Math 110 – Quizzes 1 & 2
Professor Busken

Name: Key

Directions: You may NOT use a calculator. The use of any other electronic devices are strictly prohibited. Show your work on ALL of the questions. Do NOT work together. Tutor help NOT okay. Due Wednesday, November 13th at 10:15 am., with no exceptions. Late work will not be accepted.

Use $f(x) = x^4 - 5x^3 + 6x^2 + 4x - 8$ **for questions 1 through 8.**

1. (5 points) Find all the zeros of $f(x)$. What is the multiplicity of each root?

The divisors of the constant, -8, form the set $\{\pm 1, \pm 2, \pm 4, \pm 8\}$ from which we select numbers, p. The divisors of the leading coeff. form the set $\{\pm 1\}$ from which we select numbers, q. Then, $p/q \in \{\pm 1, \pm 2, \pm 4, \pm 8\}$ is the set of possible rational zeros.

1. ______

$$\begin{array}{r|rrrrr} -1 & 1 & -5 & 6 & 4 & -8 \\ & \downarrow & -1 & 6 & -12 & 8 \\ \hline & 1 & -6 & 12 & -8 & 0 \end{array}$$

$$\boxed{\begin{array}{lll} \text{Zeros} & & \\ x=-1 & \text{multiplicity} & 3 \\ x=2 & \text{multiplicity} & 1 \end{array}}$$

$\Rightarrow f(x) = (x+1)(x^3 - 6x^2 + 12x - 8)$

$$\begin{array}{r|rrrr} 2 & 1 & -6 & 12 & -8 \\ & \downarrow & 2 & -8 & 8 \\ \hline & 1 & -4 & 4 & 0 \end{array}$$

$\Rightarrow f(x) = (x-2)(x+1)(x^2-4x+4)$

$= (x-2)(x+1)(x-2)(x-2) = (x+1)(x-2)^3$

2. (3 points) Write the complete factorization of $f(x)$ here.

$$\boxed{f(x) = (x+1)(x-2)^3}$$

3. (2 points) What is the domain of $f(x)$?

3. $\boxed{(-\infty, \infty)}$

4. (2 points) Find the y-intercept of $f(x)$

$$f(0) = -8$$

4. $\boxed{(0,-8)}$

5. (4 points) Write an end behavior description for $f(x)$

$$\begin{bmatrix} n=4 \text{ (even)} \\ \text{and } a_n = 1 > 0 \end{bmatrix} \Rightarrow$$

5. $\boxed{y \to \infty \text{ as } x \to \pm\infty}$

6. (2 points) Find the solution set to $f(x) > 0$

+ odd − odd +

-1 (0), 2 (0), x

6. $\boxed{(-\infty,-1) \cup (2,\infty)}$

7. (2 points) Find the solution set to $f(x) < 0$

7. $\boxed{(-1,2)}$

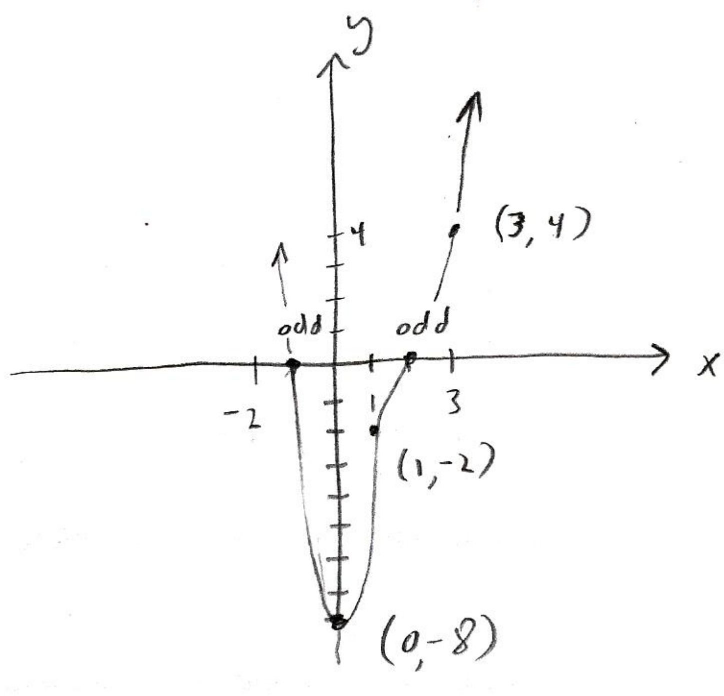
8. (4 points) Graph $f(x) = x^4 - 5x^3 + 6x^2 + 4x - 8$. Label some points and the axes for full credit.

$$\begin{array}{r|rrrrr} -2 & 1 & -5 & 6 & 4 & -8 \\ & \downarrow & -2 & 14 & -40 & 72 \\ \hline & 1 & -7 & 20 & -36 & 64 \end{array}$$

$$\begin{array}{r|rrrrr} 1 & 1 & -5 & 6 & 4 & -8 \\ & \downarrow & 1 & -4 & 2 & 6 \\ \hline & 1 & -4 & 2 & 6 & -2 \end{array}$$

$$\begin{array}{r|rrrrr} 3 & 1 & -5 & 6 & 4 & -8 \\ & \downarrow & 3 & -6 & 0 & 12 \\ \hline & 1 & -2 & 0 & 4 & 4 \end{array}$$

x	y
-2	64
1	-2
3	4



Express the quadratic function in standard (vertex) form.

9. (5 points) $g(x) = 2x^2 + 3x - 7$

$$\boxed{g(x) = 2\left(x + \tfrac{3}{4}\right)^2 - \tfrac{65}{8}}$$

9

$g(x) = (2x^2 + 3x) - 7$

$= 2\left(x^2 + \frac{3}{2}x\right) - 7$

$= 2\left(x^2 + \frac{3}{2}x + \underline{\frac{9}{16}}\right) - 7 - \underline{2 \cdot \frac{9}{16}}$

$= 2\left(x + \frac{3}{4}\right)^2 - \frac{65}{8}$

$-7 - 2 \cdot \frac{9}{16}$

$= -\frac{7}{1} - \frac{9}{8}$

$= \frac{-56}{8} - \frac{9}{8} = -\frac{65}{8}$

A quadratic function is given. Express the quadratic function in standard form. Find its vertex and its $x-$ and y- intercept(x). Sketch its graph. Then identify the vertex, axis of symmetry, domain and range.

10. (8 points) $C(x) = -x^2 - 4x + 4$

$C(x) = -(x^2 + 4x) + 4$

$= -(x^2 + 4x + \underline{4}) + 4 + \underline{4}$

$= -(x+2)^2 + 8$

$\Rightarrow$ vertex $(x, y) = (-2, 8)$,

axis of symmetry $x = -2$,

quadratic opens downward

range: $y \in (-\infty, 8]$

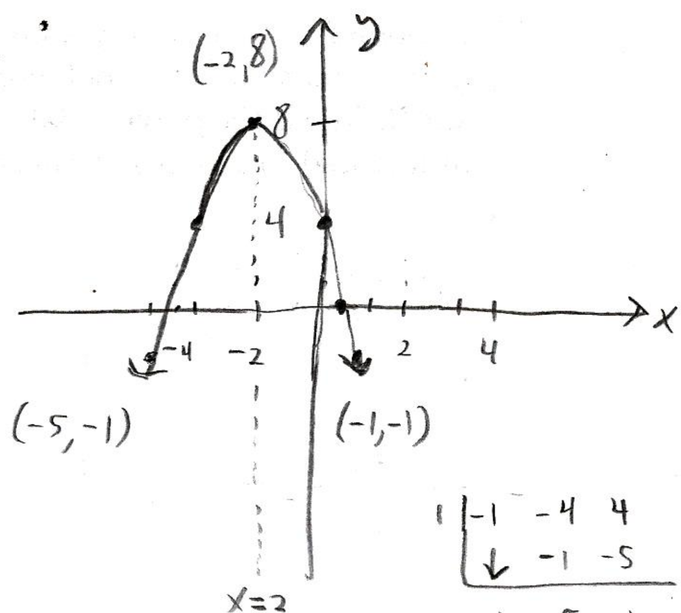
domain: $x \in (-\infty, \infty)$

y-int: $(x, y) = (0, 4)$

x-ints: solns to $0 = -(x+2)^2 + 8$

$(x+2)^2 = 8$

$x + 2 = \pm\sqrt{8} \Rightarrow \boxed{x = -2 \pm 2\sqrt{2}}$



$$\begin{array}{r|rrr} 1 & -1 & -4 & 4 \\ & \downarrow & -1 & -5 \\ \hline & -1 & -5 & -1 \end{array}$$

11. (5 points) Find the quotient and the remainder for $\dfrac{9x^3 - x + 5}{3x^2 - 7x}$

$$
\begin{array}{r|l}
 & 3x + 7 \\
\hline
3x^2 - 7x & 9x^3 + 0x^2 - x + 5 \\
 & -(9x^3 - 21x^2) \\
 & 21x^2 - x \\
 & -(21x^2 - 49x) \\
 & 48x + 5
\end{array}
$$

11. ________

quotient $3x+7$

remainder $48x+5$

$48x+5$ has a degree (1) less than the degree of the divisor, $3x^2-7x$, so we are finished with the long divisíon.

So, $\dfrac{9x^3-x+5}{3x^2-7x} = 3x-7 + \dfrac{48x+5}{3x^2-7x}$

12. (4 points) Find a polynomial with integer coefficients that satisfies the given conditions. The polynomial is degree 3 and has a zeros at $x = 2, 3,$ and 4. Write the polynomial in descending order (leaving your polynomial in factored form doesn't constitute a full credit answer).

12. ________

$f(x) = (x-2)(x-3)(x-4)$

$= (x-2)(x^2-4x-3x+12) = (x-2)(x^2-7x+12)$

$= x(x^2-7x+12) - 2(x^2-7x+12)$

$= x^3 - 7x^2 + 12x - 2x^2 + 14x - 24$

$$\boxed{f(x) = x^3 - 9x^2 + 26x - 24}$$

13. (3 points) Use the Factor Theorem to show that $x - 1$ is a factor of

$$f(x) = x^3 - 7x - 6.$$

$$\begin{array}{r|rrrr} 1 & 1 & 0 & -7 & -6 \\ & \downarrow & 1 & 1 & -6 \\ \hline & 1 & 1 & -6 & -12 \end{array}$$

$\Rightarrow f(1) = -12$

$\Rightarrow$ $(x-1)$ is not a factor of f

14. (5 points) Find a polynomial with integer coefficients that satisfies the given conditions. The polynomial is degree 4 and has a zero $1 + 2i$ with multiplicity 2. Write the polynomial in descending order (leaving your polynomial in factored form doesn't constitute a full credit answer).

14. $f(x) = x^4 - 4x^3 + 14x^2 - 20x + 25$

We know that $x = 1+2i$ is a zero of $f(x)$.

Hence, $x-1-2i$ is a factor of $f(x)$. (with multiplicity 2)

We know that complex roots of a polynomial come in conjugate pairs. Thus, we know $x = 1-2i$ is also a root of $f(x)$.

Since $x = 1-2i$ is a root of $f(x)$, $x-1+2i$ is a factor of $f(x)$

$$f(x) = (x-1+2i)^2(x-1-2i)^2 = \left[(x-1+2i)(x-1-2i)\right]^2$$

$$= \left[((x-1)+2i)((x-1)-2i)\right]^2 = \left[(x-1)^2 - 2i^2\right]^2 = (x^2-2x+1+4)^2$$

$$= (x^2-2x+5)^2 = (x^2-2x+5)(x^2-2x+5) = x^2(x^2-2x+5) - 2x(x^2-2x+5) + 5(x^2-2x+5)$$

$$= x^4 - 2x^3 + 5x^2 - 2x^3 + 4x^2 - 10x + 5x^2 - 10x + 25$$

$$= x^4 - 4x^3 + 14x^2 - 20x + 25$$