Test 4 Name: Key

No Calculators or Computing Devices allowed! Use Algebraic Notation AND Show All of Your Work.

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1. (6 points) Find the inverse of $C = \begin{bmatrix} 1 & -2 & -4 \\ 2 & -3 & -6 \\ -3 & 6 & 15 \end{bmatrix}$ if it exists.

1. ________

$$[C \mid I_3] = \left[\begin{array}{ccc|ccc} 1 & -2 & -4 & 1 & 0 & 0 \\ 2 & -3 & -6 & 0 & 1 & 0 \\ -3 & 6 & 15 & 0 & 0 & 1 \end{array}\right] \begin{array}{l} \\ -2R_1+R_2 \to R_2 \\ 3R_1+R_3 \to R_3 \end{array}$$

$$= \left[\begin{array}{ccc|ccc} 1 & -2 & -4 & 1 & 0 & 0 \\ 0 & 1 & 2 & -2 & 1 & 0 \\ 0 & 0 & 3 & 3 & 0 & 1 \end{array}\right] \begin{array}{l} \\ 2R_2+R_1 \to R_1 \\ \frac{1}{3}R_3 \to R_3 \end{array} = \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -3 & 2 & 0 \\ 0 & 1 & 2 & -2 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1/3 \end{array}\right]$$

Now $-2R_3+R_2 \to R_2$

$$= \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -3 & 2 & 0 \\ 0 & 1 & 0 & -4 & 1 & -2/3 \\ 0 & 0 & 1 & 1 & 0 & 1/3 \end{array}\right] = [I_3 \mid A^{-1}]$$

So, $A^{-1} = \begin{bmatrix} -3 & 2 & 0 \\ -4 & 1 & 2/3 \\ 1 & 0 & 1/3 \end{bmatrix}$

2. (a) (2 points) Write a matrix equation equivalent to the following system.

$$\begin{cases} 3x + 2y = 14 \\ x - 2y = 2 \end{cases}$$

$\begin{bmatrix} 3 & 2 \\ 1 & -2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 14 \\ 2 \end{bmatrix}$

(a) ________

(b) (4 points) Find the inverse of the coefficient matrix, and use it to solve the system.

$(x,y) = (-4,1)$

(b) ________

$A^{-1} = \frac{1}{-6-2}\begin{bmatrix} -2 & -2 \\ -1 & 3 \end{bmatrix} = -\frac{1}{8}\begin{bmatrix} -2 & -2 \\ -1 & 3 \end{bmatrix}$

and $\begin{bmatrix} x \\ y \end{bmatrix} = -\frac{1}{8}\begin{bmatrix} -2 & -2 \\ -1 & 3 \end{bmatrix}\begin{bmatrix} 14 \\ 2 \end{bmatrix} = \frac{-1}{8}\begin{bmatrix} -2(14) - 2(2) \\ -1(14) + 3(2) \end{bmatrix}$

$= \frac{-1}{8}\begin{bmatrix} -32 \\ -8 \end{bmatrix} = \begin{bmatrix} -32/_{-8} \\ -8/_{-8} \end{bmatrix} = \begin{bmatrix} 4 \\ 1 \end{bmatrix}$

3. (5 points) Solve $\left\{\begin{array}{l} 2x + y = 1 \\ 3x + 4y = 14 \end{array}\right\}$ using Cramer's Rule.

3. ________

$x = \frac{D_x}{D}$ and $y = \frac{D_y}{D}$ where $D = \begin{vmatrix} 2 & 1 \\ 3 & 4 \end{vmatrix} = 8-3 = 5$

And $D_x = \begin{vmatrix} 1 & 1 \\ 14 & 4 \end{vmatrix} = 4 - 14 = -10$ and

$D_y = \begin{vmatrix} 2 & 1 \\ 3 & 14 \end{vmatrix} = 28 - 3 = 25$. Thus, $x = \frac{-10}{5} = -2$

and $y = \frac{25}{5}$

4. Let $A = \begin{bmatrix} 1 & -5 \\ -3 & 7 \end{bmatrix}$, $B = \begin{bmatrix} -2 & -6 \\ 2 & 7 \\ 1 & 0 \end{bmatrix}$, $C = \begin{bmatrix} 1 & 3 & 1 \\ -2 & 7 & 2 \\ 0 & 2 & 4 \end{bmatrix}$

Carry out the indicated operation, or explain, using complete sentences, why it cannot be performed.

(a) (2 points) $A + B$ A and B don't have the same dimension so addn is not defined for A+B.

(b) (2 points) AB

A B

2×2 3×2

Can't be done the number of columns in A don't equal the number of rows in B.

(c) (2 points) $BA - 3A$

$$BA-3A = \begin{bmatrix} -2 & -6 \\ 2 & 7 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 1 & -5 \\ -3 & 7 \end{bmatrix} - 3\begin{bmatrix} 1 & -5 \\ -3 & 7 \end{bmatrix} = \begin{bmatrix} -2+18 & 10-42 \\ 2-21 & -10+49 \\ 1+0 & -5+0 \end{bmatrix} - 3\begin{bmatrix} 1 & -5 \\ -3 & 7 \end{bmatrix}$$

3×2 2×2

Subtraction is not defined; can't be done!

(d) (2 points) B^{-1} Inverses are only for square matrices. B doesn't have an inverse

(e) (2 points) $\det(B)$ Det(B) doesn't exist.

Determinants are for square matrices

5. (6 points) Find the partial fraction decomposition of $\dfrac{7x-2}{x^2-4}$.

5. ____________

$$\frac{7x-2}{(x-2)(x+2)} = \frac{A}{x-2} + \frac{B}{x+2}$$

$$7x-2 = A(x+2) + B(x-2)$$

$$7x-2 = Ax + 2A + Bx - 2B$$

$$7x-2 = (A+B)x + (2A-2B)$$

$$\Rightarrow \left\{\begin{array}{l} 7 = A+B \\ -2 = 2A-2B \end{array}\right\} = \left\{\begin{array}{l} A+B = 7 \\ A-B = -1 \end{array}\right\}$$

$$= \left\{\begin{array}{l} A+B = 7 \\ 2A = 6 \end{array}\right\} = \left\{\begin{array}{l} B = 4 \\ A = 3 \end{array}\right\}$$

Check $\dfrac{3}{x-2} + \dfrac{4}{x+2} = \dfrac{3(x+2)+4(x-2)}{x^2-4} = \dfrac{3x+6+4x-8}{x^2-4}$ ✓

So $\boxed{\dfrac{7x-2}{x^2-4} = \dfrac{3}{x-2} + \dfrac{4}{x+2}}$

6. Only one of the following two matrices has an inverse.

$$\begin{bmatrix} + & - & + \\ - & + & - \\ + & - & + \end{bmatrix} \qquad A = \begin{bmatrix} 2 & 3 & -1 \\ 0 & 2 & 4 \\ -2 & 5 & 6 \end{bmatrix}, \qquad B = \begin{bmatrix} 1 & 3 & 7 \\ 2 & 0 & 8 \\ 0 & 2 & 2 \end{bmatrix}$$

$$\boxed{\begin{array}{l} \det(A) = -44 \\ \det(B) = 0 \end{array}}$$

(a) (5 points) Find the determinant of each matrix. (a) $\det(B) = 0$

$$\det(A) = -0 + 2\begin{vmatrix} 2 & -1 \\ -2 & 6 \end{vmatrix} - 4\begin{vmatrix} 2 & 3 \\ -2 & 5 \end{vmatrix}$$

$$= 2(12-2)\ 4(10+6) = 2(10) - 4(16)$$

$$= 20 - 64 = -44$$

$$\det(B) = -2\begin{vmatrix} 3 & 7 \\ 2 & 2 \end{vmatrix} + 0 - 8\begin{vmatrix} 1 & 3 \\ 0 & 2 \end{vmatrix}$$

$$= -2(6-14) - 8(2)$$

$$= -2(-8) - 16$$

$$= 16 - 16 = 0$$

(b) (1 point) Use the determinants from part (a) to identify which matrix has an inverse.

A has an Inverse

(b) ________

B doesn't have an inverse.

7. Let $A = \begin{bmatrix} 2 & -5 \\ -6 & 2 \\ 2 & -8 \end{bmatrix}$, $B = \begin{bmatrix} -1 & 3 \\ 3 & -4 \\ 1 & 0 \end{bmatrix}$, $C = \begin{bmatrix} 3 & 0 & 1 \\ -2 & 4 & 6 \\ 2 & 2 & 5 \end{bmatrix}$

Carry out the indicated operation, or explain, using complete sentences, why it cannot be performed.

$3 \times 3,\ 3 \times 2$

(a) (4 points) CA

$$= \begin{bmatrix} 3 & 0 & 1 \\ -2 & 4 & 6 \\ 2 & 2 & 5 \end{bmatrix} \begin{bmatrix} 2 & -5 \\ -6 & 2 \\ 2 & -8 \end{bmatrix} = \begin{bmatrix} d_{11} & d_{12} \\ d_{21} & d_{22} \\ d_{31} & d_{32} \end{bmatrix}$$

$d_{11} = 6 + 0 + 2 = 8$

$d_{12} = -15 + 0 - 8 = -23$

$d_{21} = -4 - 24 + 12 = -16$

$d_{22} = 10 + 8 - 48 = -30$

$d_{31} = 4 - 12 + 10 = 2$

$d_{32} = -10 + 4 + -40 = -46$

$$= \begin{bmatrix} 8 & -23 \\ -16 & -30 \\ 2 & -46 \end{bmatrix}$$

(b) (4 points) $2B - 3A$

$$2\begin{bmatrix} -1 & 3 \\ 3 & -4 \\ 1 & 0 \end{bmatrix} - 3\begin{bmatrix} 2 & -5 \\ -6 & 2 \\ 2 & -8 \end{bmatrix} = \begin{bmatrix} -2 & 6 \\ 6 & -8 \\ 2 & 0 \end{bmatrix} + \begin{bmatrix} -6 & 15 \\ 18 & -6 \\ -6 & 24 \end{bmatrix}$$

$$= \begin{bmatrix} -8 & 21 \\ 24 & -14 \\ -4 & 24 \end{bmatrix}$$

8. (6 points) Find the complete solution of the system, or show that no solution exists.

$$\begin{cases} x - y + 5z = -2 \\ 2x + y + 4z = 2 \\ 2x + 4y - 2z = 8 \end{cases}$$

8. ____________

$$= \left[\begin{array}{ccc|c} 1 & -1 & 5 & -2 \\ 2 & 1 & 4 & 2 \\ 2 & 4 & -2 & 8 \end{array}\right] \begin{array}{l} -2R_1 + R_2 \to R_2 \\ -2R_1 + R_3 \to R_3 \end{array}$$

$$= \left[\begin{array}{ccc|c} 1 & -1 & 5 & -2 \\ 0 & 3 & -6 & 6 \\ 0 & 6 & -12 & 12 \end{array}\right] \begin{array}{l} \frac{1}{3}R_2 \to R_2 \\ \frac{1}{6}R_3 \to R_3 \end{array}$$

$$= \left[\begin{array}{ccc|c} 1 & -1 & 5 & -2 \\ 0 & 1 & -2 & 2 \\ 0 & 1 & -2 & 2 \end{array}\right] -R_2 + R_3 \to R_3 \quad = \left[\begin{array}{ccc|c} 1 & -1 & 5 & -2 \\ 0 & 1 & -2 & 2 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

zero row $\Rightarrow$ inf. number of solns.

$$= \begin{Bmatrix} x - y + 5z = -2 \\ y - 2z = 2 \end{Bmatrix} = \begin{Bmatrix} x = -2 + y - 5z \\ y = 2 + 2z \end{Bmatrix}$$

$$= \begin{Bmatrix} x = -2 + (2+2z) - 5z \\ y = 2 + 2z \end{Bmatrix} = \begin{Bmatrix} x = -3z \\ y = 2 + 2z \end{Bmatrix}$$

$\boxed{\underline{\text{Soln set}} \ \{ (x,y,z) = (-3z,\ 2+2z,\ z) \mid z \text{ is any real number} \}}$

9. (6 points) Sketch the graph (and label the vertices, or boundary intersections) of the solution set of ordered pairs of the system.

$$\begin{cases} x \geq 0 \\ y \geq 0 \\ x \leq 5 \\ x + y \leq 7 \end{cases}$$