

College Algebra - Test 1**Name:** _____

1. (6 points) Suppose $g(x) = \begin{cases} -3x & \text{if } x < 0 \\ \sqrt{16 - x^2} & \text{if } 0 \leq x < 4 \\ (x - 4)^2 & \text{if } x \geq 4 \end{cases}$.

Evaluate the piecewise defined function at the values indicated below.

(a) $g(-1)$ (a) _____

(b) $g(-3)$ (b) _____

(c) $g(0)$ (c) _____

(d) $g(4)$ (d) _____

(e) $g(6)$ (e) _____

(f) $g(8)$ (f) _____

2. (4 points) Sketch the graph of the piecewise function defined above.

3. (5 points) Write the domain of $f(x) = \frac{1}{4-x}$ using interval notation.

3. _____

4. (5 points) Write the domain of $f(x) = \sqrt{2x+3}$ using interval notation.

4. _____

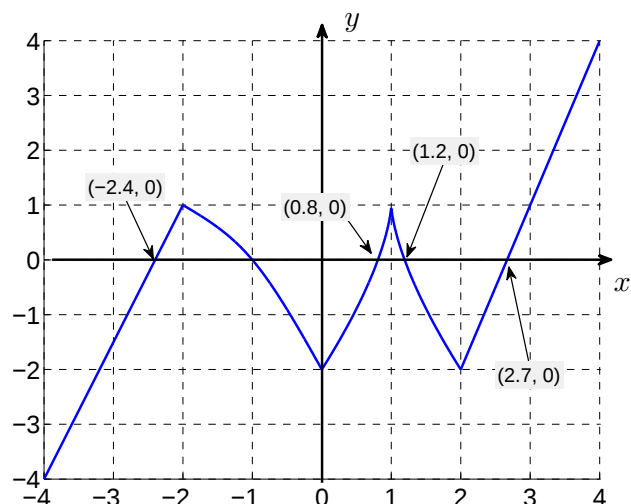
5. (5 points) Find f/g and its domain. $f(x) = \sqrt{25-x^2}$ and $g(x) = \sqrt{2+x}$

5. _____

6. (5 points) Find the average rate of change of $f(x) = 2x^2 - 3x$ from $x_1 = 2$ to $x_2 = 3$

6. _____

7. (12 points) The graph of a function f is given. Assume the entire graph of f is shown in the figure.



(a) Find all *local* and absolute maximum and minimum values of the function and the value of x at which each occurs.

(b) State the x intervals for which $f(x) > 0$.

(c) State the x intervals for which $f(x) < 0$.

(d) Find the x intervals on which the function is *increasing*.

(e) Find the x intervals on which the function is *decreasing*.

(f) Find $f(4)$.

(f) _____

(g) Find $f(-1)$.

(g) _____

Directions: Sketch the graph of the function, not by plotting points, but by starting with the graph of a standard function and applying transformations. Label at least 3 points on your final graph.

8. (5 points) $h(x) = -3\sqrt{x-4} + 1$

Find $f \circ g$ its domain.

9. (5 points) $f(x) = \frac{2}{1-x}$ and $g(x) = 2 + 7x$.

10. (5 points) Find the inverse function of $f(x) = \frac{2x}{x+3}$

10. _____

11. (3 points) Find the vertex of $g(x) = -3(x+4)^2 - 7$. Does f open up or down?

11. _____

12. (3 points) What is the range of $g(x) = 3(x-5)^2 + 7$?

12. _____

Express the quadratic function in standard (vertex) form.

13. (5 points) $g(x) = 2x^2 + 4x - 7$

13. _____

Directions: You may not use a calculator. The use of any other electronic devices are strictly prohibited. Show your work on ALL of the questions. Scratch paper is not allowed. You will not be allowed to leave to use the restroom.

Use $f(x) = x^4 + 10x^3 + 35x^2 + 50x + 24$ for questions 1 through 7.

1. (5 points) Find all the zeros of $f(x)$. What is the multiplicity of each root?

1. _____

2. (2 points) Write the complete factorization of $f(x)$ here.

2. _____

3. (2 points) What is the domain of $f(x)$?

3. _____

4. (2 points) Find the y -intercept of $f(x)$

4. _____

5. (2 points) Write an end behavior description for $f(x)$

5. _____

6. (2 points) Find the solution set to $f(x) > 0$

6. _____

7. (2 points) Find the solution set to $f(x) < 0$

7. _____

For questions 8 through 14, use $f(x) = \frac{x - 2}{x^2 - 17x - 18}$

8. (2 points) Find the vertical asymptote(s) of $f(x)$ 8. _____

9. (2 points) Find the domain of $f(x)$ 9. _____

10. (2 points) Find the x -intercept(s) of $f(x)$ 10. _____

11. (2 points) Find the y -intercept of $f(x)$ 11. _____

12. (2 points) Find the horizontal asymptote of $f(x)$ 12. _____

13. (2 points) Find all x values for which $f(x) > 0$ 13. _____

14. (2 points) Describe the behavior of the graph of f around its vertical asymptote(s).

14. _____

15. (4 points) Find the quotient and the remainder for $\frac{x^5 - 2x^3 + 2x + 1}{x^2 + 1}$

15. _____

16. (4 points) Find a polynomial with integer coefficients that satisfies the given conditions. The polynomial is degree 3 and has a zeros at $x = 1$, -2 , and that -2 is a zero with multiplicity of 2. Write the polynomial in descending order (leaving your polynomial in factored form doesn't constitute a full credit answer).

16. _____

Find a mathematical model for the verbal statement.

17. (2 points) y varies directly as the cube of x and inversely as the square of s .

17. _____

Find a mathematical model that represents the statement. Then determine the value of the constant of proportionality, k .

18. P varies directly as x and inversely as the square of y . It is known from experimental results that ($P = \frac{28}{3}$ when $x = 42$ and $y = 9$.)

18. _____

No Calculators or Computing Devices on this section. Once you turn this section in, you may NOT have it back! Use Algebraic Notation AND Show All of Your Work.

1. (5 points) Find the standard form of the equation of the ellipse with the given characteristic(s) and center at the origin.

Foci: $(x, y) = (\pm 2, 0)$; major axis of length 10

1. _____

2. (5 points) Write the equation of a circle in standard form, and then find its center and radius.

$$x^2 + y^2 - 16x - 4y + 59 = 0$$

2. _____

3. (5 points) This is a ***Matching question*** associated with the theory on graphical translations of functions. Suppose $f(x) = 3^x$. Relative to the graph of $f(x)$ the graphs of the following functions have been changed in what way?

_____ $g(x) = - \cdot 3^x$	a.) shifted 5 units right
_____ $g(x) = 3^{(x+5)}$	b.) reflected about the x axis
_____ $g(x) = 3^x + 5$	c.) shifted 5 units up
_____ $g(x) = 3^{(x-5)}$	d.) shifted 5 units left
_____ $g(x) = 3^x - 5$	e.) shifted 5 units vertically down

4. (4 points) Use the **One-to-One Property** to solve the equation for x .

$$2^{2x-3} = \frac{1}{4}$$

4. _____

5. (1 point) What number is $\log_3(1)$ equal to? 5. _____

6. (1 point) What number is $\log_5(25)$ equal to? 6. _____

7. (1 point) What number is $\ln(e)$ equal to? 7. _____

8. (4 points) Graph the function $f(x) = -\log_3(x + 1)$

9. (1 point) What is the domain of $f(x)$?

9. _____

10. (1 point) What equation represents the vertical asymptote of $f(x)$?

10. _____

11. (4 points) Solve the equation.

$$\log_4(x - 3) = 2$$

11. _____

12. (5 points) Solve the system $\begin{cases} 2x + 3y = 17 \\ 5x - y = 17 \end{cases}$

12. _____

Calculator Section

Name: _____

Directions: After you turn this in, please pick up the no-calculator section of the exam, which has 12 questions. You are allowed to take THIS paper back to work on or double check your work, **AFTER** you turn in the no-calculator section of the exam.

13. (5 points) The number of bacteria in a culture is increasing according to the law of exponential growth. After 3 hours, there are 100 bacteria, and after 5 hours there are 600 bacteria. How many bacteria will there be after 8 hours

13. _____

14. (5 points) Use DeMoivre's Theorem to find $(4 - 4i)^5$

14. _____

Test 4

Name: _____

No Calculators or Computing Devices allowed! Use Algebraic Notation AND Show All of Your Work.

1. (6 points) Find the inverse of $C = \begin{bmatrix} 1 & -2 & -4 \\ 2 & -3 & -6 \\ -3 & 6 & 15 \end{bmatrix}$ if it exists.

1. _____

2. (a) (2 points) Write a matrix equation equivalent to the following system.

$$\begin{cases} 3x + 2y = 14 \\ x - 2y = 2 \end{cases}$$

(a) _____

- (b) (4 points) Find the inverse of the coefficient matrix, and use it to solve the system.

(b) _____

3. (5 points) Solve $\begin{cases} 2x + y = 1 \\ 3x + 4y = 14 \end{cases}$ using Cramer's Rule.

3. _____

4. Let $A = \begin{bmatrix} 1 & -5 \\ -3 & 7 \end{bmatrix}$, $B = \begin{bmatrix} -2 & -6 \\ 2 & 7 \\ 1 & 0 \end{bmatrix}$, $C = \begin{bmatrix} 1 & 3 & 1 \\ -2 & 7 & 2 \\ 0 & 2 & 4 \end{bmatrix}$

Carry out the indicated operation, or explain, using complete sentences, why it cannot be performed.

(a) (2 points) $A + B$

(b) (2 points) AB

(c) (2 points) $BA - 3A$

(d) (2 points) B^{-1}

(e) (2 points) $\det(B)$

5. (6 points) Find the partial fraction decomposition of $\frac{7x - 2}{x^2 - 4}$.

5. _____

6. Only one of the following two matrices has an inverse.

$$A = \begin{bmatrix} 2 & 3 & -1 \\ 0 & 2 & 4 \\ -2 & 5 & 6 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 3 & 7 \\ 2 & 0 & 8 \\ 0 & 2 & 2 \end{bmatrix}$$

(a) (5 points) Find the determinant of each matrix. (a) _____

(b) (1 point) Use the determinants from part (a) to identify which matrix has an inverse.

(b) _____

7. Let $A = \begin{bmatrix} 2 & -5 \\ -6 & 2 \\ 2 & -8 \end{bmatrix}$, $B = \begin{bmatrix} -1 & 3 \\ 3 & -4 \\ 1 & 0 \end{bmatrix}$, $C = \begin{bmatrix} 3 & 0 & 1 \\ -2 & 4 & 6 \\ 2 & 2 & 5 \end{bmatrix}$

Carry out the indicated operation, or explain, using complete sentences, why it cannot be performed.

(a) (4 points) CA

(b) (4 points) $2B - 3A$

8. (6 points) Find the complete solution of the system, or show that no solution exists.

$$\begin{cases} x - y + 5z = -2 \\ 2x + y + 4z = 2 \\ 2x + 4y - 2z = 8 \end{cases}$$

8. _____

9. (6 points) Sketch the graph (and label the vertices, or boundary intersections) of the solution set of ordered pairs of the system.

$$\begin{cases} x \geq 0 \\ y \geq 0 \\ x \leq 5 \\ x + y \leq 7 \end{cases}$$