the study.

A random sample is drawn and a sample statistic is calculated. The researcher then measures the difference between the sample statistic and the hypothesized value of the population parameter as stated in the null.

If this difference is large enough, the statement of the null hypothesis is rejected as being the truth. Otherwise, if the difference is small, then the researcher concludes that the sample evidence does not contradict the assumed value of the population parameter as stated in the null, and that the difference is due to sampling error.

Hypothesis Testing Outcomes

In the hypothesis-testing situation, there are four possible outcomes. In reality, the null hypothesis may or may not be true, and a decision is made to reject or not to reject it on the basis of the data obtained from a sample.

Definition

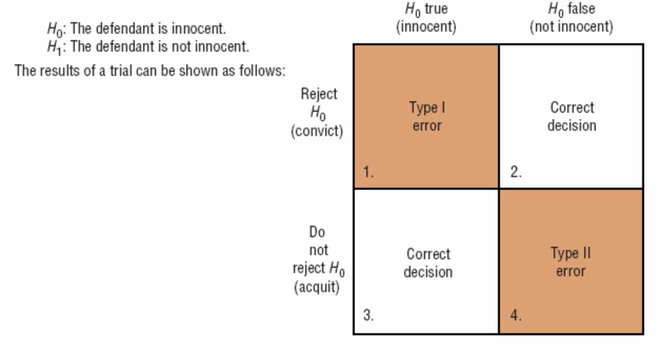
A **type I (or α type) error** occurs if you reject the null hypothesis when it is true. A **type II (or β type) error** occurs if you do not reject the null hypothesis when it is false.

	H_0 true	H_0 false
Reject H_0	**Error** Type I	Correct decision
Do not reject H_0	Correct decision	**Error** Type II

8.2 Basics of Hypothesis Testing
Type I and II Errors
Level of Significance
P-Value Method
Identifying H_0 and H_1
Works Cited

Hypothesis Testing Outcomes

The hypothesis-testing situation can be likened to a jury trial.[1] In a jury trial, there are four possible outcomes. The defendant is either guilty or innocent, and he or she will be convicted or acquitted.



8.2 Basics of Hypothesis Testing
Type I and II Errors
Level of Significance
P-Value Method
Identifying H_0 and H_1
Works Cited

Jury Decision

The decision of the jury does not prove that the defendant did or did not commit the crime. The decision is based on the evidence presented. If the evidence is strong enough, the defendant will be convicted in most cases. If the evidence is weak, the defendant will be acquitted in most cases. Nothing is proved absolutely. Likewise, the decision to reject or not reject the null hypothesis does not prove anything.

The only way to prove anything statistically is to use the entire population, which, in most cases, is not possible. The decision, then, is made on the basis of probabilities. That is, when there is a large difference between the mean obtained from the sample and the hypothesized mean, the null hypothesis is probably not true.

The question is, How large a difference is necessary to reject the null hypothesis? Here is where the **level of significance** is used. [1]

Level of Significance, α

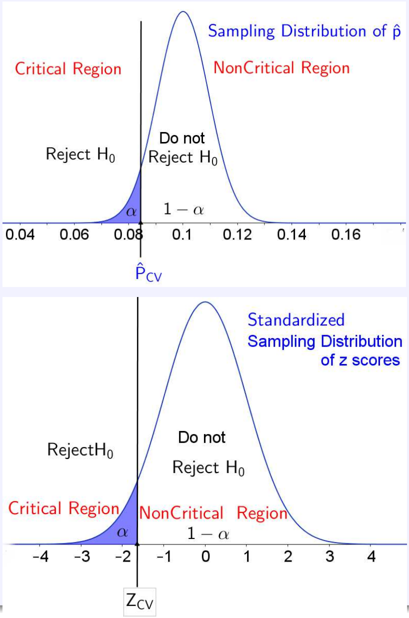
Definition

The **level of significance** is the maximum probability of committing a type I error. This probability is symbolized by a (Greek letter alpha). That is,

$$P(\textit{type I error}) = \alpha.$$

- The researcher preselects what level of significance to use before conducting the hypothesis test.
- Typical significance levels are: 0.10, 0.05, and 0.01
- For example, when $\alpha = 0.10$, there is a 10% chance of rejecting a true null hypothesis.
- Significance levels can be any level depending on the seriousness of a type I error.
- After a significance level is chosen, a **critical value** is selected for the appropriate test.

The Critical Value Method – A Left-Tailed Test



Recall the situation with the airline company that wants to lower the percentage of flights that arrive late, which is currently 10%, by making some changes in company policy.

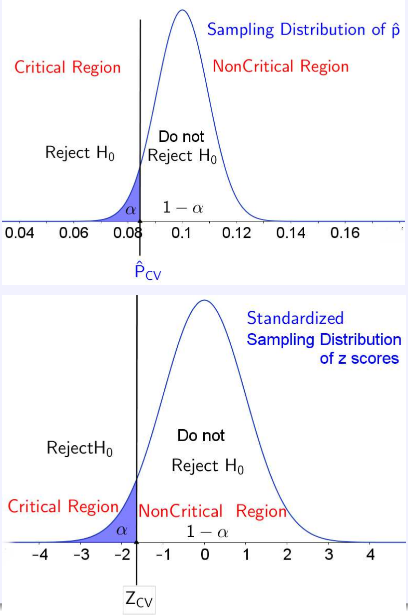
The hypotheses for this situation were

$$H_0: \quad p = 0.10 \qquad\qquad H_1: \quad p < 0.10$$

When the company hypothesizes in the null that the population proportion of late arrivals is 0.10, they also hypothesize that the theoretical sampling distribution of sample proportions in the Cartesian coordinate system is centered at $p = 0.10$. (The x-axis is a $\hat{p}$-axis in this context.)

A **level of significance** is then selected, say $\alpha = 5\% = 0.05$. This determines a **critical value of** $\hat{p}$ along the horizontal axis, left of center, that provides a border between acceptable and unacceptable sample proportions. In addition, the critical z-score value associated with the critical value of $\hat{p}$ is located under the standardized sampling distribution of z-scores.

The Critical Value Method – A Left-Tailed Test



Definition

The **critical value** separates the critical region from the noncritical region. The symbol for critical value is C.V. (A critical value is a standardized score (*z*-score, *t*-score, etc.) that is used to distinguish between sample statistics that are likely to occur from those that are unlikely to occur.)

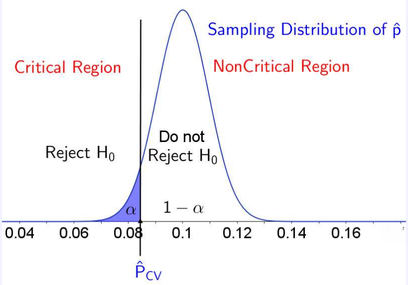
Definition

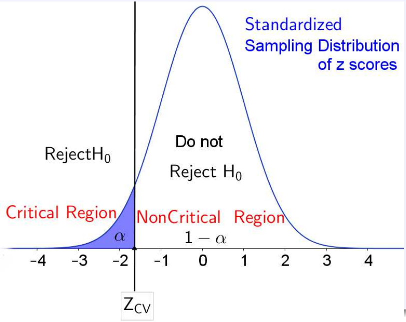
The **critical or rejection region** is the range of values of the test statistic that indicates that there is a significant difference and that the null hypothesis should be rejected.

Definition

The **noncritical region (sometimes called the "fail to reject" region)** is the range of values of the test statistic that indicates that the difference was probably due to chance and that the null hypothesis should not be rejected.

The Critical Value Method – A Left-Tailed Test



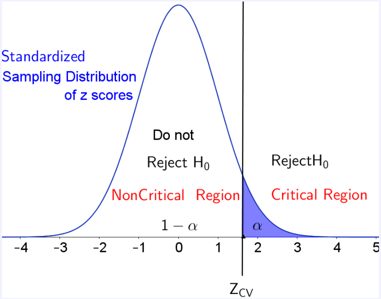


After a level of significance is chosen and the critical value of z is determined, the airline company applies a treatment to the situation in the form of some changes in company policy. To determine whether the changes lowered the late arrival rates, the company draws a random sample of say, 1000 flights from it's population and measures the sample percentage, $\hat{p}$, of flights arriving late.

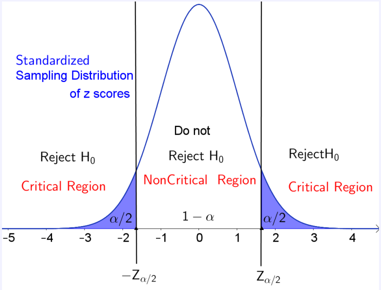
The test statistic, the value of z associated with this sample percentage is calculated. Afterwards, this test value of z is located along the horizontal axis under the standard normal curve. If the test value of z is located in the critical region, the company can reject H_0 and conclude that their change in company policy led to a decrease in late arrival rates among the population of all flights.

Otherwise, if the test value of z is located along the horizontal axis in the noncritical region, then the company does not have enough evidence to reject H_0 and conclude that their change in company policy was effective.

The Critical Value Method



Whenever you have a right-tailed test (that is, when the symbol used in the statement of H_1 is a $>$ symbol), the critical value of z is located right of the center. In a right-tailed test, the null hypothesis is rejected when the test value lies in the critical region.



Or it could be that the test is a two-tailed test (that is, the symbol used in the statement of H_1 is a $\neq$ symbol), in which case their are two critical values of z, one left of center and one located right of center. In a two-tailed test, the null hypothesis should be rejected when the test value is in either of the two critical regions.

The P-Value Method for Hypothesis Testing

Statisticians usually test hypotheses at the common α levels of 0.05 or 0.01 and sometimes at 0.10. Recall that the choice of the α level depends on the seriousness of the type I error. Besides listing an α value, many computer statistical packages give a P-value for hypothesis tests.

Definition (P–value)

The P–value is the probability of getting a value of the test statistic that is at least as extreme as the one representing the given sample data, assuming that the null hypothesis is true. One often "rejects the null hypothesis" when the P–value is less than the predetermined significance level (α), indicating that the observed result would be highly unlikely under the null hypothesis.

In other words, the P-value is the actual area under the standard normal distribution curve (or other curve, depending on what statistical test is being used) representing the probability of a particular sample statistic or a more extreme sample statistic occurring if the null hypothesis is true.[1]

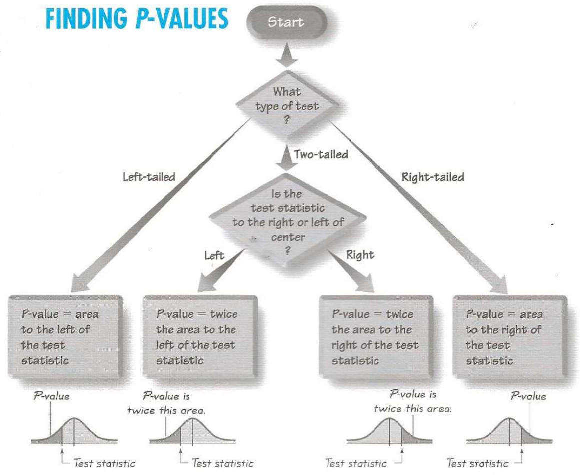
For example, suppose that an alternative hypothesis is $H_1 : p < 0.50$ and the proportion from a sample is 0.38. If the computer printed a P-value of 0.0356 for a statistical test, then the probability of getting a sample percentage of 38% or less is 3.56% if the true population proportion is 50%. The relationship between the P-value and the α value can be explained in this manner. For a P-value $= 0.0356$, the null hypothesis would be rejected at a $\alpha = 0.05$ but not at $\alpha = 0.01$.

8.2 Basics of Hypothesis Testing
Type I and II Errors
Level of Significance
P-Value Method
Identifying H_0 and H_1

Works Cited

Using the *P* Value Method [2]

DO NOT LOCATE CRITICAL VALUES OF Z IN YOUR PICTURE OF THE STANDARDIZED SAMPLING DISTRIBUTION.



IF THE *P*-VALUE $\leq \alpha$, REJECT H_0. IF THE *P*-VALUE $> \alpha$, FAIL TO REJECT H_0.

8.2 Basics of Hypothesis Testing

Type I and II Errors

Level of Significance

P-Value Method

Identifying H_0 and H_1

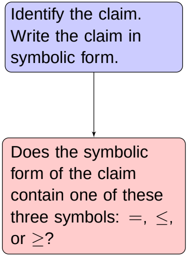
Works Cited

Identifying H_0 and H_1

Identify the claim.
Write the claim in symbolic form.

8.2 Basics of Hypothesis Testing
Type I and II Errors
Level of Significance
P-Value Method
Identifying H_0 and H_1
Works Cited

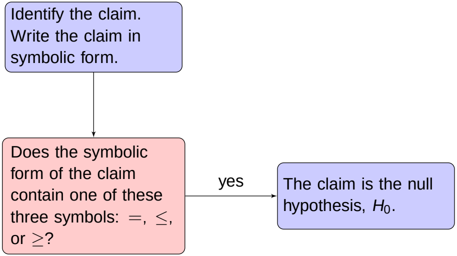
Identifying H_0 and H_1



8.2 Basics of Hypothesis Testing
Type I and II Errors
Level of Significance
P-Value Method
Identifying H_0 and H_1

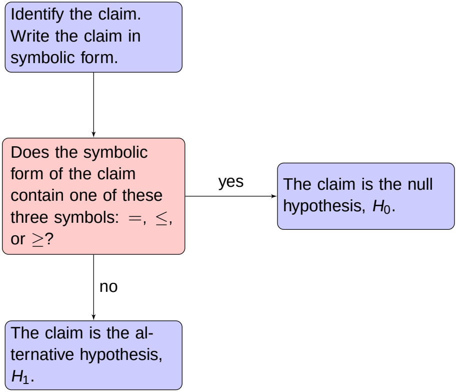
Works Cited

Identifying H_0 and H_1



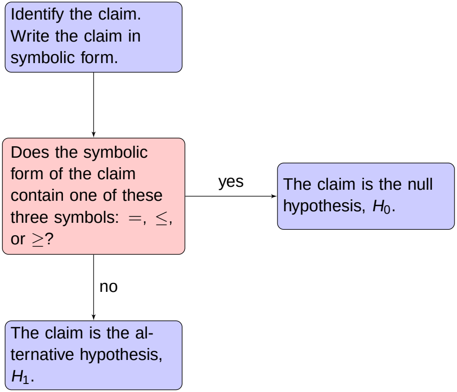
8.2 Basics of Hypothesis Testing
Type I and II Errors
Level of Significance
P-Value Method
Identifying H_0 and H_1
Works Cited

Identifying H_0 and H_1



Identifying H_0 and H_1

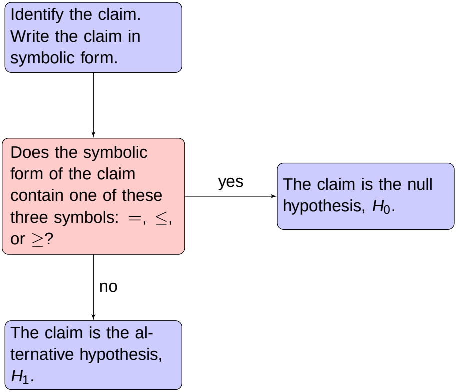
The claim can be either the null or alternative hypothesis — but the null hypothesis must always have the $=$ (or $\geq$ or $\leq$) symbol.



Identifying H_0 and H_1

Example: Express the null hypothesis and the alternative hypothesis in symbolic form. Also, state whether the test is a left-tailed, right-tailed, or two-tailed test.

Claim: less than 15% of teens smoke.



Identifying H_0 and H_1

Example: Express the null hypothesis and the alternative hypothesis in symbolic form. Also, state whether the test is a left-tailed, right-tailed, or two-tailed test.

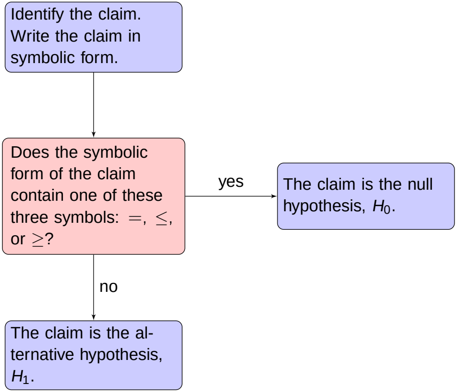
Claim: less than 15% of teens smoke.

Answer:

$$H_0: \ p = 0.15$$

$$H_1: \ p < 0.15 \quad \text{(claim)}$$

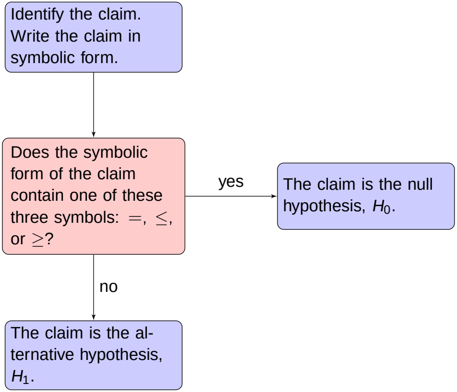
Left-Tailed test since the symbol used in H_1 is a less than symbol.



Identifying H_0 and H_1

Example: Express the null hypothesis and the alternative hypothesis in symbolic form. Also, state whether the test is a left-tailed, right-tailed, or two-tailed test.

Cash Motor Company claims that its new sedan will average better than 36 miles per gallon in the city.



Identifying H_0 and H_1

Example: Express the null hypothesis and the alternative hypothesis in symbolic form. Also, state whether the test is a left-tailed, right-tailed, or two-tailed test.

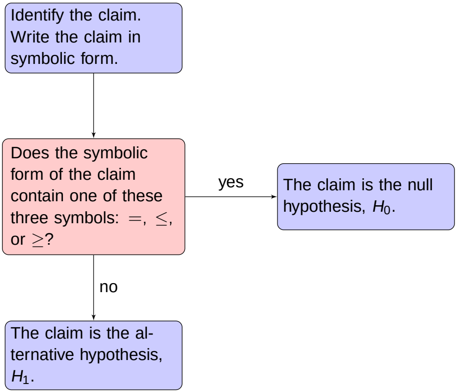
Cash Motor Company claims that its new sedan will average better than 36 miles per gallon in the city.

Answer:

$$H_0: \ \mu = 36 \ mpg$$

$$H_1: \ \mu > 36 \quad \text{(claim)}$$

Right-Tailed test since the symbol used in H_1 is a greater than symbol.



8.2 Basics of Hypothesis Testing

Type I and II Errors

Level of Significance

P-Value Method

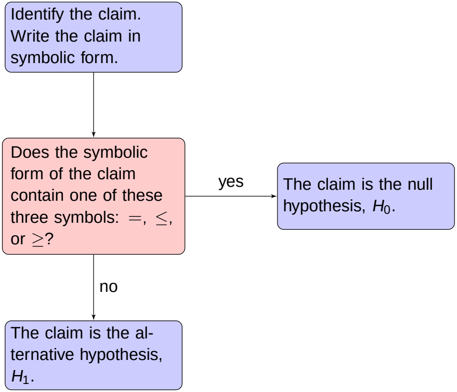
Identifying H_0 and H_1

Works Cited

Identifying H_0 and H_1

Example: Express the null hypothesis and the alternative hypothesis in symbolic form. Also, state whether the test is a left-tailed, right-tailed, or two-tailed test.

Claim: 59% of Americans are optimistic that coming technological and scientific changes will make life in the future better



Identifying H_0 and H_1

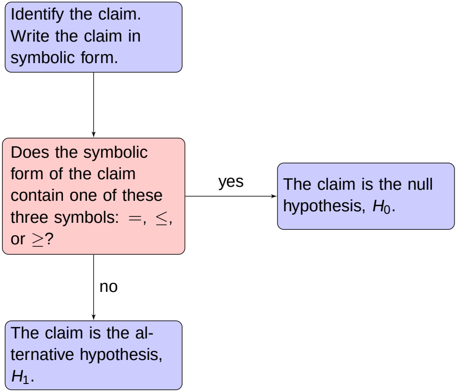
Example: Express the null hypothesis and the alternative hypothesis in symbolic form. Also, state whether the test is a left-tailed, right-tailed, or two-tailed test.

Claim: 59% of Americans are optimistic that coming technological and scientific changes will make life in the future better

Answer:

$$H_0: \; p = 0.59 \quad \text{(claim)}$$

$$H_1: \; p \neq 0.59$$



8.2 Basics of Hypothesis Testing
Type I and II Errors
Level of Significance
P-Value Method
Identifying H_0 and H_1

Works Cited

The 5 Steps of Hypothesis Testing

1. State the null hypothesis, H_0
2. State the alternative hypothesis, H_1
3. Identify which test statistic formula is to be used from the formula card. Calculate the value of the test statistic.
4. Draw a picture of the sampling distribution being used. Apply either the Traditional Method or P-value Method of analysis.
5. State the full sentence conclusion (result) of the hypothesis test. Use the wording from the flow chart given on the formula card.

The Test Statistic

Definition

The **test statistic** is a value used in making a decision about the null hypothesis. The test statistic is found by converting (standardizing) the sample statistic (such as $\hat{p}$, $\overline{x}$ or s) to a score (such as z, t, or χ^2), with the assumption that the null hypothesis is true.

Ch. 8: Test Statistics (one population)

$z = \dfrac{\hat{p} - p}{\sqrt{\dfrac{pq}{n}}}$ Proportion—one population

$z = \dfrac{\bar{x} - \mu}{\sigma/\sqrt{n}}$ Mean—one population (σ known)

$t = \dfrac{\bar{x} - \mu}{s/\sqrt{n}}$ Mean—one population (σ unknown)

$\chi^2 = \dfrac{(n-1)s^2}{\sigma^2}$ Standard deviation or variance—one population

Ch. 9: Test Statistics (two populations)

$z = \dfrac{(\hat{p}_1 - \hat{p}_2) - (p_1 - p_2)}{\sqrt{\dfrac{\bar{p}\bar{q}}{n_1} + \dfrac{\bar{p}\bar{q}}{n_2}}}$ Two proportions $\bar{p} = \dfrac{x_1 + x_2}{n_1 + n_2}$

$t = \dfrac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\dfrac{s_1^2}{n_1} + \dfrac{s_2^2}{n_2}}}$ df = smaller of $n_1 - 1$, $n_2 - 1$

Two means—independent; σ_1 and σ_2 unknown, and not assumed equal.

$t = \dfrac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\dfrac{s_p^2}{n_1} + \dfrac{s_p^2}{n_2}}}$ (df = $n_1 + n_2 - 2$)

$s_p^2 = \dfrac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}$

Two means—independent; σ_1 and σ_2 unknown, but assumed equal.

$z = \dfrac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\dfrac{\sigma_1^2}{n_1} + \dfrac{\sigma_2^2}{n_2}}}$ Two means—independent; σ_1, σ_2 known.

$t = \dfrac{\bar{d} - \mu_d}{s_d/\sqrt{n}}$ Two means—matched pairs (df = $n - 1$)

Ch. 11: Goodness-of-Fit and Contingency Tables

$\chi^2 = \Sigma \dfrac{(O - E)^2}{E}$ Goodness-of-fit (df = $k - 1$)

$\chi^2 = \Sigma \dfrac{(O - E)^2}{E}$ Contingency table [df = $(r - 1)(c - 1)$]

where $E = \dfrac{(\text{row total})(\text{column total})}{(\text{grand total})}$

Ch. 10: Linear Correlation/Regression

Correlation $r = \dfrac{n\Sigma xy - (\Sigma x)(\Sigma y)}{\sqrt{n(\Sigma x^2) - (\Sigma x)^2}\sqrt{n(\Sigma y^2) - (\Sigma y)^2}}$

or $r = \dfrac{\sum (z_x z_y)}{n - 1}$ where z_x = z score for x, z_y = z score for y

Slope: $b_1 = \dfrac{n\Sigma xy - (\Sigma x)(\Sigma y)}{n(\Sigma x^2) - (\Sigma x)^2}$

or $b_1 = r\dfrac{s_y}{s_x}$

y-Intercept:

$b_0 = \bar{y} - b_1\bar{x}$ or $b_0 = \dfrac{(\Sigma y)(\Sigma x^2) - (\Sigma x)(\Sigma xy)}{n(\Sigma x^2) - (\Sigma x)^2}$

$\hat{y} = b_0 + b_1 x$ Estimated eq. of regression line

$r^2 = \dfrac{\text{explained variation}}{\text{total variation}}$

$s_e = \sqrt{\dfrac{\Sigma(y - \hat{y})^2}{n - 2}}$ or $\sqrt{\dfrac{\Sigma y^2 - b_0\Sigma y - b_1\Sigma xy}{n - 2}}$

$\hat{y} - E < y < \hat{y} + E$ Prediction interval

where $E = t_{\alpha/2}s_e\sqrt{1 + \dfrac{1}{n} + \dfrac{n(x_0 - \bar{x})^2}{n(\Sigma x^2) - (\Sigma x)^2}}$

$r_s = 1 - \dfrac{6\Sigma d^2}{n(n^2 - 1)}$ Rank correlation

$\left(\text{critical value for } n > 30\text{: } \dfrac{\pm z}{\sqrt{n - 1}}\right)$

Ch. 11: One-Way Analysis of Variance

Procedure for testing H_0: $\mu_1 = \mu_2 = \mu_3 = \cdots$

1. Use software or calculator to obtain results.
2. Identify the P-value.
3. Form conclusion:

   If P-value $\le \alpha$, reject the null hypothesis of equal means.

   If P-value $> \alpha$, fail to reject the null hypothesis of equal means.

Step 3: The Test Statistic

- Identify and calculate the correct test statistic.

Ch. 8: Test Statistics (one population)

$z = \dfrac{\hat{p} - p}{\sqrt{\dfrac{pq}{n}}}$ Proportion—one population

$z = \dfrac{\bar{x} - \mu}{\sigma/\sqrt{n}}$ Mean—one population (σ known)

$t = \dfrac{\bar{x} - \mu}{s/\sqrt{n}}$ Mean—one population (σ unknown)

$\chi^2 = \dfrac{(n-1)s^2}{\sigma^2}$ Standard deviation or variance—one population

Ch. 9: Test Statistics (two populations)

$z = \dfrac{(\hat{p}_1 - \hat{p}_2) - (p_1 - p_2)}{\sqrt{\dfrac{\bar{p}\bar{q}}{n_1} + \dfrac{\bar{p}\bar{q}}{n_2}}}$ Two proportions, $\bar{p} = \dfrac{x_1 + x_2}{n_1 + n_2}$

$t = \dfrac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\dfrac{s_1^2}{n_1} + \dfrac{s_2^2}{n_2}}}$ df = smaller of $n_1 - 1, n_2 - 1$

Two means—independent; σ_1 and σ_2 unknown, and not assumed equal.

$t = \dfrac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\dfrac{s_p^2}{n_1} + \dfrac{s_p^2}{n_2}}}$ (df $= n_1 + n_2 - 2$)

$s_p^2 = \dfrac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}$

Two means—independent; σ_1 and σ_2 unknown, but assumed equal.

$z = \dfrac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\dfrac{\sigma_1^2}{n_1} + \dfrac{\sigma_2^2}{n_2}}}$ Two means—independent; σ_1, σ_2 known.

$t = \dfrac{\bar{d} - \mu_d}{s_d/\sqrt{n}}$ Two means—matched pairs (df $= n - 1$)

Ch. 11: Goodness-of-Fit and Contingency Tables

$\chi^2 = \Sigma \dfrac{(O - E)^2}{E}$ Goodness-of-fit (df $= k - 1$)

$\chi^2 = \Sigma \dfrac{(O - E)^2}{E}$ Contingency table [df $= (r - 1)(c - 1)$]

where $E = \dfrac{(\text{row total})(\text{column total})}{(\text{grand total})}$

Ch. 10: Linear Correlation/Regression

Correlation $r = \dfrac{n\Sigma xy - (\Sigma x)(\Sigma y)}{\sqrt{n(\Sigma x^2) - (\Sigma x)^2}\sqrt{n(\Sigma y^2) - (\Sigma y)^2}}$

or $r = \dfrac{\Sigma(z_x z_y)}{n - 1}$ where z_x = z score for x, z_y = z score for y

Slope: $b_1 = \dfrac{n\Sigma xy - (\Sigma x)(\Sigma y)}{n(\Sigma x^2) - (\Sigma x)^2}$

or $b_1 = r\dfrac{s_y}{s_x}$

y-Intercept:

$b_0 = \bar{y} - b_1\bar{x}$ or $b_0 = \dfrac{(\Sigma y)(\Sigma x^2) - (\Sigma x)(\Sigma xy)}{n(\Sigma x^2) - (\Sigma x)^2}$

$\hat{y} = b_0 + b_1 x$ Estimated eq. of regression line

$r^2 = \dfrac{\text{explained variation}}{\text{total variation}}$

$s_e = \sqrt{\dfrac{\Sigma(y - \hat{y})^2}{n - 2}}$ or $\sqrt{\dfrac{\Sigma y^2 - b_0\Sigma y - b_1\Sigma xy}{n - 2}}$

$\hat{y} - E < y < \hat{y} + E$ Prediction interval

where $E = t_{\alpha/2} s_e \sqrt{1 + \dfrac{1}{n} + \dfrac{n(x_0 - \bar{x})^2}{n(\Sigma x^2) - (\Sigma x)^2}}$

$r_s = 1 - \dfrac{6\Sigma d^2}{n(n^2 - 1)}$ Rank correlation

$\left(\text{critical value for } n > 30\text{: } \dfrac{\pm z}{\sqrt{n - 1}}\right)$

Ch. 11: One-Way Analysis of Variance

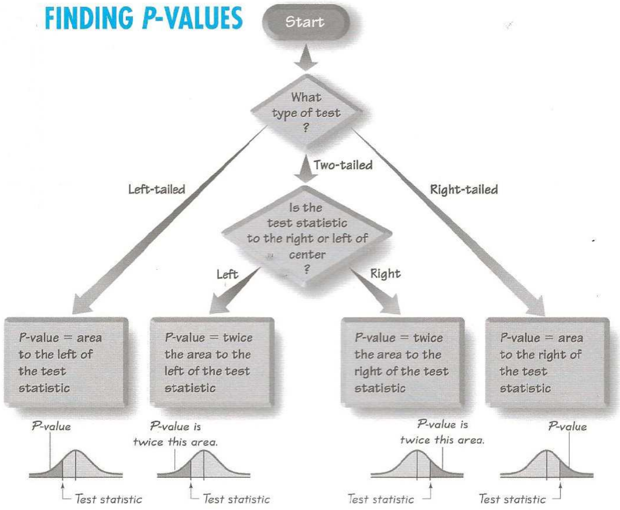
Procedure for testing H_0: $\mu_1 = \mu_2 = \mu_3 = \cdots$

1. Use software or calculator to obtain results.
2. Identify the P-value.
3. Form conclusion:

   If P-value $\le \alpha$, reject the null hypothesis of equal means.

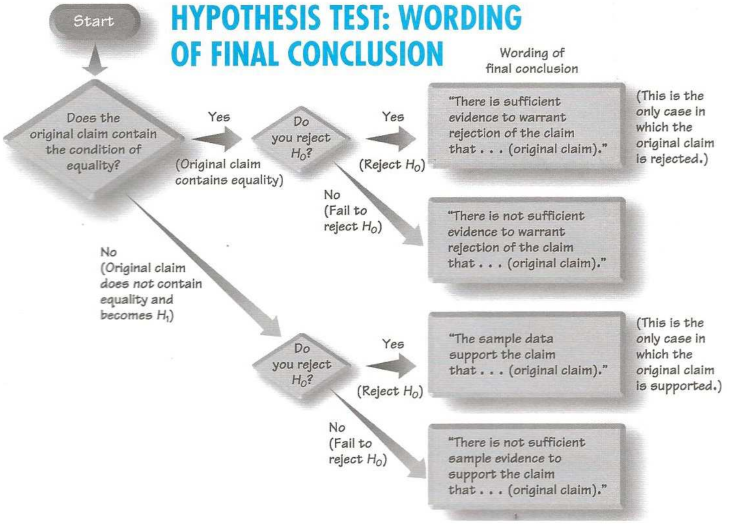
   If P-value $> \alpha$, fail to reject the null hypothesis of equal means.

Step 4: Using the *P* Value Method [2]



If the *P*-value $\leq \alpha$, reject H_0. If the *P*-value $> \alpha$, fail to reject H_0.

Step 5 of the Hypothesis Test [2]



A. G. Bluman, *Elementary Statistics*, Magraw-Hill, sixth ed., 2013.

M. F. Triola, *Essentials of Statistics*, Addison-Wesley, fourth ed., 2011.