Hypothesis Testing Algorithm

STEP 1: Write the null hypothesis, H_0.

STEP 2: Write the alternative hypothesis, H_1.

STEP 3: Identify which **test statistic** should be used from the formula card. Calculate the value of the test statistic.

STEP 4: Draw a picture of the appropriate *standardized* sampling distribution. Apply either the Critical Value (CV) Method or the *P*-value Method of analysis.

STEP 5: Write the full sentence conclusion (result) of the hypothesis test. Use the wording from the flow chart given on page 3.

Critical Value (CV) Method

☞ Draw a picture of the *standardized* sampling distribution. This is either a z distribution, t distribution, or χ^2 distribution, depending on which population parameter (p, μ, σ or σ^2) is being tested.

☞ Determine if the test is a left-tailed, right-tailed or two-tailed test according to this rule: If the symbol used in the statement of the alternative hypothesis, H_1, is

$>$ then you have a right-tailed test

$<$ then you have a left-tailed test

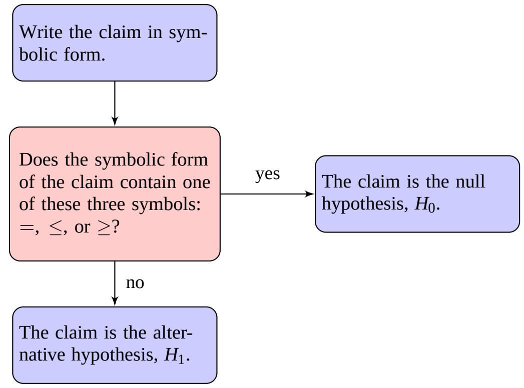
$\neq$ then you have a two-tailed test

☞ The critical value separates the critical region (also called the "reject H_0 region") from the non-critical or "fail to reject H_0 region." (Critical values (CVs) are denoted by z_α or $z_{\alpha/2}$ if the z distribution is used, t_α or $t_{\alpha/2}$ if the t distribution is used, and χ^2_L and χ^2_R if the Chi-Square distribution is used. Alternatively, we sometimes just label or denote critical values as "CV" instead.)

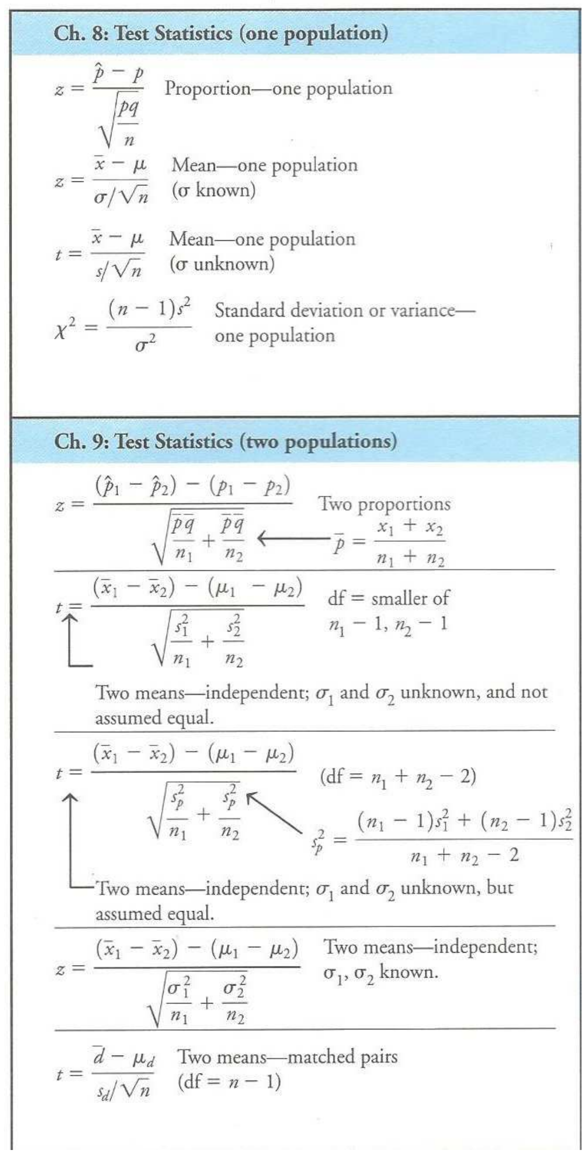
☞ Mark the location of the CV or CV's along the horizontal axis of your *standardized* sampling distribution picture (see Cases 1, 2 and 3 on the next page).

☞ Determine which region the test statistic falls in.

Use this flow chart to do Steps 1 and 2.



Use this for Step 3.



Ch. 8: Test Statistics (one population)

$z = \dfrac{\hat{p} - p}{\sqrt{\dfrac{pq}{n}}}$ Proportion—one population

$z = \dfrac{\bar{x} - \mu}{\sigma/\sqrt{n}}$ Mean—one population (σ known)

$t = \dfrac{\bar{x} - \mu}{s/\sqrt{n}}$ Mean—one population (σ unknown)

$\chi^2 = \dfrac{(n-1)s^2}{\sigma^2}$ Standard deviation or variance—one population

Ch. 9: Test Statistics (two populations)

$z = \dfrac{(\hat{p}_1 - \hat{p}_2) - (p_1 - p_2)}{\sqrt{\dfrac{\bar{p}\bar{q}}{n_1} + \dfrac{\bar{p}\bar{q}}{n_2}}}$ Two proportions $\bar{p} = \dfrac{x_1 + x_2}{n_1 + n_2}$

$t = \dfrac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\dfrac{s_1^2}{n_1} + \dfrac{s_2^2}{n_2}}}$ df = smaller of $n_1 - 1$, $n_2 - 1$

Two means—independent; σ_1 and σ_2 unknown, and not assumed equal.

$t = \dfrac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\dfrac{s_p^2}{n_1} + \dfrac{s_p^2}{n_2}}}$ (df $= n_1 + n_2 - 2$)

$s_p^2 = \dfrac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}$

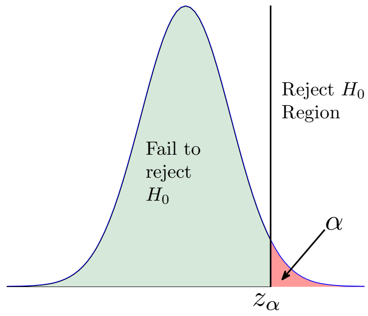
Two means—independent; σ_1 and σ_2 unknown, but assumed equal.

$z = \dfrac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\dfrac{\sigma_1^2}{n_1} + \dfrac{\sigma_2^2}{n_2}}}$ Two means—independent; σ_1, σ_2 known.

$t = \dfrac{\bar{d} - \mu_d}{s_d/\sqrt{n}}$ Two means—matched pairs (df $= n - 1$)

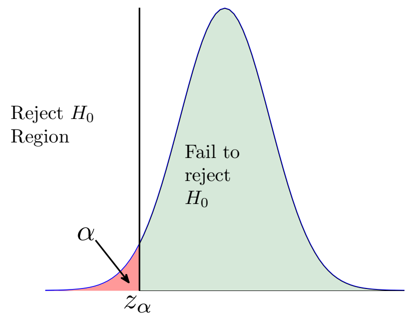
Critical Value (CV) Method

Case 1: The Right-Tailed Test



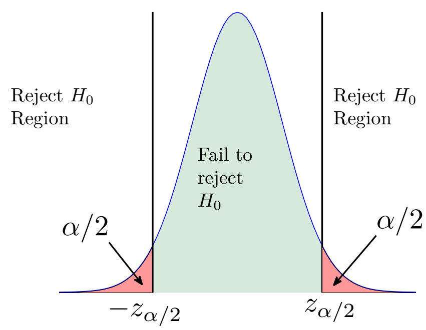
- ✎ Locate the critical value (CV) along the horizontal axis right of center. Label the CV either z_α, t_α or χ^2_R, depending on which parameter (μ, p, σ or σ^2) is being tested.
- ✎ Draw a vertical line segment at the CV. Label the region right of the segment as the critical or "reject H_0 region." Label the region left of the vertical line as the non-critical or "fail to reject H_0 region."
- ✎ Determine the specific value of the CV by assigning the level of significance, α, to be the area under the *standardized* sampling distribution curve, right of the CV. Next, locate the test statistic (from STEP 3) along the horizontal axis in your picture. If the test statistic lies in the critical region, then we reject H_0 and conclude that H_1 is true. Otherwise, if the test statistic lies in the non-critical region, then we do not reject H_0.

Case 2: The Left-Tailed Test



- ✎ Locate the critical value (CV) along the horizontal axis left of center. Label the CV either z_α, t_α or χ^2_L, depending on which parameter (μ, p, σ or σ^2) is being tested.
- ✎ Draw a vertical line segment at the CV. Label the region left of the vertical line as the critical or "reject H_0 region." Label the region right of the vertical line as the non-critical or "fail to reject H_0 region."
- ✎ Determine the specific value of the CV by assigning the level of significance, α, to be the area under the *standardized* sampling distribution curve, left of the CV. Next, locate the test statistic (from STEP 3) along the horizontal axis in your picture. If the test statistic lies in the critical region, then we reject H_0 and conclude that H_1 is true. Otherwise, if the test statistic lies in the non-critical region, then we do not reject H_0.

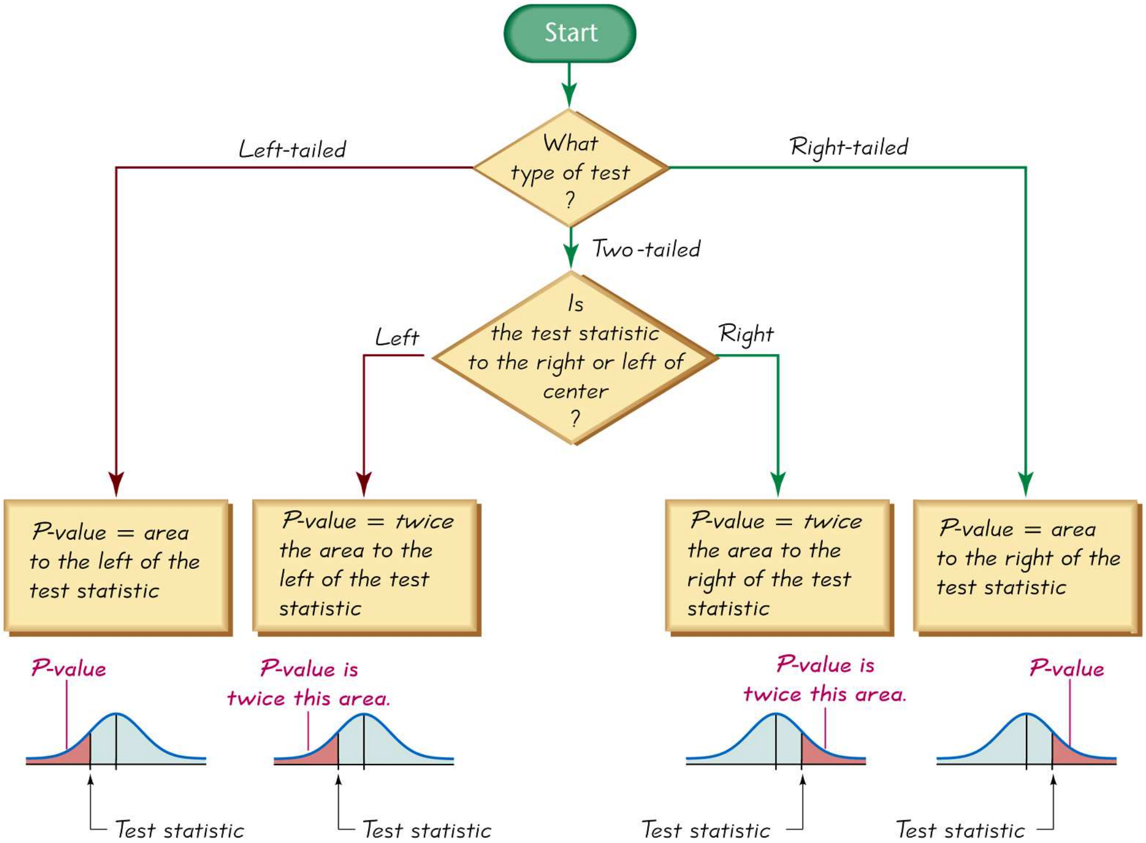
Case 3: The Two-Tailed Test



- ✎ Locate the two critical values (CV's) along the horizontal axis, one left of center and one right of center. Label the CV's as either $\pm z_\alpha/2$, $\pm t_\alpha/2$ or χ^2_L and χ^2_R, depending on which parameter (μ, p, σ or σ^2) is being tested.
- ✎ Draw vertical line segments at the CV's. Label the region between the CV's as the noncritical or "fail to reject H_0 region." Label the tail regions as the "reject H_0 regions."
- ✎ Determine the specific value of the leftmost CV by assigning $\alpha/2$ to be the area under the *standardized* sampling distribution curve, left of the leftmost CV. The rightmost CV is the *opposite* of the leftmost CV (unless you are using the χ^2 distribution). Next, locate the test statistic (from STEP 3) along the horizontal axis in your picture. If the test statistic lies between the CV's, then we do not reject H_0; otherwise, we reject H_0 and support H_1.

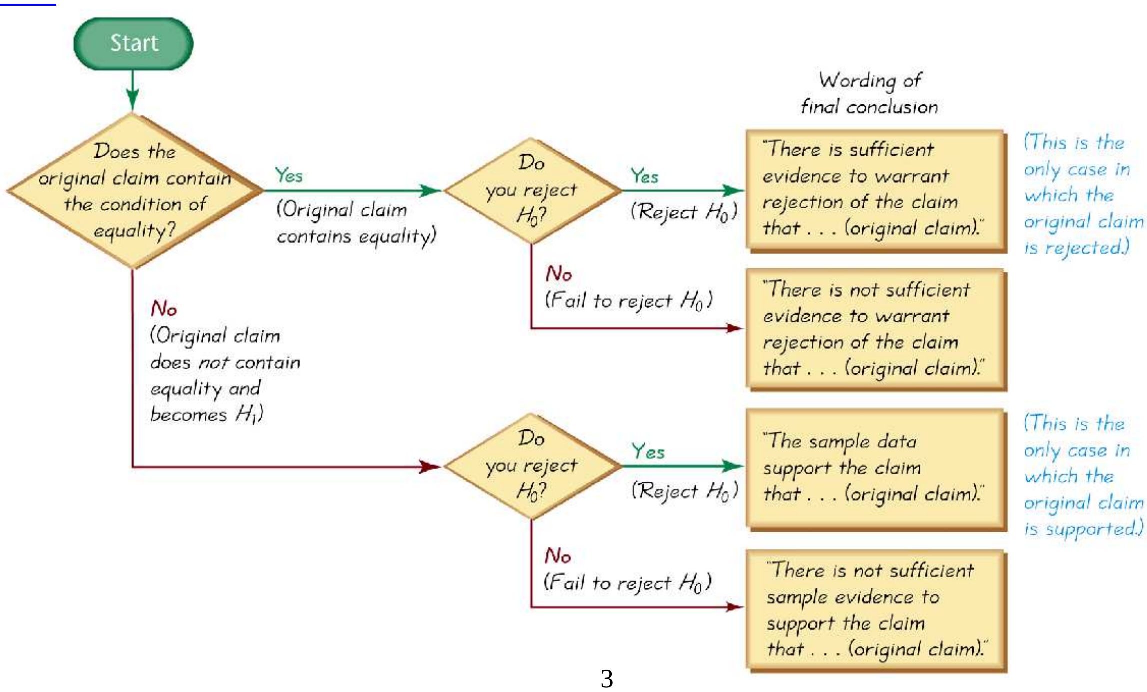
P–value Method

Do not locate any CV's along the horizontal axis in your picture. Locate only the test statistic along the horizontal axis in your picture of your standardized sampling distribution.



If the P-value $\leq \alpha$, reject H_0. If the P-value $> \alpha$, fail to reject H_0.

Step 5: the Wording of the Conclusion of the Hypothesis Test



Definition 1 (Statistical Hypothesis)**.** *A statistical hypothesis is a conjecture about a population parameter (for example* $\mu, p, \sigma or \sigma^2$*). This conjecture may or may not be true.*

Definition 2 (Null Hypothesis, H_0)**.** *The null hypothesis (denoted by* H_0*) is a statement that the value of a population parameter (such as proportion, mean, or standard deviation) is equal (*$=, \geq$ *or* $\leq$*) to some specific value.*

Definition 3 (Alternative Hypothesis, H_1)**.** *The alternative hypothesis (denoted by* H_a *or* H_1*) is a statement that the value of a population parameter has a value that somehow differs from the null hypothesis. The symbolic form of the alternative hypothesis must use one of these symbols:* ≠ *(the "not equals" symbol),* $>$ *or* $<$*.*

Definition 4 (Statistical Test)**.** *A statistical test uses the data obtained from a sample to make a decision about whether the null hypothesis should be rejected.*

Definition 5 (Test Value or Test Statistic)**.** *The numerical value obtained from a statistical test is called the test value or test statistic. The test statistic has the form*

$$Test\ Statistic = \frac{(observed\ value) - (expected\ value)}{standard\ error}$$

where the observed value is the value of the sample statistic, the expected value is the hypothesized value of the parameter (stated in H_0*), and the denominator is the standard error of the statistic being tested.*

Definition 6 (Type I or Type II Error)**.** *A type I (or* α *type) error occurs if you reject the null hypothesis when it is true. A type II (or* β *type) error occurs if you do not reject the null hypothesis when it is false.*

Definition 7 (Level of Significance, α)**.** *The level of significance is the maximum probability of committing a type I error. This probability is symbolized by a (Greek letter alpha). That is,* $P(type\ I\ error) = \alpha$*.*

Definition 8 (Critical Value)**.** *The critical value separates the critical region from the noncritical region. The symbol for critical value is C.V. (A critical value is a standardized score (z-score, t-score, etc.) that is used to distinguish between sample statistics that are likely to occur from those that are unlikely to occur.)*

Definition 9 (Critical or Rejection Region)**.** *The critical or rejection region is the range of values of the test statistic that indicates that there is a significant difference and that the null hypothesis should be rejected.*

Definition 10 (Non-Critical or Fail to Reject Region)**.** *The noncritical region (sometimes called the "fail to reject" region) is the range of values of the test statistic that indicates that the difference was probably due to chance and that the null hypothesis should not be rejected.*

Definition 11 (One-Tailed Test)**.** *A* ***one-tailed test*** *indicates that the null hypothesis should be rejected when the test value is in the critical region on one side of the mean. A one-tailed test is either a* ***right-tailed test*** *or* ***left-tailed test****, depending on the direction of the inequality of the alternative hypothesis.*

Definition 12 (Two-Tailed Test)**.** *In a two-tailed test, the null hypothesis should be rejected when the test value is in either of the two critical regions.*

Definition 13 (*P*–value)**.** *The P–value is the probability of getting a value of the test statistic that is at least as extreme as the one representing the given sample data, assuming that the null hypothesis is true. One often "rejects the null hypothesis" when the P–value is less than the predetermined significance level (*α*), indicating that the observed result would be highly unlikely under the null hypothesis.*