College Algebra - Review 1 **Name:** Key

line, Half circle, upwards opening parabola

1. Suppose $g(x) = \begin{cases} 1-3x & \text{if } x<0 \\ \sqrt{4-x^2} & \text{if } 0 \le x \le 2 \\ \dfrac{x^2}{4}-4 & \text{if } x>2 \end{cases}$. Evaluate the piecewise defined function at the values indicated below.

(a) $g(-9) = 1-3(-9) = 1+(-3)(-9) = 1+27$ (a) 28

(b) $g(0) = \sqrt{4-0^2} = \sqrt{4} = 2$ (b) 2

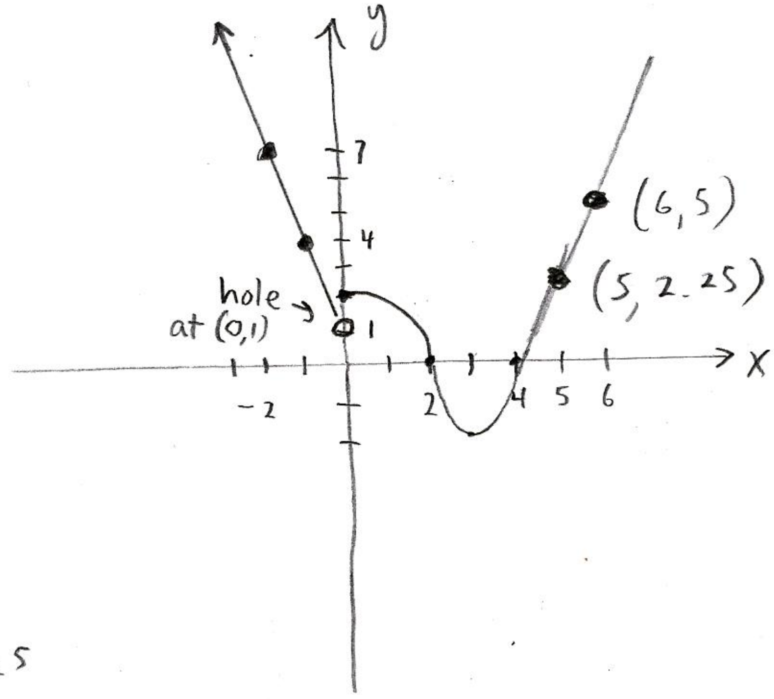
(c) $g(1) = \sqrt{4-1} = \sqrt{3}$ (c) $\sqrt{3}$

(d) $g(-4) = 1-3(-4) = 1+12 = 13$ (d) 13

(e) $g(4) = \dfrac{4^2}{4} - 4 = 4-4 = 0$ (e) 0

2. Sketch the graph of the piecewise function defined above.

x	y
-9	28
-2	7
-1	4
0	2
2	0
3	$\frac{9}{4} - 4 = 2\frac{1}{4} - 4 = -1\frac{3}{4}$
4	0
5	$\frac{25}{4} - 4 = 6.25 - 4 = 2.25$
6	$\frac{36}{4} - 4 = 9 - 4 = 5$



3. Write the domain of $f(x) = \dfrac{1}{x-5}$ using interval notation. 3. $\boxed{(-\infty, 5) \cup (5, \infty)}$

$x \neq 5$

4. Write the domain of $f(x) = \sqrt{x-5}$ using interval notation. 4. $\boxed{[5, \infty)}$

The quantity under the radical must be positive or zero

Set $x - 5 \geq 0$ and solve for x

$x - 5 + 5 \geq 0 + 5$

$x \geq 5$

5. Evaluate and simplify the expression $\dfrac{f(x+h) - f(x)}{h}$ for $f(x) = 2x^2 - 1$. Assume x, h and $x + h$ are real numbers in the domain of f and that $h \neq 0$. Hint: You know you are finished when you can replace h with zero and not get a division by zero.

$f(x+h) = 2(x+h)^2 - 1$ and 5. $\boxed{4x + 2h}$

$$\frac{f(x+h) - f(x)}{h} = \frac{\left[2(x+h)^2 - 1\right] - \left[2x^2 - 1\right]}{h} = \frac{2(x^2 + 2xh + h^2) - \cancel{1} - 2x^2 + \cancel{1}}{h}$$

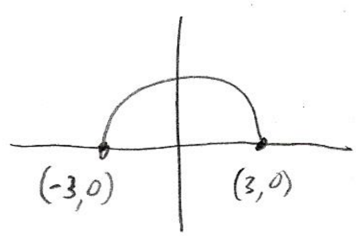
$$= \frac{\cancel{2x^2} + 4xh + 2h^2 - \cancel{2x^2}}{h} = \frac{4xh + 2h^2}{h} = \frac{h(4x + 2h)}{h} = 4x + 2h$$

6. Find the zeros of $g(x) = \sqrt{9 - x^2}$. ← half circle 6. $\boxed{(-3,0) \text{ and } (3,0)}$

Set $g(x) = 0$ and solve for x

$0 = \sqrt{9 - x^2}$

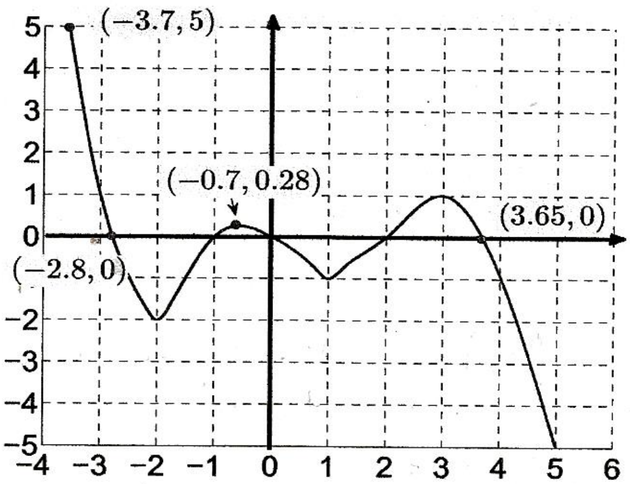
$0^2 = \left(\sqrt{9 - x^2}\right)^2$



$0 = 9 - x^2$

$x^2 = 9 \Rightarrow x = \pm 3$

7. (12 points) The graph of a function f is given in the figure (right). Assume the entire graph of the function is shown.



(a) Find all local and absolute maximum and minimum values of the function and the value of x at which each occurs.

Abs max: $y=5$ at $x=-3.7$

Abs min: $y=-5$ at $x=5$

Local max: 0.28 at $x=-0.7$ and $y=1$ at $x=3$

Local min: $y=-2$ at $x=-2$ and $y=-1$ at $x=1$

(b) State the x intervals for which $f(x) > 0$.

The graph of y-values is above the x-axis for all x values in the interval $[-3.7, -2.8) \cup (-1, 0) \cup (2, 3.65)$

(c) State the x intervals for which $f(x) < 0$.

x in the interval $(-2.8, -1) \cup (0, 2) \cup (3.65, 5]$

(d) Find the intervals on which the function is ***increasing***.

$y \uparrow$ for x in $(-2, -0.7) \cup (1, 3)$

(e) Find the intervals on which the function is ***decreasing***.

$y \downarrow$ for x in $[-3.7, -2) \cup (-0.7, 1) \cup (3, 5]$

(f) Find $f(4)$. (f) -1

(g) Find $f(5)$. (g) -5

8. Find the average rates of change Of $f(x) = x^3 - 3x$ from $x_1 = -2$ to $x_2 = -1$

8. 4

$$\frac{f(x_2) - f(x_1)}{x_2 - x_1} = \frac{f(-1) - f(-2)}{-1-(-2)}$$

$$= \frac{[(-1)^3 - 3(-1)] - [(-2)^3 - 3(-2)]}{-1+2} = \frac{[-1+3] - [-8+6]}{1} = 2-(-2)$$

9. Determine if $f(x) = -x+x^3$ is even, odd or neither. If the graph of the function has symmetry, state which kind it has.

9. $\boxed{\text{odd, origin Symmetry}}$

$$f(-x) = -(-x) + (-x)^3$$
$$= x + (-x)(-x)(-x)$$
$$= x - x^3$$
$$= -1(-x + x^3) = -1 \cdot f(x),$$

$f(-x) = -1 \cdot f(x) \Rightarrow f$ is odd

f is odd $\Rightarrow$ f's graph has origin Symmetry

10. What is the domain and range of $f(x) = \sqrt{x}$? Use interval notation to answer.

$\text{dom}(f) = [0, \infty)$

10. ________

$\text{rng}(f) = [0, \infty)$

Write an equation for the function described by the given characteristics.

11. The shape of $f(x) = \sqrt{x}$, but shifted five units right, reflected in the x axis and 12 units down.

11. $\boxed{y = 12 - \sqrt{x-5} \text{ or } y = -\sqrt{x-5} + 12}$

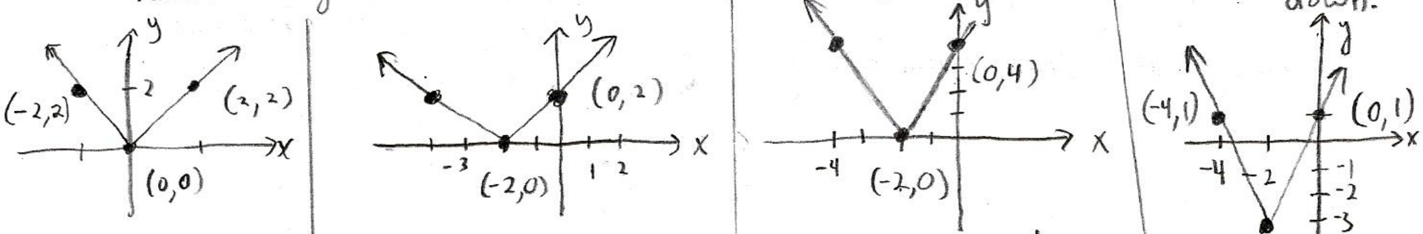
$y = \sqrt{x-5}$ horizontal shift

$y = -\sqrt{x-5}$ reflect ron

Directions: Sketch the graph of the function, not by plotting points, but by starting with the graph of a standard function and applying transformations. Label at least 3 points on your final graph.

12. $C(x) = 2|x + 2| - 3$

$y = 2|x+2| - 3$ is the function that translates $f(x) = |x|$ horizontally 2 units left, magnifies it by 2 and shifted 3 units down.



(-2,-3)

$f(x) = |x|$

parent function graph

Apply horizontal shift. Graph $y = |x+2|$. Subtract 2 from the x-coordinates of your key points

Graph $y = 2|x+2|$ Apply reflection/magnification. Multiply the y-coordinates of your key points by 2.

Shift key points down 3 units.

Find $f \circ g$ and $g \circ f$ and their domains.

13. $f(x) = \dfrac{1}{x}$ and $g(x) = 4x - 9$.

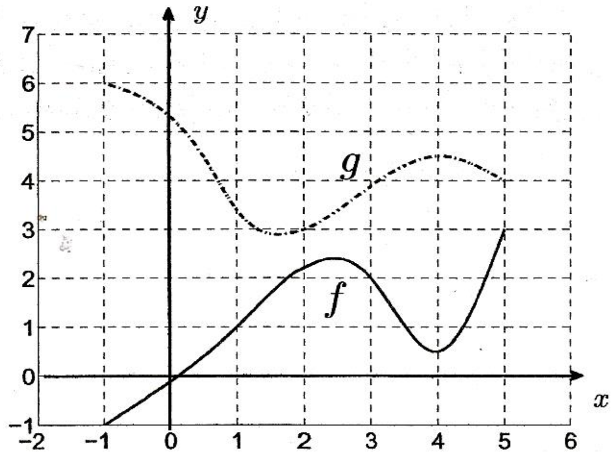
$f \circ g = f(g(x)) = f(4x-9) = \dfrac{1}{4x-9}$.

$\text{dom}(f \circ g) = \{x \mid x = \frac{9}{4}\}$ or $(-\infty, \frac{9}{4}) \cup (\frac{9}{4}, \infty)$

$g \circ f = g(f(x)) = g\left(\frac{1}{x}\right) = 4 \cdot \frac{1}{x} - 9 = \frac{4}{x} - 9$

$\text{dom}(g \circ f) = \{x \mid x \neq 0\}$ or $(-\infty, 0) \cup (0, \infty)$

14. Use the graph to find the indicated functional values.



(a) $(f+g)(3) = f(3) + g(3)$

$= 2 + 4 = 6$

(b) $(f-g)(-1) = f(-1) - g(-1)$

$= -1 - 6 = -7$

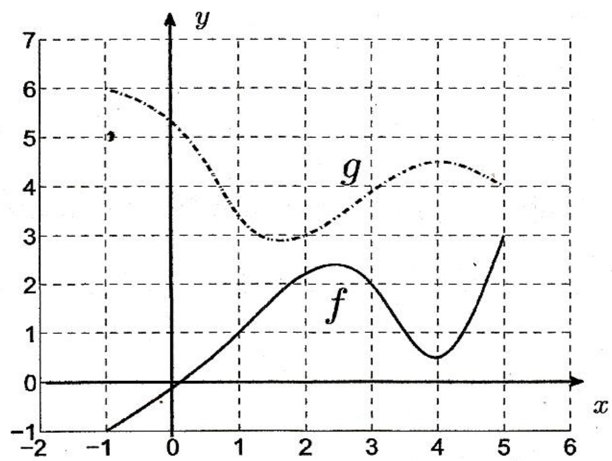
(c) $\dfrac{f}{g}(5) = \dfrac{f(5)}{g(5)} = \dfrac{3}{4}$

(d) Find the domain and range of f

$\text{dom}(f) = [-1, 5]$

$\text{rng}(f) = [-1, 3]$

15. Use the graph to find the indicated functional values.



(a) $f(g(3)) = f(4) = 1/2$

(b) $f(g(-1)) = f(6)$ not in the range of f, Therefore, $x = -1$ is not in the domain of $f \circ g$.

(c) $g(f(3))$

$= g(2) = 3$

(d) $g(g(3)) = g(4) = 4.5$

16. Find ~~and~~ f/g and ~~their~~ its domains. $f(x) = \sqrt{7-x}$ and $g(x) = \sqrt{7+x}$

$\boxed{y = \dfrac{f}{g} = \dfrac{\sqrt{7-x}}{\sqrt{7+x}}}$

$\boxed{\text{dom}\left(\frac{f}{g}\right) = (-7, 7]}$

dom(f) set $7 - x \geq 0$, solve for x and write the soln as an interval $7 \geq x$, $(-\infty, 7]$

7 x, dom(f)

dom(g) set $7 + x \geq 0$ and solve for x.

$x \geq -7$, or $[-7, \infty)$

−7 x, dom(g)

−7 7 x, dom$\left(\frac{f}{g}\right)$

dom(f/g): Find the intersection set. Remove from that interval value(s) of x that cause division by zero.