Key

70 total pts

Key

College Algebra - Test 1 **Name:** Key

1. (6 points) Suppose $g(x) = \begin{cases} -3x & \text{if } x < 0 \\ \sqrt{16 - x^2} & \text{if } 0 \le x < 4 \\ (x-4)^2 & \text{if } x \ge 4 \end{cases}$.

Evaluate the piecewise defined function at the values indicated below.

(a) $g(-1) = -3(-1) = 3$ (a) 3

(b) $g(-3) = -3(-3) = 9$ (b) 9

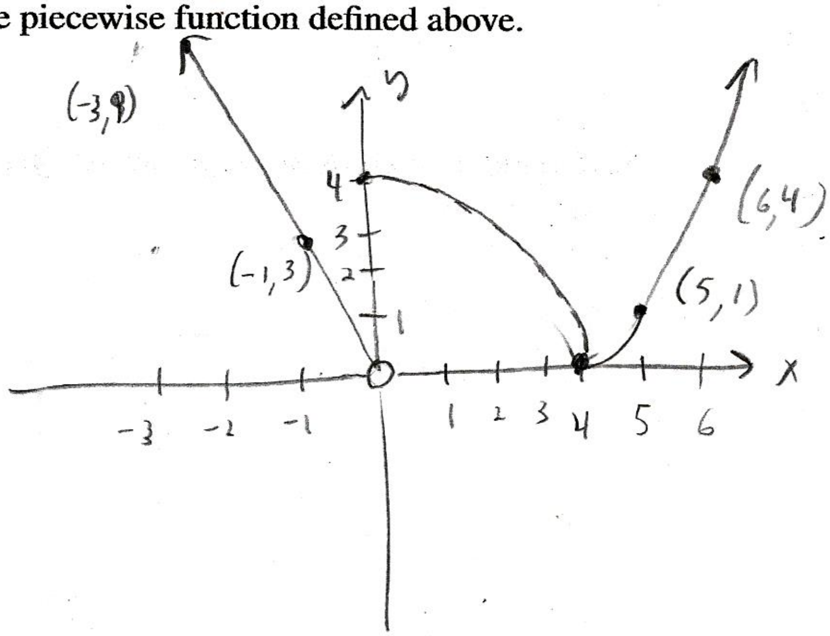
(c) $g(0) = 4$ (c) 4

(d) $g(4) = 0$ (d) 0

(e) $g(6) = (6-4)^2 = 2^2 = 4$ (e) 4

(f) $g(8) = (8-4)^2 = 4^2 = 16$ (f) 16

2. (4 points) Sketch the graph of the piecewise function defined above.

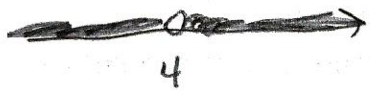


Key

3. (5 points) Write the domain of $f(x) = \dfrac{1}{4-x}$ using interval notation.

$4 - x = 0$ when $x = 4$

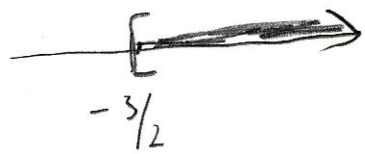
3. $(-\infty, 4) \cup (4, \infty)$



4. (5 points) Write the domain of $f(x) = \sqrt{2x+3}$ using interval notation.

4. $[-3/2, \infty)$

$2x + 3 \geq 0$



$2x \geq -3$

$x \geq -3/2$

5. (5 points) Find f/g and its domain. $f(x) = \sqrt{25 - x^2}$ and $g(x) = \sqrt{2 + x}$

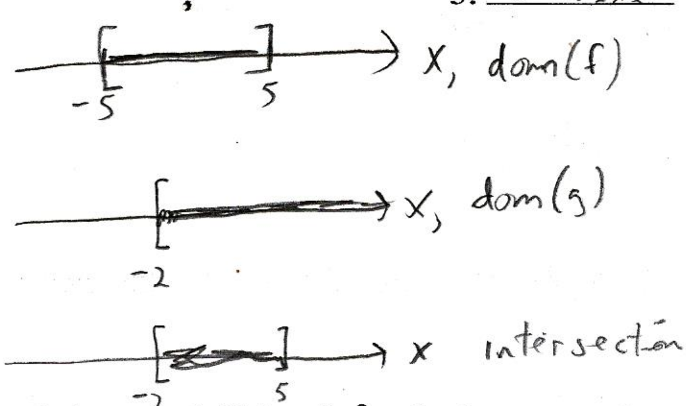
5. $f/g = \dfrac{\sqrt{25-x^2}}{\sqrt{2+x}}$

$$\boxed{\begin{aligned} (f/g)(x) &= \frac{\sqrt{25-x^2}}{\sqrt{2+x}} \\ \mathrm{dom}(f/g) &= (-2, 5] \end{aligned}}$$

$2 + x \geq 0$

$x \geq -2$

but $x = -2$ gives division by zero



6. (5 points) Find the average rate of change of $f(x) = 2x^2 - 3x$ from $x_1 = 2$ to $x_2 = 3$

$$\frac{f(x_2) - f(x_1)}{x_2 - x_1} = \frac{9 - 2}{3 - 2} = \frac{7}{1}$$

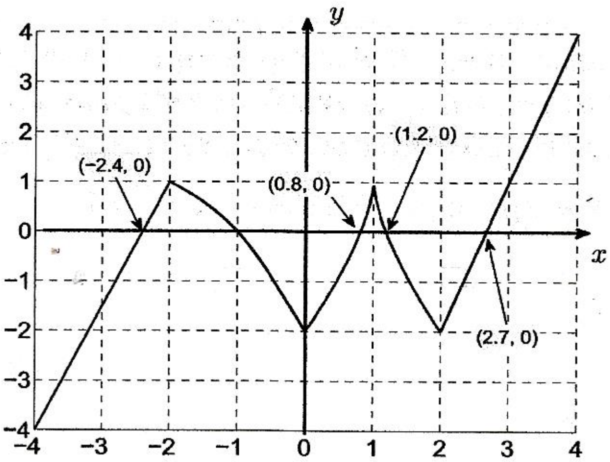
6. 7

$f(3) = 2 \cdot 9 - 3(3)$
$= 18 - 9 = 9$

$f(2) = 2 \cdot 4 - 3(2)$
$= 8 - 6 = 2$

Key

7. (12 points) The graph of a function f is given. **Assume the entire graph of f is shown in the figure.**



(a) Find all ***local*** and absolute maximum and minimum values of the function and the value of x at which each occurs.

abs max $(4,4)$ abs min $(-4,-4)$

local max $(-2,1)$, $(1,1)$

local min $(0,-2)$, $(2,-2)$

(b) State the x intervals for which $f(x) > 0$.

$(-2.4,-1) \cup (0.8,1.2) \cup (2.7,4]$

(c) State the x intervals for which $f(x) < 0$.

$[-4,-2.4) \cup (-1,0.8) \cup (1.2,2.7)$

(d) Find the x intervals on which the function is ***increasing***.

$[-4,-2) \cup (0,1) \cup (2,4]$

(e) Find the x intervals on which the function is ***decreasing***.

$(-2,0) \cup (1,2)$

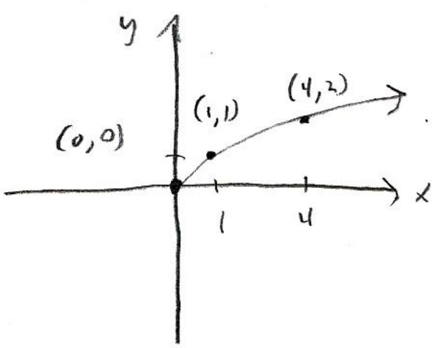
(f) Find $f(4)$. (f) 4

(g) Find $f(-1)$. (g) 0

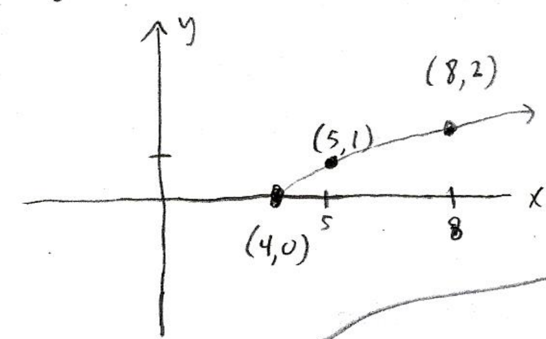
Directions: Sketch the graph of the function, not by plotting points, but by starting with the graph of a standard function and applying transformations. Label at least 3 points on your final graph.

8. (5 points) $h(x) = -3\sqrt{x-4} + 1$

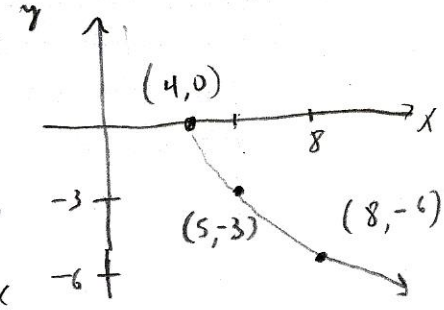
$f(x) = \sqrt{x}$



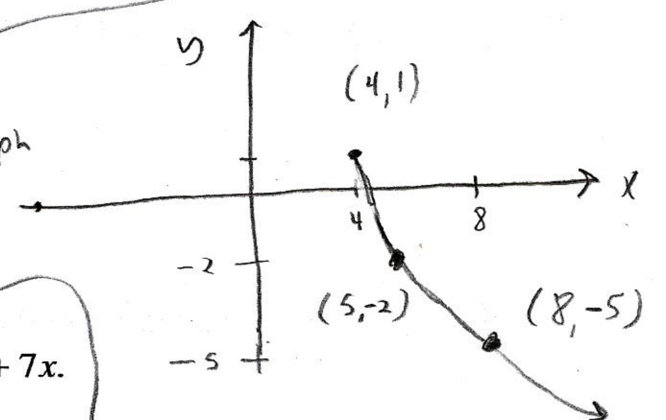
$y = \sqrt{x-4}$



$y = -3\sqrt{x-4}$



Final graph



Find $f \circ g$ and its domain.

9. (5 points) $f(x) = \dfrac{2}{1-x}$ and $g(x) = 2 + 7x$.

$$f(g(x)) = f(2+7x) = \frac{2}{1-(2+7x)} = \frac{2}{1-2-7x} = \frac{2}{-1-7x}$$

and $-1-7x=0$ when $-1 = 7x$ or when $x = -1/7$

$$\boxed{f \circ g = \frac{2}{-1-7x} \qquad \text{dom}(f \circ g) = \{x \mid x \neq -1/7\} \text{ or } (-\infty, -1/7) \cup (-1/7, \infty)}$$

10. (5 points) Find the inverse function of $f(x) = \dfrac{2x}{x+3}$

$\boxed{f^{-1}(x) = \dfrac{-3x}{x-2}}$

10. ________

$y = \dfrac{2x}{x+3}$

$x = \dfrac{2y}{y+3}$

$x(y+3) = 2y$

$xy + 3x = 2y$

$-2y \quad -2y$

$xy - 2y + 3x = 0$

$-3x \quad -3x$

$xy - 2y = -3x$

$xy - 2y = -3x$

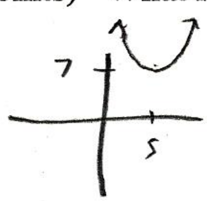
$(x-2)y = -3x$

$y = \dfrac{-3x}{x-2}$

11. (3 points) Find the vertex of $g(x) = -3(x+4)^2 - 7$. Does f open up or down?

11. down

12. (3 points) What is the range of $g(x) = 3(x-5)^2 + 7$?



12. $[7, \infty)$

Express the quadratic function in standard (vertex) form.

13. (5 points) $g(x) = 2x^2 + 4x - 7$

$\boxed{g(x) = 2(x+1)^2 - 9}$

13. ________

$= 2(x^2 + 2x) - 7$

$= 2(x^2 + 2x + \_\_) - 7 - \_\_$

$= 2(x^2 + 2x + \underline{1}) - 7 - \underline{2}$

$= 2(x+1)^2 - 9$

44 pts total

Math 110 – Exam 2 Name: Key

Directions: You may not use a calculator. The use of any other electronic devices are strictly prohibited. Show your work on ALL of the questions. Scratch paper is not allowed. You will not be allowed to leave to use the restroom.

Use $f(x) = x^4 + 10x^3 + 35x^2 + 50x + 24$ for questions 1 through 7.

1. (5 points) Find all the zeros of $f(x)$. What is the multiplicity of each root?

1. ________

$\frac{p}{q} \in \{\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 8, \pm 12, \pm 24\}$

$$\begin{array}{r|rrrrr} -1 & 1 & 10 & 35 & 50 & 24 \\ & \downarrow & -1 & -9 & -26 & -24 \\ \hline & 1 & 9 & 26 & 24 & 0 \end{array}$$

$\Rightarrow f(x) = (x+1)(x^3+9x^2+26x+24)$

$$\begin{array}{r|rrrr} -2 & 1 & 9 & 26 & 24 \\ & \downarrow & -2 & -14 & -24 \\ \hline & 1 & 7 & 12 & 0 \end{array}$$

$\Rightarrow f(x) = (x+1)(x+2)(x^2+7x+12)$

$\Rightarrow \boxed{f(x) = (x+1)(x+2)(x+3)(x+4)}$

roots

$x \in \{-1, -2, -3, -4\}$

all have multiplicity equal to 1

Key

2. (2 points) Write the complete factorization of $f(x)$ here.

2. ______

$$f(x) = (x+1)(x+2)(x+3)(x+4)$$

3. (2 points) What is the domain of $f(x)$?

3. $\boxed{(-\infty, \infty)}$

4. (2 points) Find the y-intercept of $f(x)$

$f(0) = 24$

4. $\boxed{(0, 24)}$

5. (2 points) Write an end behavior description for $f(x)$

$$\left\{\begin{array}{l} n = 4 \text{ (even)} \\ \text{and } a_n = 1 > 0 \end{array}\right\} \Rightarrow \{\uparrow \ \ \uparrow\}$$

5. $y \to \infty$ as $x \to \infty$ and $y \to \infty$ as $x \to -\infty$

6. (2 points) Find the solution set to $f(x) > 0$

$+ \quad -4 \quad - \quad -3 \quad + \quad -2 \quad - \quad -1 \quad +$

6. $\boxed{(-\infty, -4) \cup (-3, -2) \cup (-1, \infty)}$

7. (2 points) Find the solution set to $f(x) < 0$

7. $\boxed{(-4, -3) \cup (-2, -1)}$

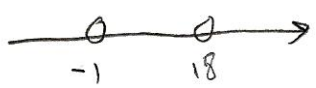
Key

For questions 8 through 14, use $f(x) = \dfrac{x-2}{x^2-17x-18} = \dfrac{x-2}{(x-18)(x+1)}$

8. (2 points) Find the vertical asymptote(s) of $f(x)$ 8. ________

$$\boxed{\begin{array}{c} x=-1 \\ x=18 \end{array}}$$

9. (2 points) Find the domain of $f(x)$ 9. ________



$\boxed{(-\infty,-1)\cup(-1,18)\cup(18,\infty)}$

10. (2 points) Find the x-intercept(s) of $f(x)$ $\boxed{10.\ (2,0)}$

$x-2=0 \Rightarrow x=2$

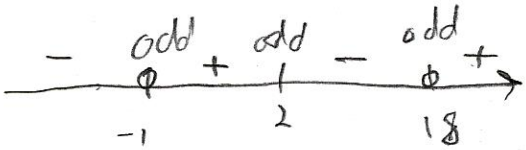
11. (2 points) Find the y-intercept of $f(x)$ 11. $\boxed{(0,\ 1/9)}$

$f(0) = \dfrac{0-2}{0-17\cdot 0-18} = \dfrac{-2}{-18} = 1/9$

12. (2 points) Find the horizontal asymptote of $f(x)$ 12. $\boxed{y=0}$

$[n<d] \Rightarrow [y=0$ is the HA$]$

13. (2 points) Find all x values for which $f(x) > 0$ 13. ________



$\boxed{(-1,2)\cup(18,\infty)}$

14. (2 points) Describe the behavior of the graph of f around its vertical asymptote(s). 14. ________

VA $x=-1$

$y \to -\infty$ as $x \to -1^-$

and $y \to \infty$ as $x \to -1^+$

VA: $x=18$

$y \to -\infty$ as $x \to 18^-$

and $y \to \infty$ as $x \to 18^+$

15. (4 points) Find the quotient and the remainder for $\dfrac{x^5 - 2x^3 + 2x + 1}{x^2 + 1}$

$$
\begin{array}{r l}
 & x^3 \qquad\qquad -3x \\
x^2+1 \Big) & x^5 + 0x^4 - 2x^3 + 0x^2 + 2x + 1 \\
 & -(x^5 \qquad\quad + x^3) \\
 & \qquad -3x^3 + 0x^2 + 2x + 1 \\
 & \qquad -(-3x^3 \qquad - 3x) \\
 & \qquad\qquad\qquad 5x + 1
\end{array}
$$

15. ________

quotient $x^3 - 3x$

remainder $5x+1$

16. (4 points) Find a polynomial with integer coefficients that satisfies the given conditions. The polynomial is degree 3 and has a zeros at $x = 1, -2$, and that -2 is a zero with multiplicity of 2. Write the polynomial in descending order (leaving your polynomial in factored form doesn't constitute a full credit answer).

$\boxed{f(x) = (x-1)(x+2)^2}$ ← initial setup

16. ________

$\boxed{f(x) = x^3 - 3x^2 - 4}$

$= (x-1)(x+2)(x+2)$

$= (x-1)\cdot(x^2+4x+4)$

$= x(x^2+4x+4) - 1(x^2+4x+4)$

$= x^3 + 4x^2 + \cancel{4x} - x^2 - \cancel{4x} - 4$

$= x^3 + 3x^2 - 4$

Find a mathematical model for the verbal statement.

17. (2 points) y varies directly as the cube of x and inversely as the square of s.

$$\boxed{y = \frac{kx^3}{s^2}}$$

17. ________

Find a mathematical model that represents the statement. Then determine the value of the constant of proportionality, k.

(3 pts)

18. P varies directly as x and inversely as the square of y. It is known from experimental results that ($P = \frac{28}{3}$ when $x = 42$ and $y = 9$.)

18. $\boxed{P = \frac{18x}{y^2}}$ final

$$\boxed{P = \frac{kx}{y^2}}$$

solve for k with ↑

$$\frac{28}{3} = \frac{k \cdot 42}{9^2}$$

28: 7, 4; 4: 2, 2

81: 9, 9; 9: 3, 3; 9: 3, 3

42: 2, 21; 21: 7, 3

$$\frac{28}{3} = \frac{42k}{81}.$$

$28 \cdot 81 = 3 \cdot 42\,k$ (now divide both sides by $3 \cdot 42$)

$$k = \frac{28 \cdot 81}{3 \cdot 42} = \frac{\cancel{7} \cdot \cancel{2} \cdot 2 \cdot \cancel{3} \cdot \cancel{3} \cdot 3 \cdot 3}{\cancel{3} \cdot \cancel{2} \cdot \cancel{7} \cdot \cancel{3}} = \boxed{18}$$

44 total pts

Math 110 **Test 3** **Name:** Key

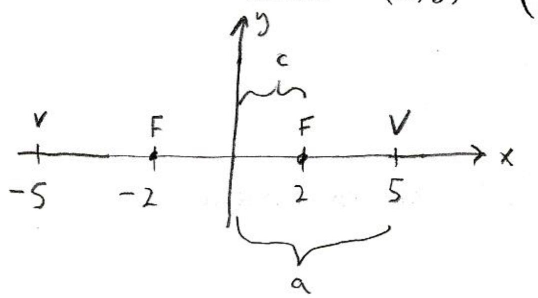
No Calculators or Computing Devices on this section. Once you turn this section in, you may NOT have it back! Use Algebraic Notation AND Show All of Your Work.

---

1. (5 points) **Find the standard form of the equation of the ellipse with the given characteristic(s) and center at the origin.**

   Foci: $(x, y) = \left(\pm 2, 0\right)$; major axis of length 10

1. $\boxed{\dfrac{x^2}{25} + \dfrac{y^2}{19} = 1}$



$$\frac{x^2}{25} + \frac{y^2}{b^2} = 1$$

Where $b^2 = a^2 - c^2$ and $c = 2$

$= 25 - 4$

$= 19 \implies \boxed{b = 19}$

2. (5 points) **Write the equation of a circle in standard form, and then find its center and radius.**

$$x^2 + y^2 - 16x - 4y + 59 = 0$$

$x^2 - 16x + \_\_\_ + y^2 - 4y + \_\_\_ = -59$

2. ________

$(x-8)^2 + (y-2)^2 = -59 + 64 + 4$

$\boxed{(x-8)^2 + (y-2)^2 = 9 \text{ ; center } (x,y) = (8,2) \text{, radius } 3}$

Key

3. (5 points) This is a ***Matching question*** associated with the theory on graphical translations of functions. Suppose $f(x) = 3^x$. Relative to the graph of $f(x)$ the graphs of the following functions have been changed in what way?

b	$g(x) = - \cdot 3^x$	a.) shifted 5 units right
d	$g(x) = 3^{(x+5)}$	b.) reflected about the x axis
c	$g(x) = 3^x + 5$	c.) shifted 5 units up
a	$g(x) = 3^{(x-5)}$	d.) shifted 5 units left
e	$g(x) = 3^x - 5$	e.) shifted 5 units vertically down

4. (4 points) **Use the One-to-One Property to solve the equation for x.**

$$2^{2x-3} = \frac{1}{4},$$

4. $\boxed{x = 1/2}$

$2^{2x-3} = 2^{-2}$

1-1 $\Rightarrow$ property $2x-3 = -2$

Then, $2x = -2+3$, or

$2x = 1$, or

$x = \frac{1}{2}$

5. (1 point) What number is $\log_3(1)$ equal to?

5. 0

6. (1 point) What number is $\log_5(25)$ equal to?

6. 2

7. (1 point) What number is $\ln(e)$ equal to?

7. 1

Key

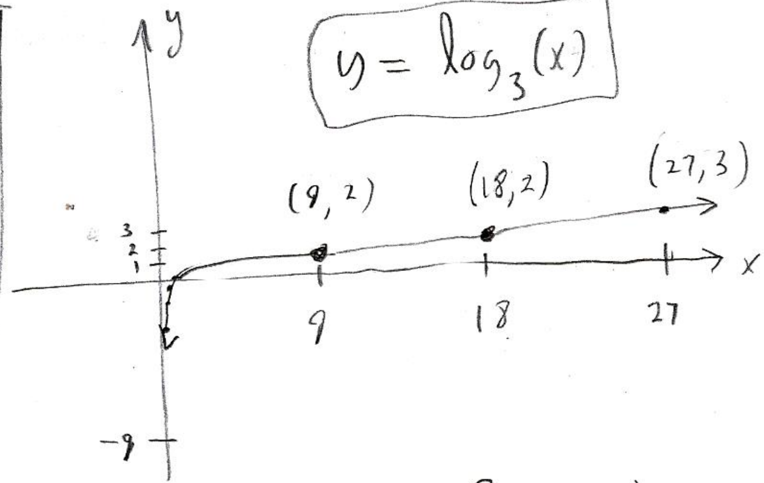
8. (4 points) Graph the function $f(x) = -\log_3(x+1)$

$y = 3^x$

x	y
-3	1/27
-2	1/9
-1	1/3
0	1
1	3
2	9
3	27

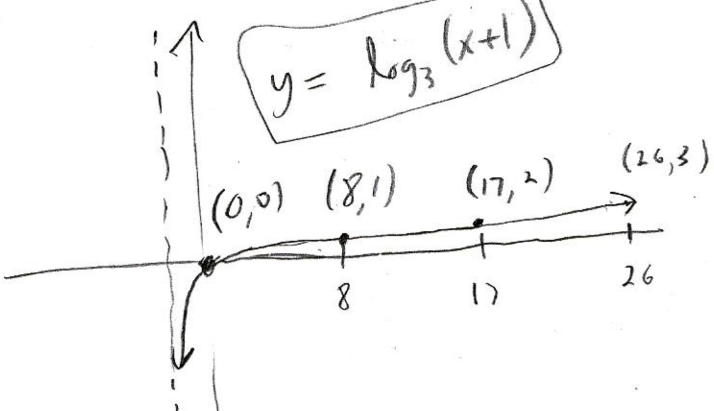
$y = \log_3(x)$

x	y
1/27	-3
1/9	-2
1/3	-1
1	0
3	1
9	2
27	3



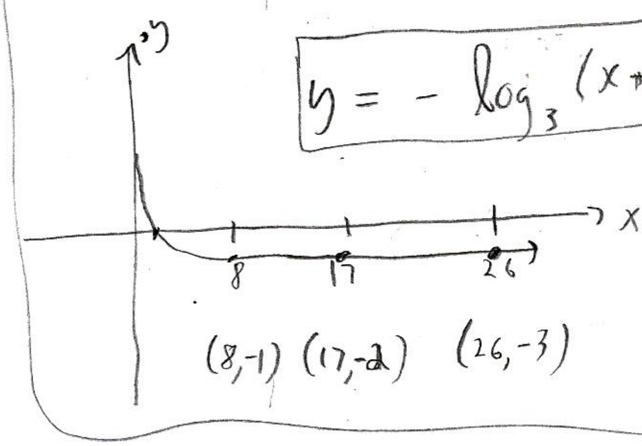
Final Graph

Apply H.S.



VA: x = -1

Apply reflection



9. (1 point) What is the domain of $f(x)$?

9. $(-1, \infty)$

10. (1 point) What equation represents the vertical asymptote of $f(x)$?

10. $x = -1$

Key

11. (4 points) Solve the equation.

$$\log_4(x-3) = 2$$

11. $\boxed{x=19}$

$$4^2 = x-3$$

$$x = 16+3$$

$$x = 19$$

12. (5 points) Solve the system $\begin{cases} 2x+3y=17 \\ 5x-y=17 \end{cases}$

12. $\boxed{(x,y)=(4,3)}$

$$\begin{Bmatrix} 2x = 17-3y \\ y = 5x-17 \end{Bmatrix}$$

$$= \begin{Bmatrix} 2x = 17-3(5x-17) \\ y = 5x-17 \end{Bmatrix}$$

$$= \begin{Bmatrix} 2x = 17-15x+51 \\ y = 5x-17 \end{Bmatrix}$$

$$= \begin{Bmatrix} 17x = 68 \\ y = 5x-17 \end{Bmatrix}$$

$$= \begin{Bmatrix} x = 68/17 \\ y = 5x-17 \end{Bmatrix}$$

$$= \begin{Bmatrix} x = 4 \\ y = 5 \cdot 4 - 17 \end{Bmatrix}$$

$$= \begin{Bmatrix} x = 4 \\ y = 3 \end{Bmatrix}$$

Key

Calculator Section **Name:** ____________________

Directions: **After you turn this in, please pick up the no-calculator section of the exam, which has 12 questions. You are allowed to take THIS paper back to work on or double check your work, AFTER you turn in the no-calculator section of the exam.**

13. (5 points) The number of bacteria in a culture is increasing according to the law of exponential growth. After 3 hours, there are 100 bacteria, and after 5 hours there are 600 bacteria. How many bacteria will there be after 8 hours

$y = ae^{bt}$

13. 8814

Given: $(t, y) = (3, 100)$ and $(t, y) = (5, 600)$

Unknown: a, b, and $y(8)$

$$\left\{ \begin{array}{l} 100 = ae^{3b} \\ 600 = ae^{5b} \end{array} \right\}$$

The givens along with the model eqn gives this non-linear system of eqns.

Divide the eqns $\frac{100}{600} = \frac{ae^{3b}}{ae^{5b}}$ (or use the subst method)

so, $\frac{1}{6} = e^{-2b}$, or $-2b = \ln(1/6)$, or $\boxed{b = -\frac{1}{2}\ln(1/6) \approx 0.8959}$

eqn 1 of non-linear system

Then, $100 = ae^{3b}$, or

$\frac{100}{e^{3b}} = a$, or $\boxed{a = 100e^{-3b} \doteq 100e^{-3(0.8959)} \approx 6.8}$

So, the model eqn is $\boxed{y(t) = 6.8e^{0.8959t}}$

and $\boxed{y(8) = 6.8e^{(0.8959)(8)} \approx 8814}$.