Paired Data and the Rectangular Coordinate System

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Paired Data and the Rectangular Coordinate System

Learning Objectives:

- Graph ordered pairs on a rectangular coordinate system.
- Graph linear equations by finding intercepts or by making a table.
- Graph horizontal and vertical lines.

Ordered Pairs

We now turn our attention to equations containing two variables, x and y. Paired data plays an important role in these type of equations.

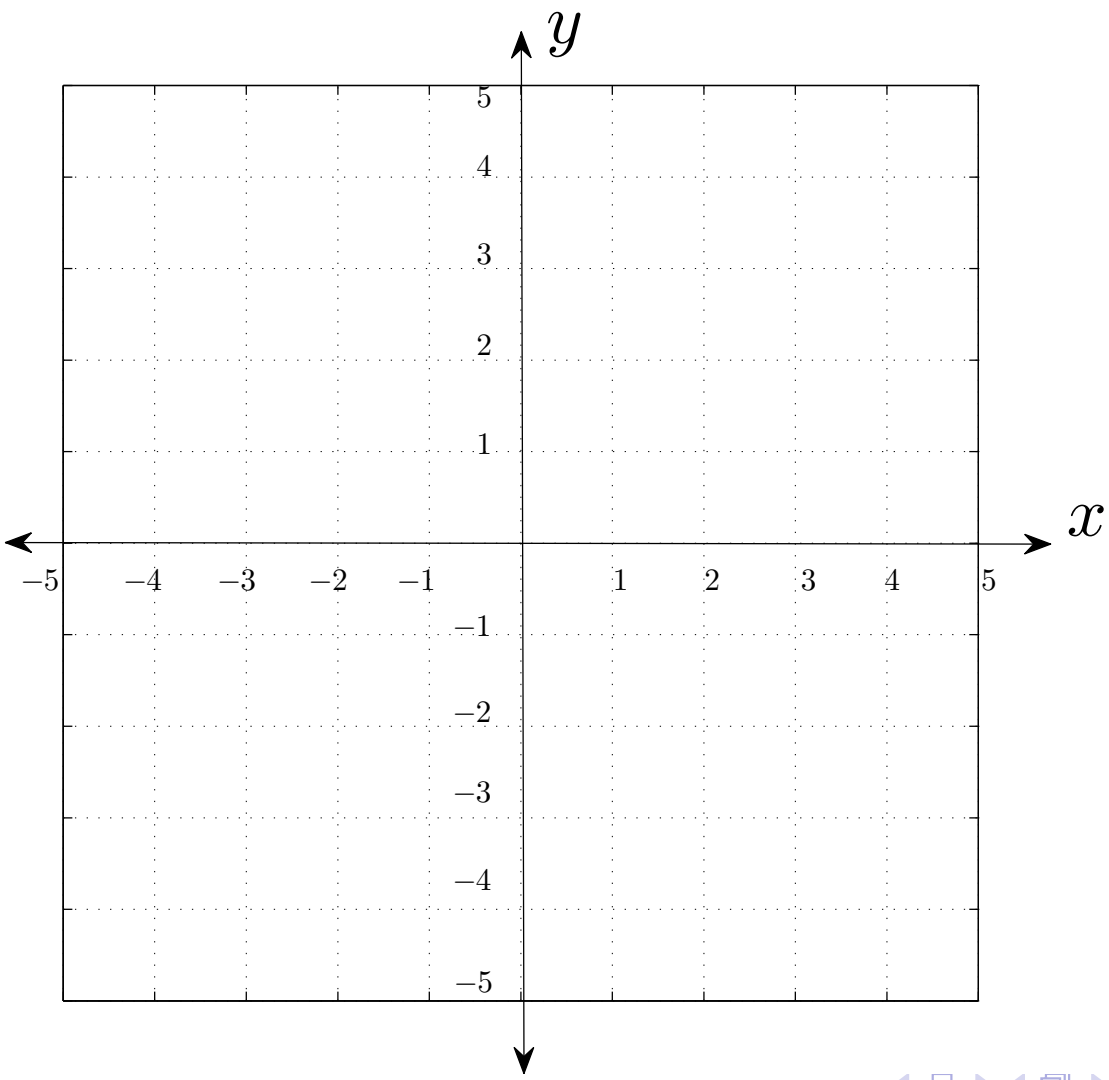
Ordered Pairs

Definition

A pair of numbers enclosed in parenthesis and separated by a comma, such as $(-2, 1)$, is called an ordered pair of numbers. The first number in the pair is called the x-coordinate of the ordered pair; the second number is called the y-coordinate. For the ordered pair $(-2, 1)$, the x-coordinate is -2 and the y-coordinate is 1.

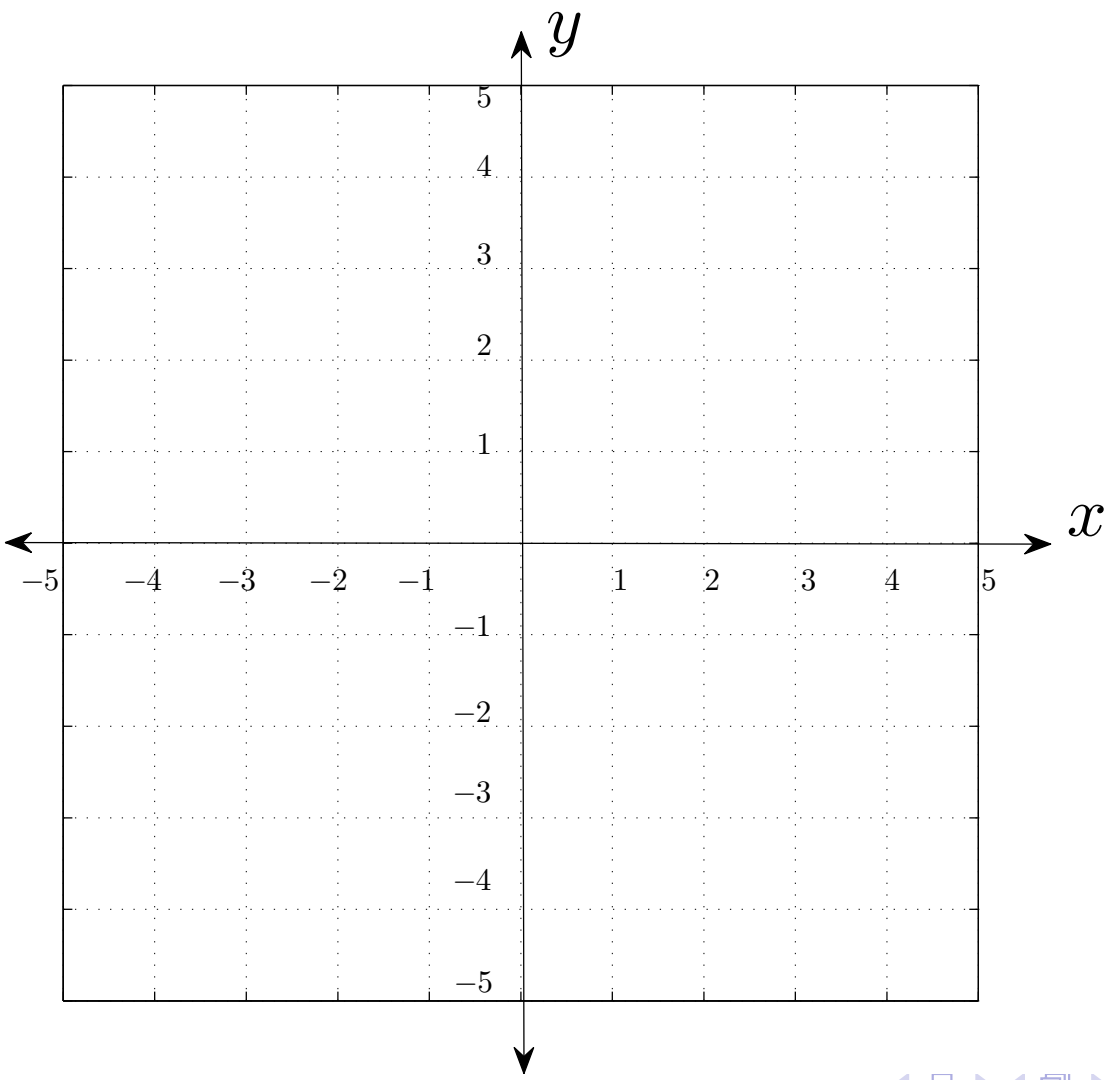
The Rectangular Coordinate System

We use a rectangular coordinate system to visualize ordered pairs.



The Rectangular Coordinate System

A rectangular coordinate system is made by drawing two real number lines at right angles to each other.



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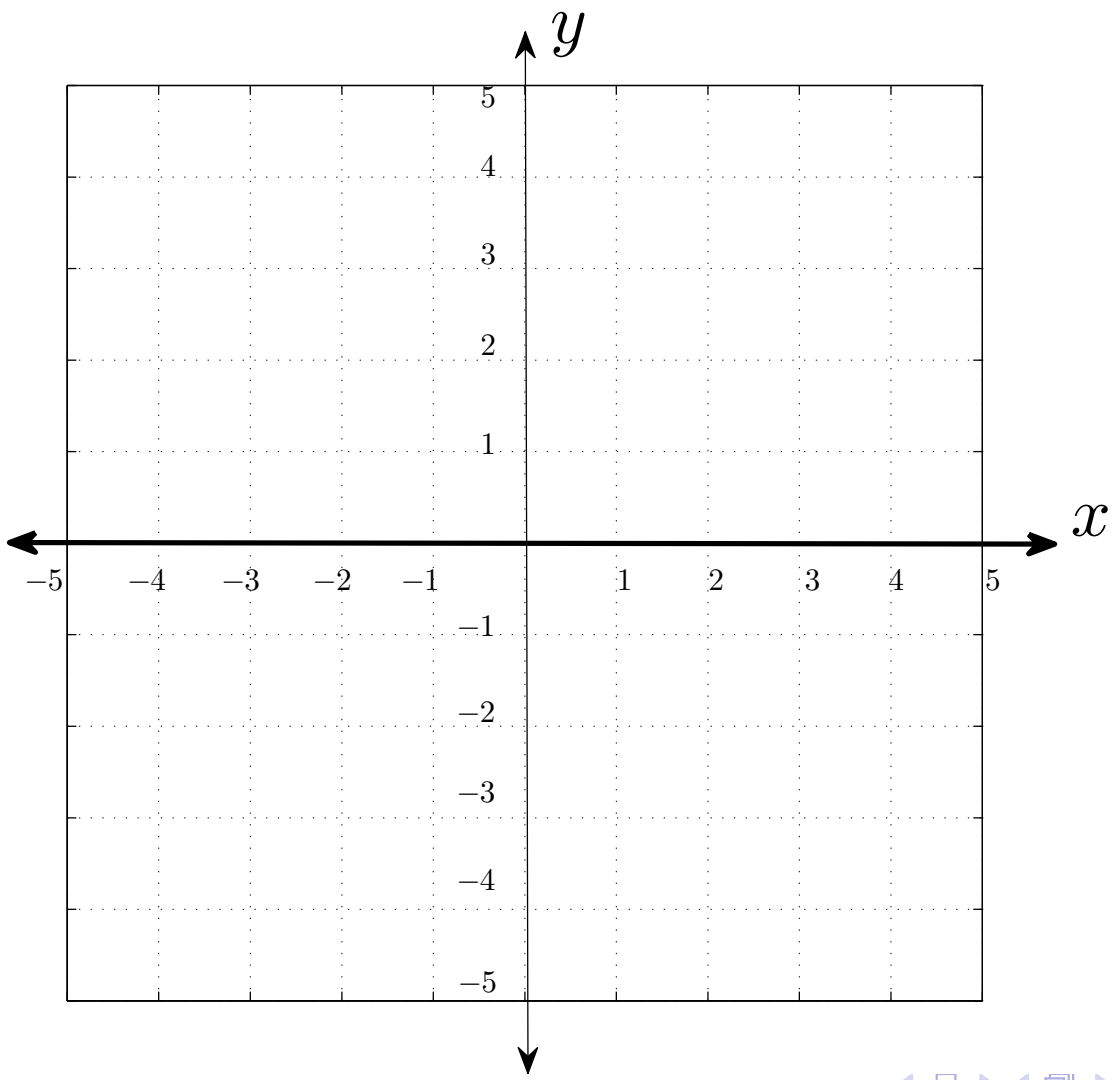

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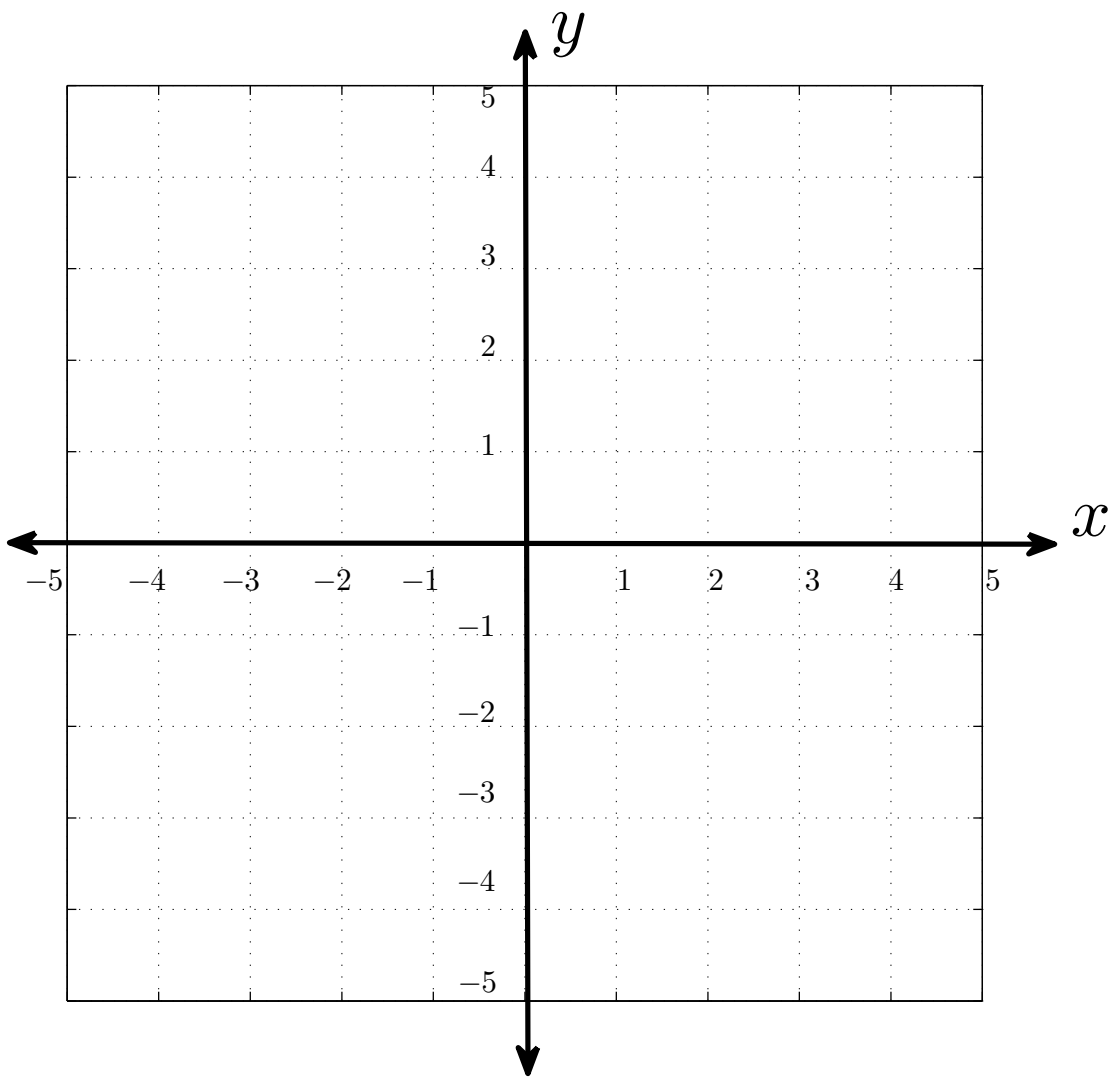

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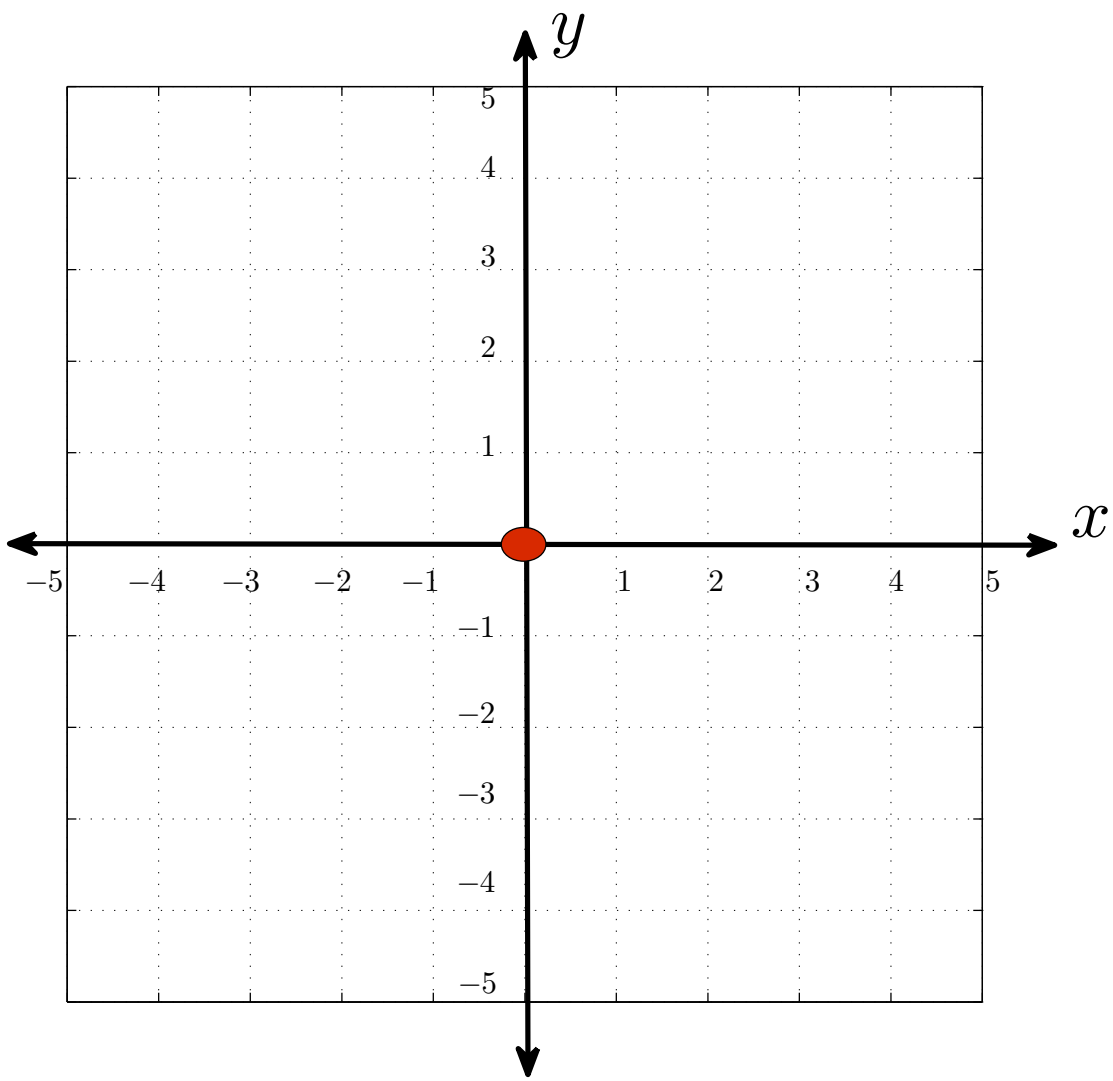
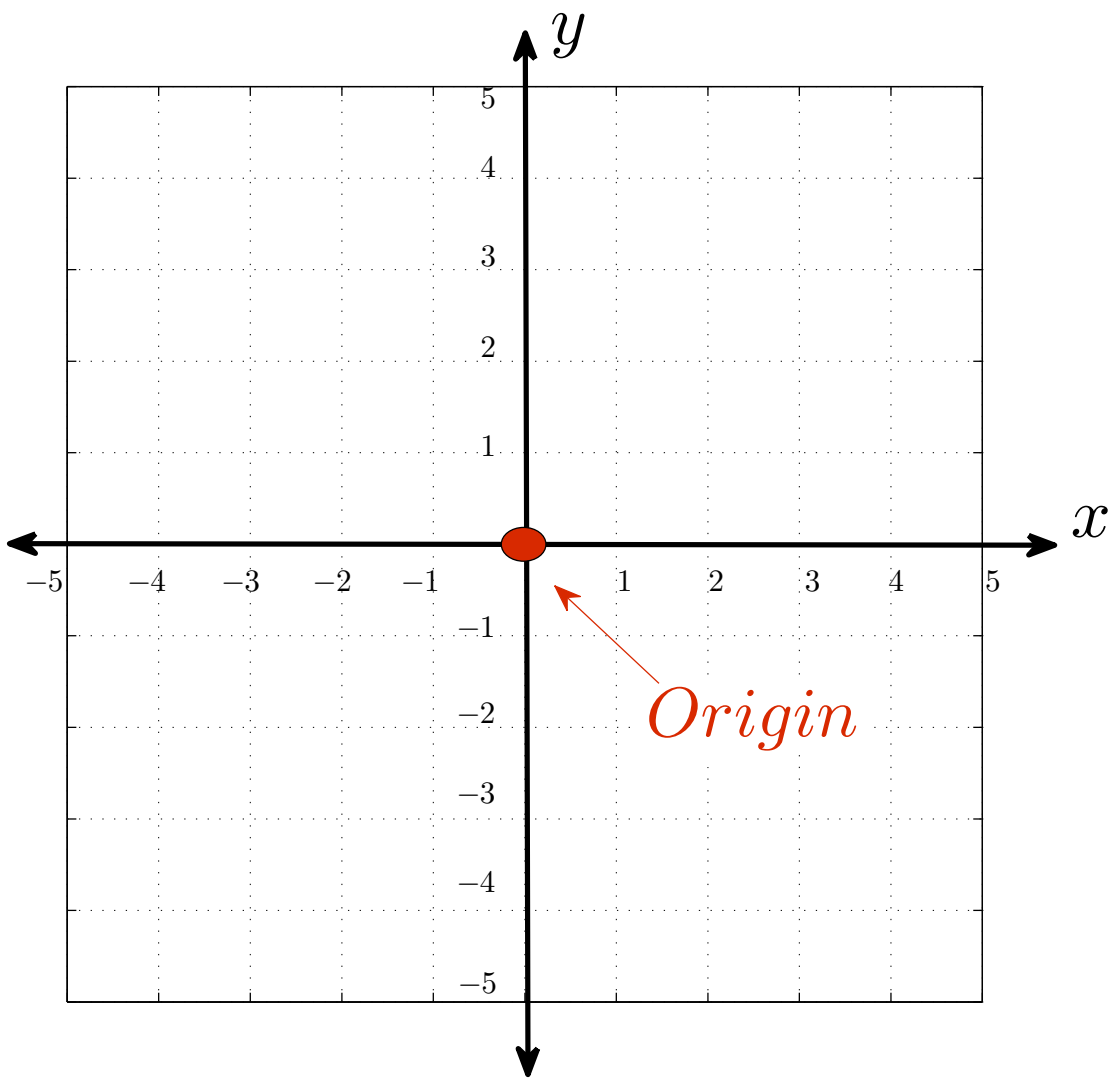
Two number lines, called axes, cross each other at zero.



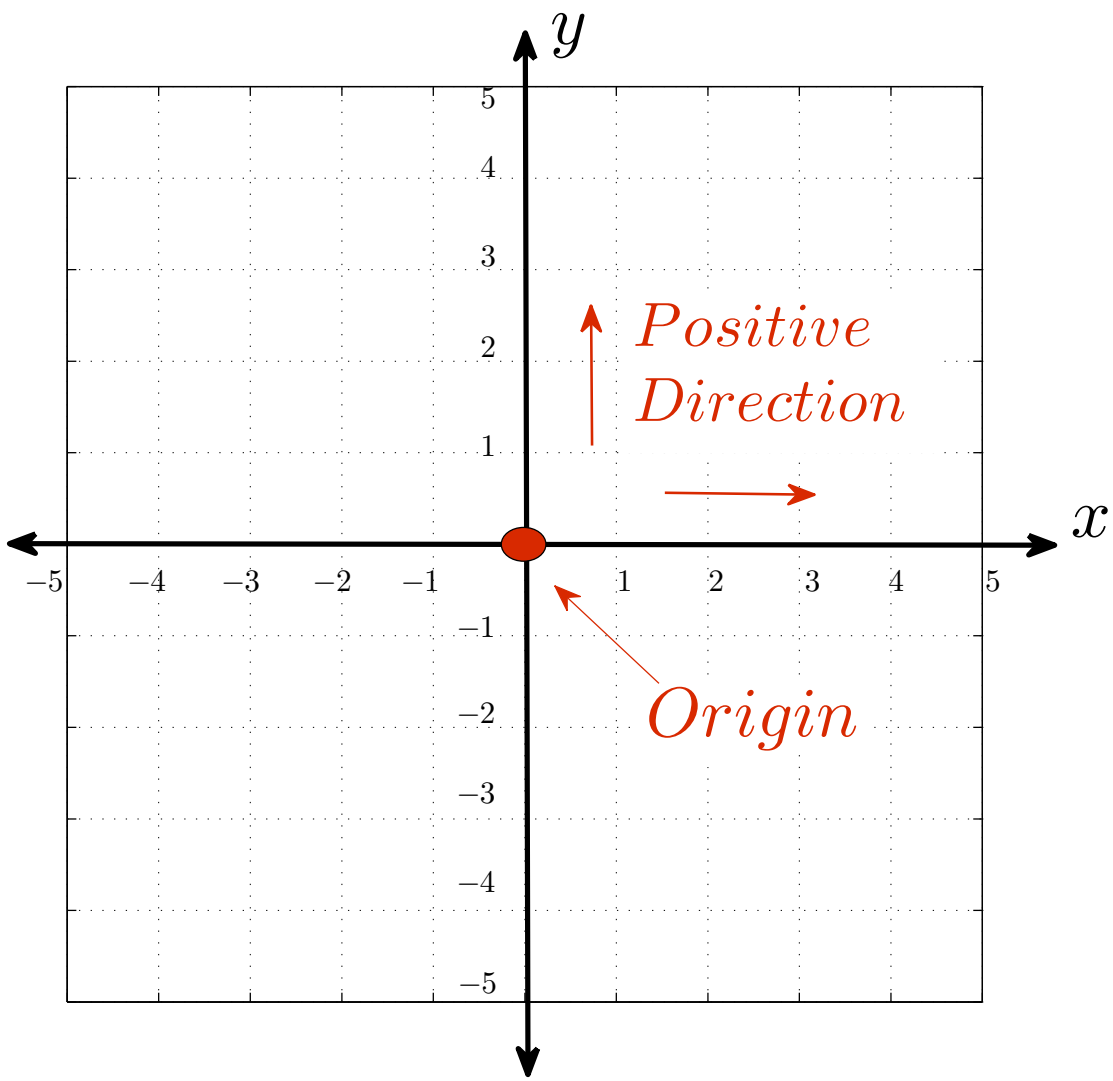
The Rectangular Coordinate System

Two number lines, called axes, cross each other at zero. This point is called the *origin*.



The Rectangular Coordinate System

Relative to the origin, positive directions are to the right and up.



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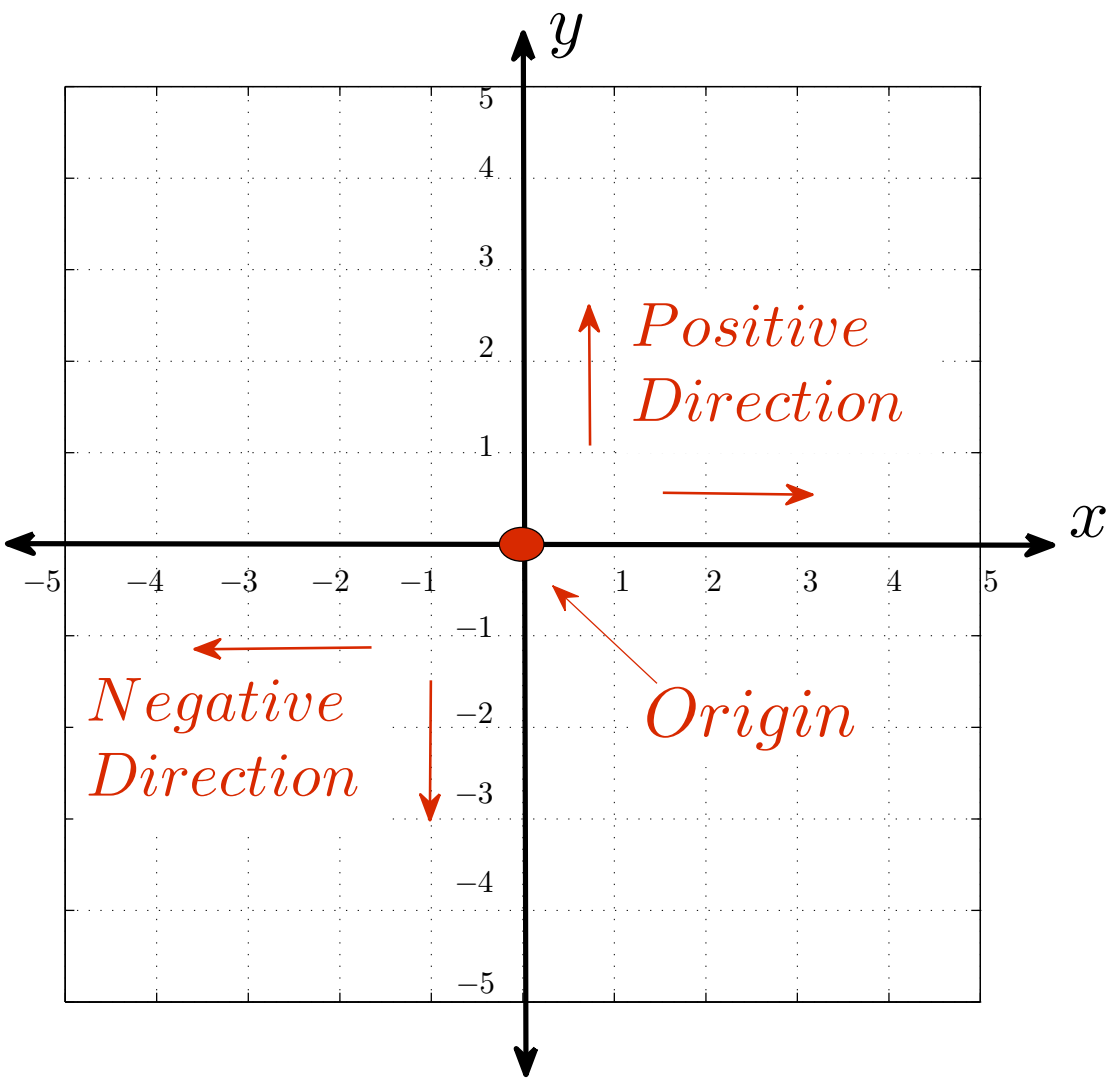
Negative directions are to the left and down.



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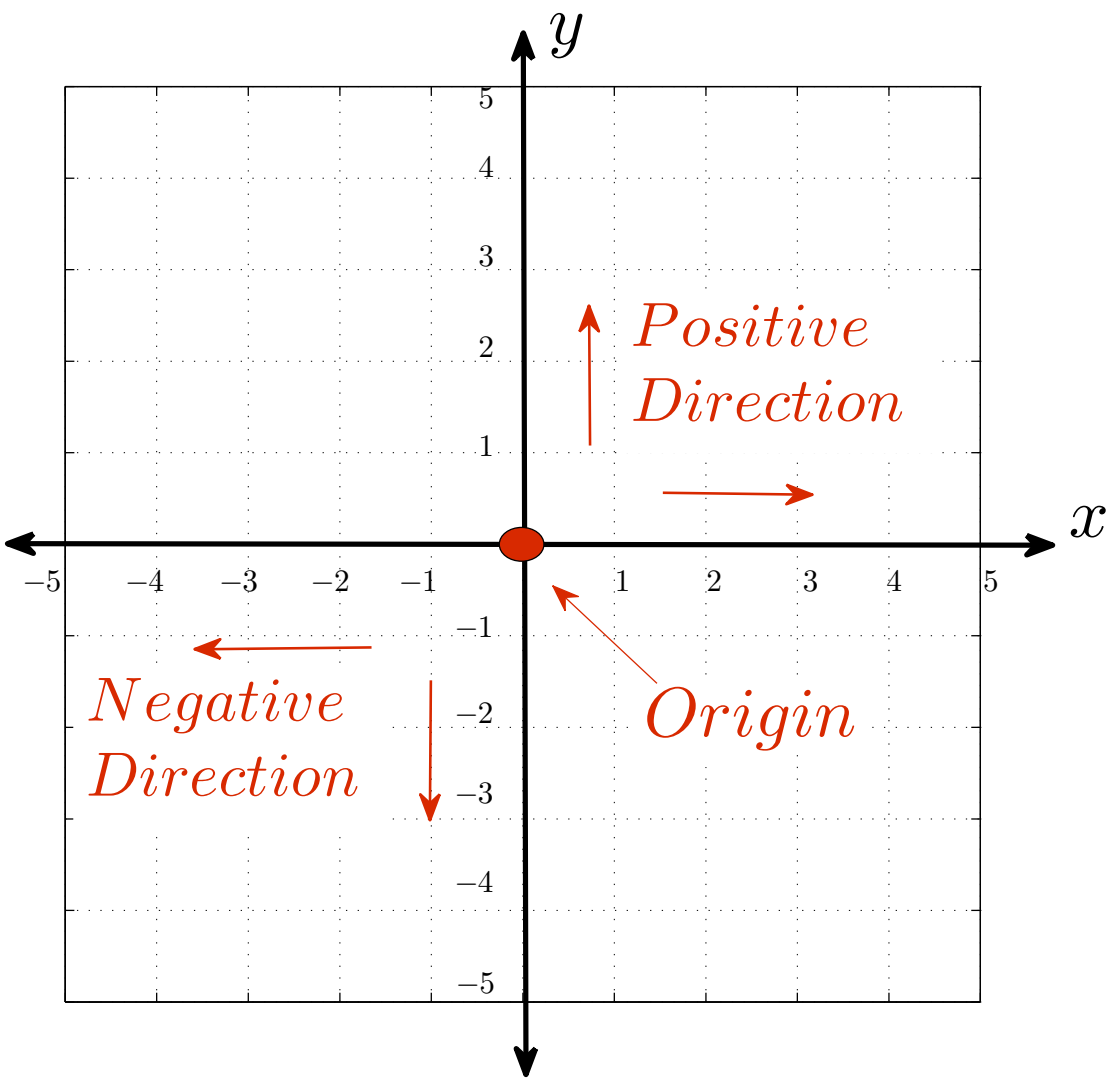

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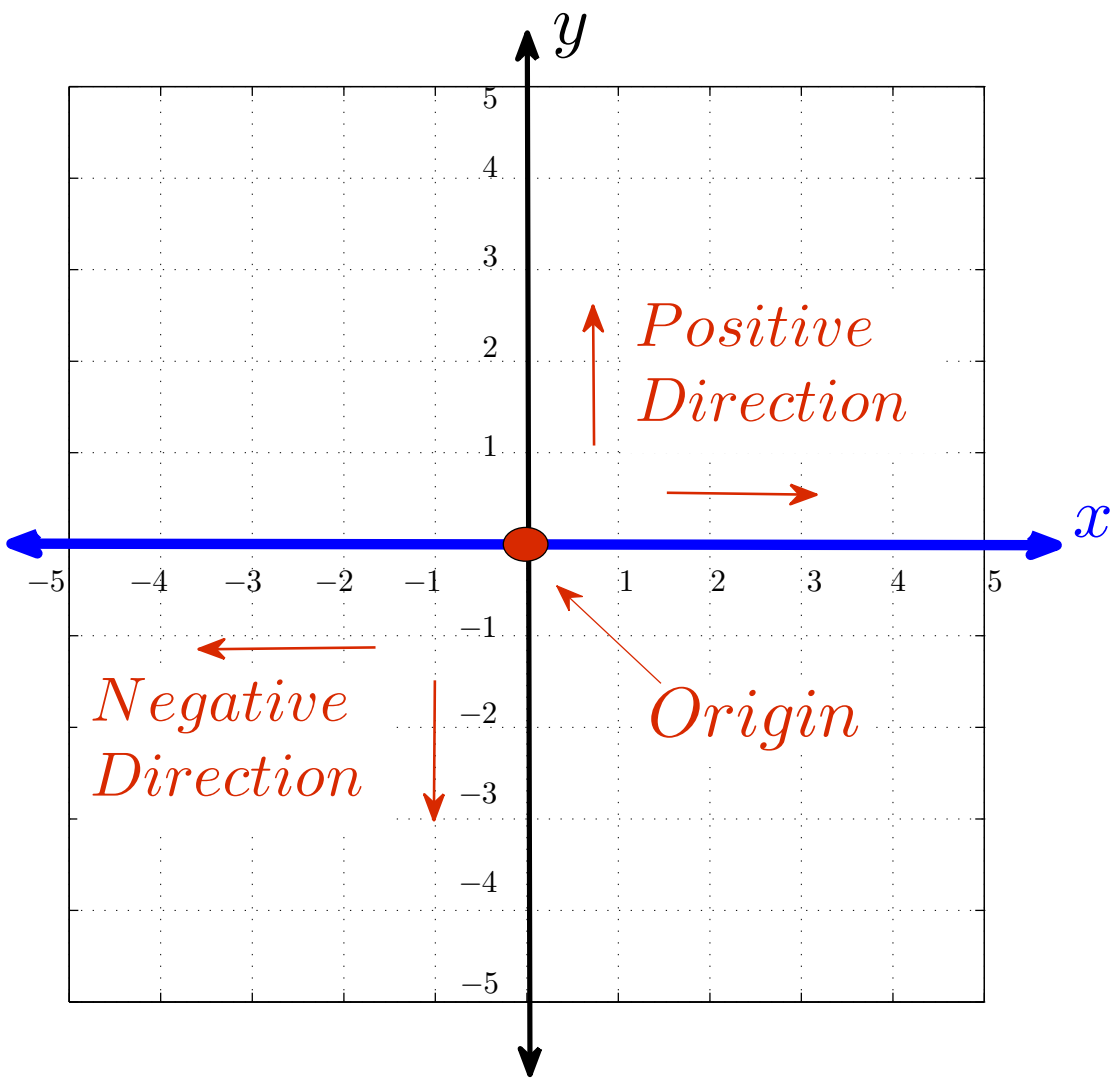

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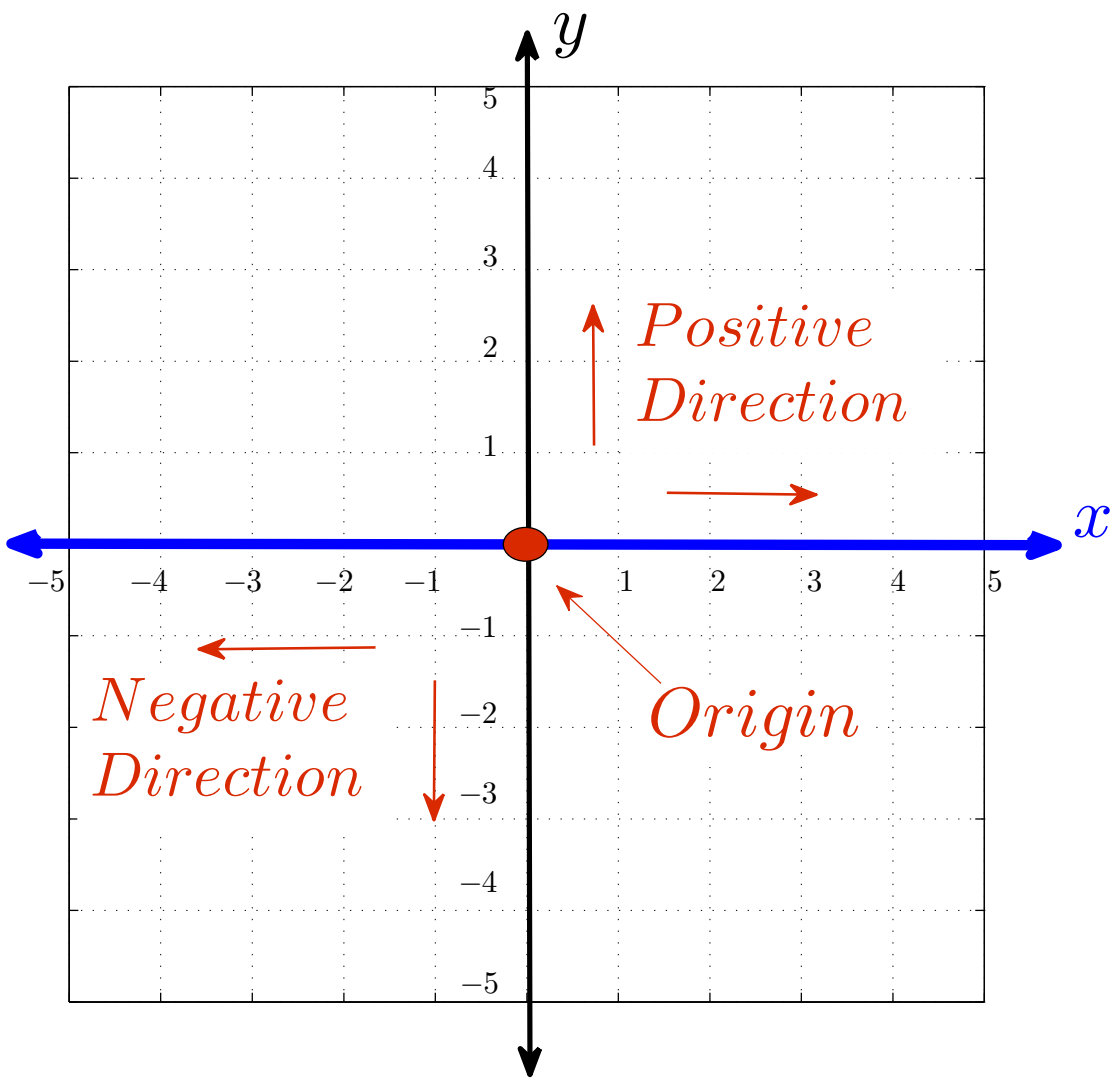

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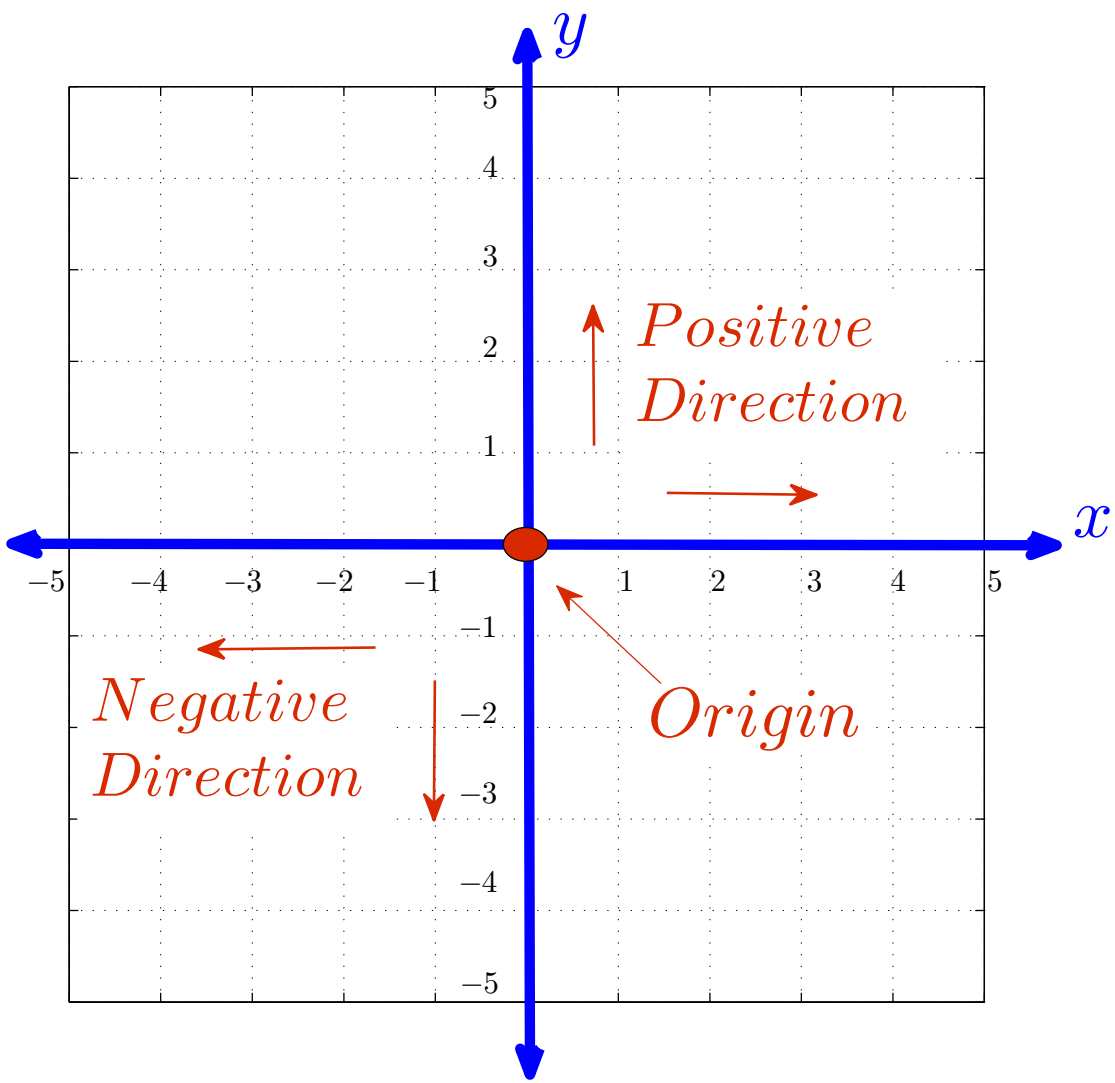
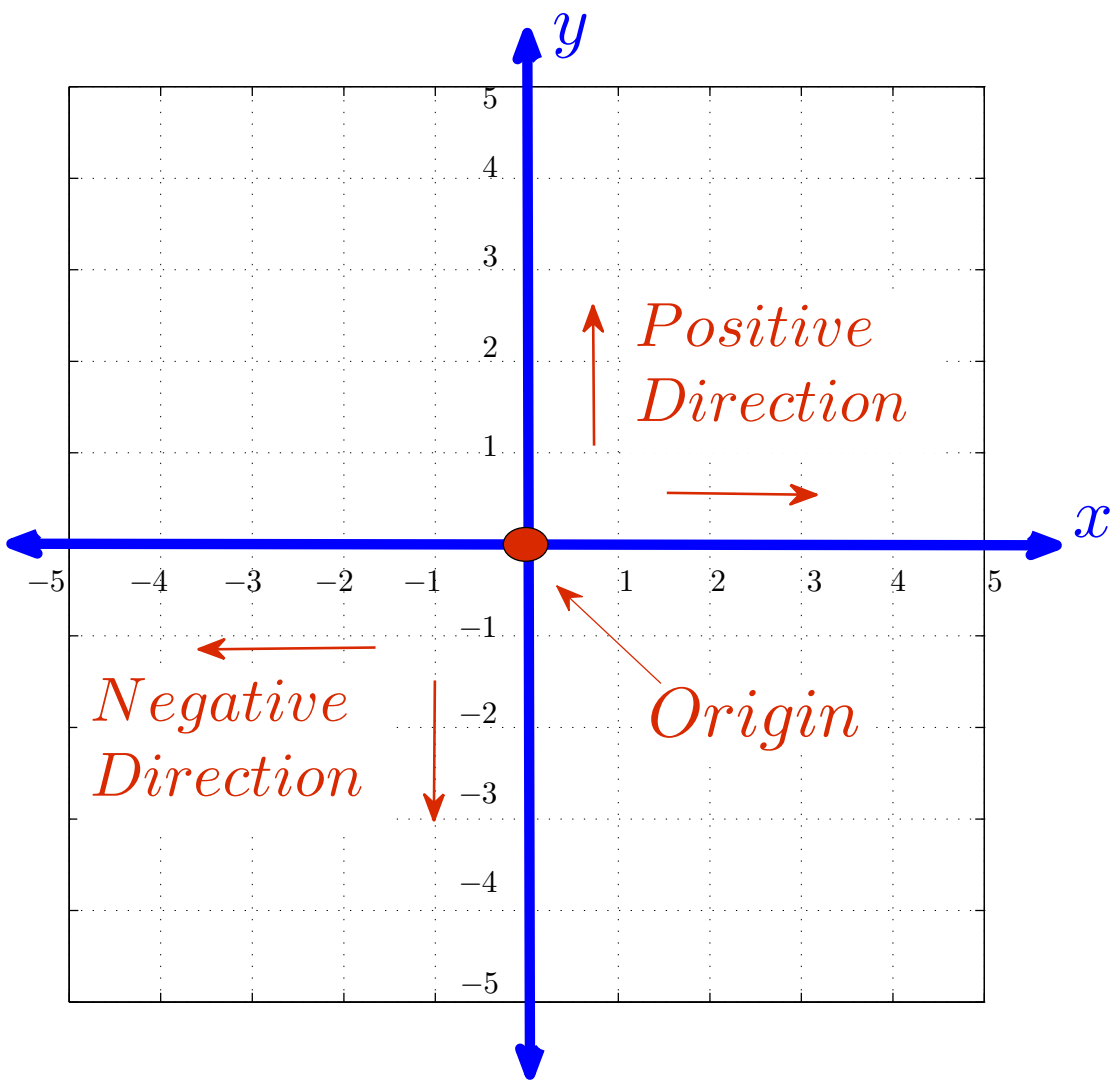


The Rectangular Coordinate System

The two number lines divide the coordinate system into four quadrants, which we number I through IV in a counterclockwise direction.



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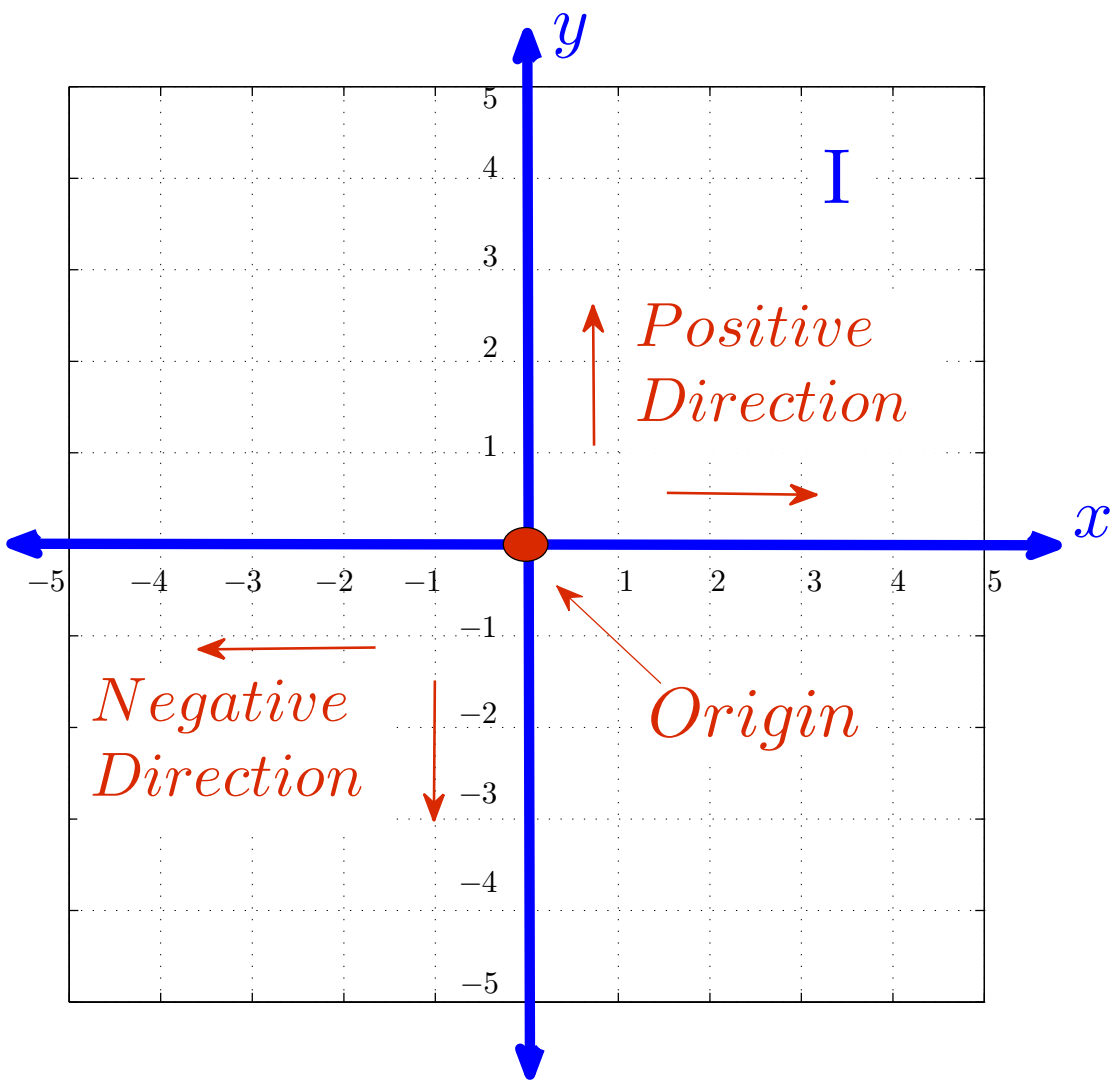

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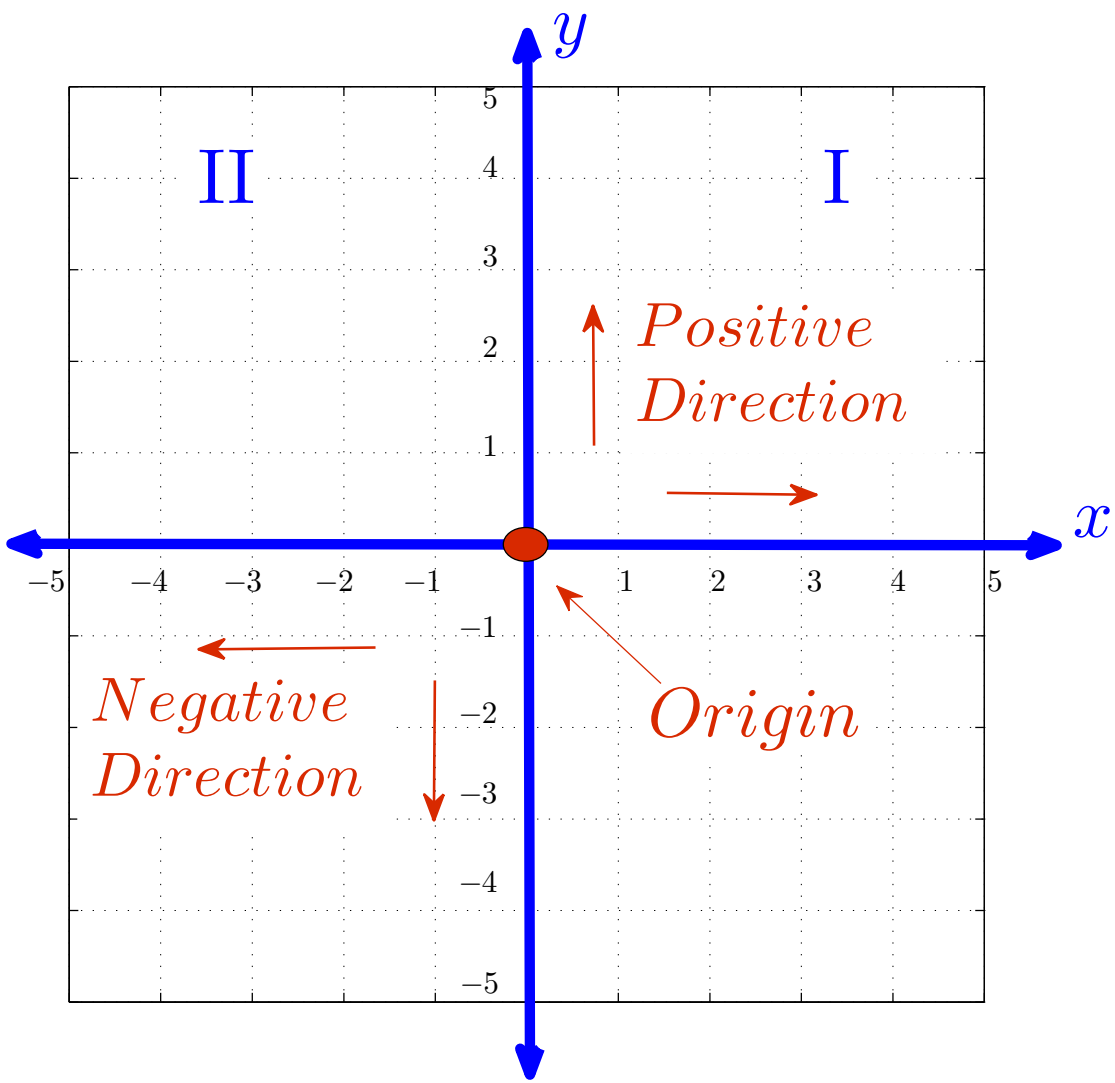

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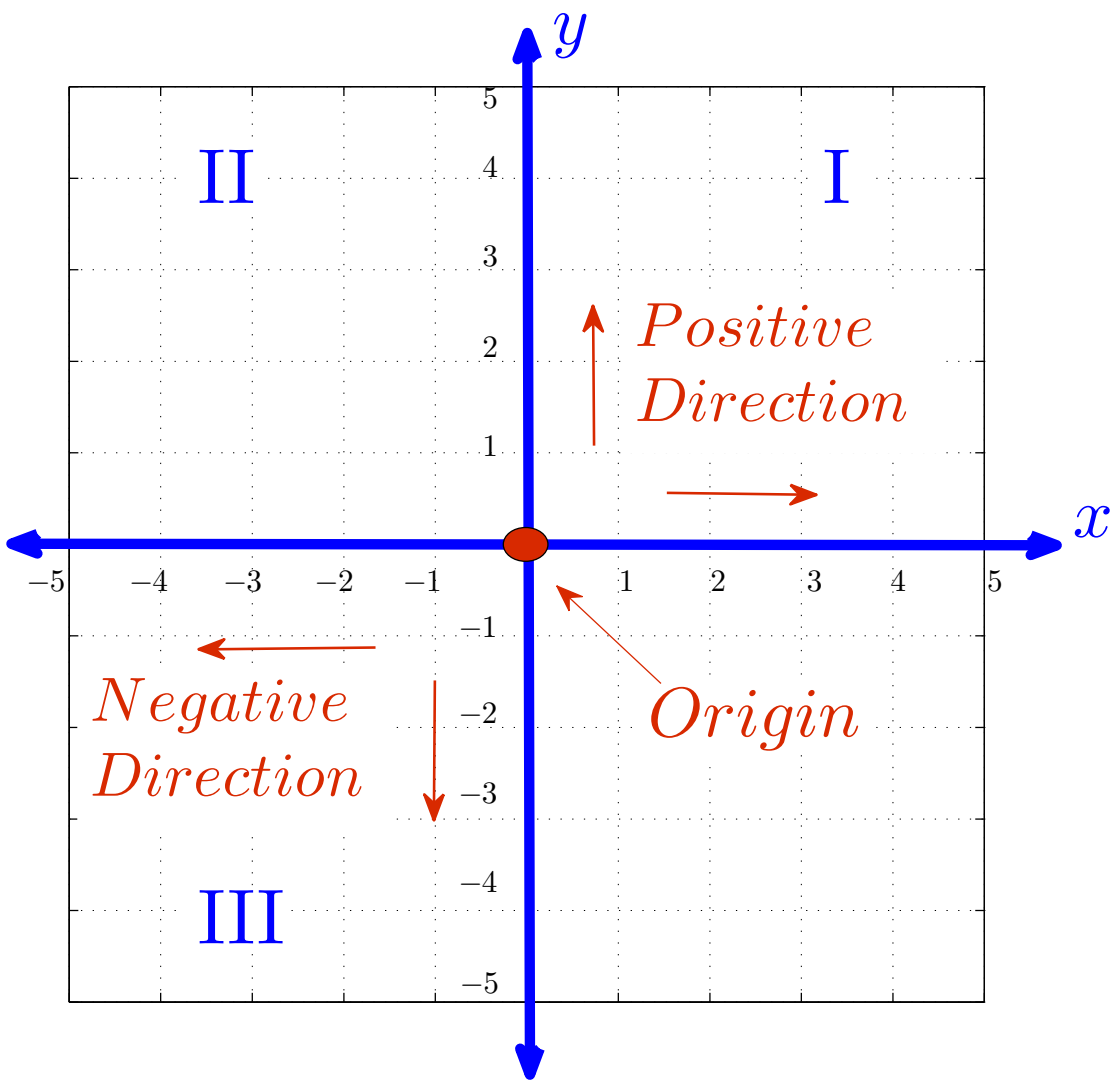

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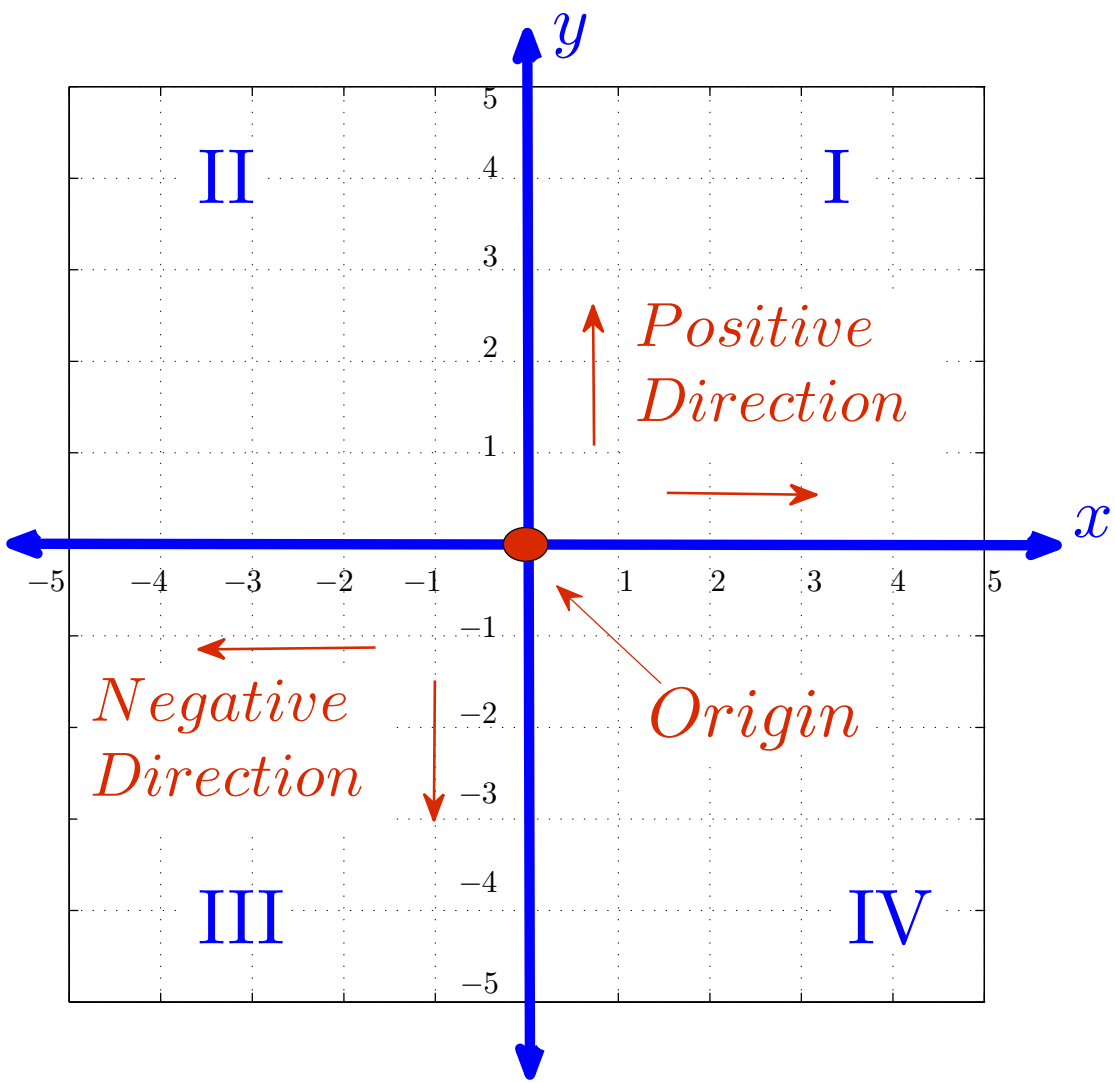

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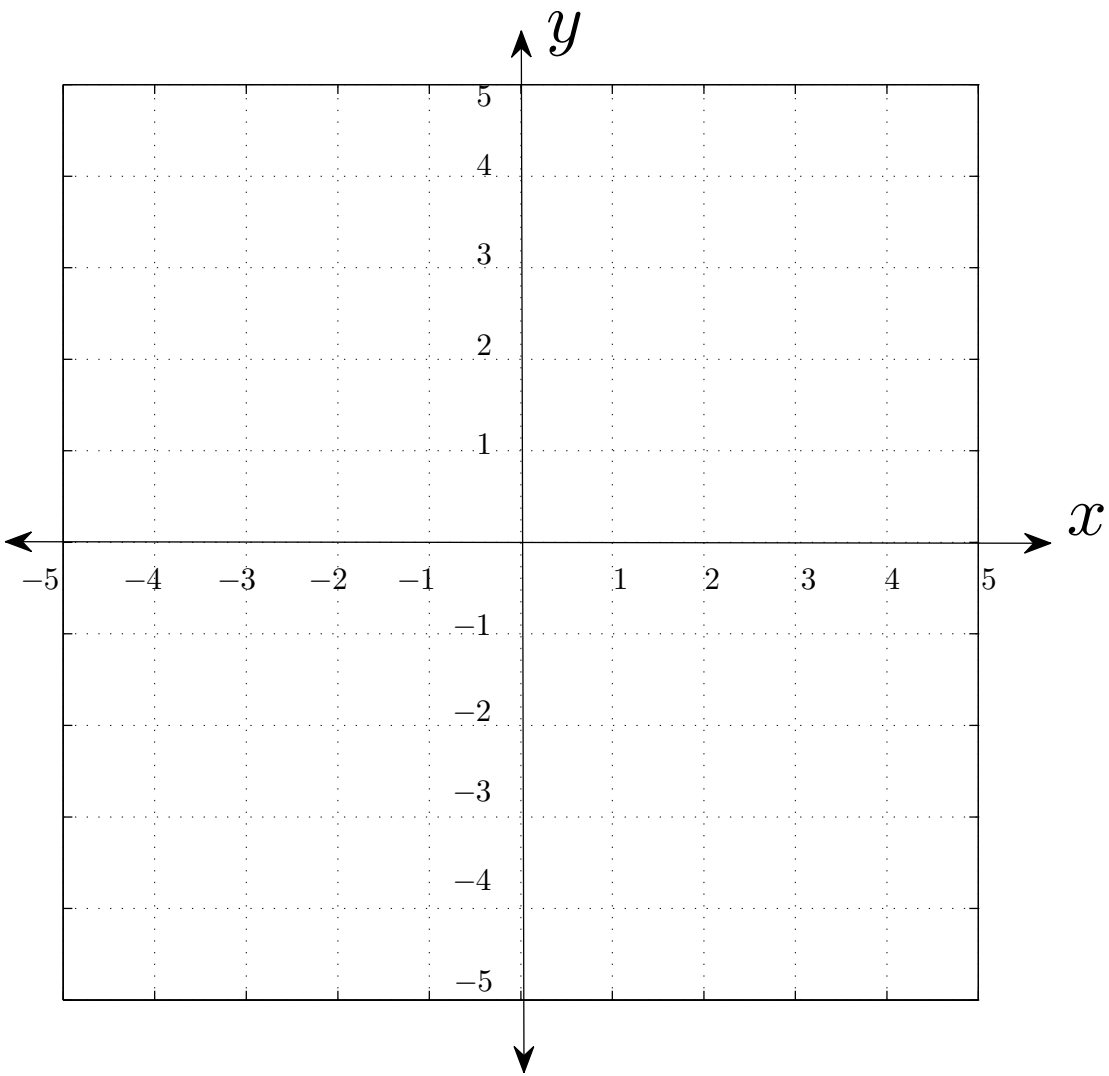
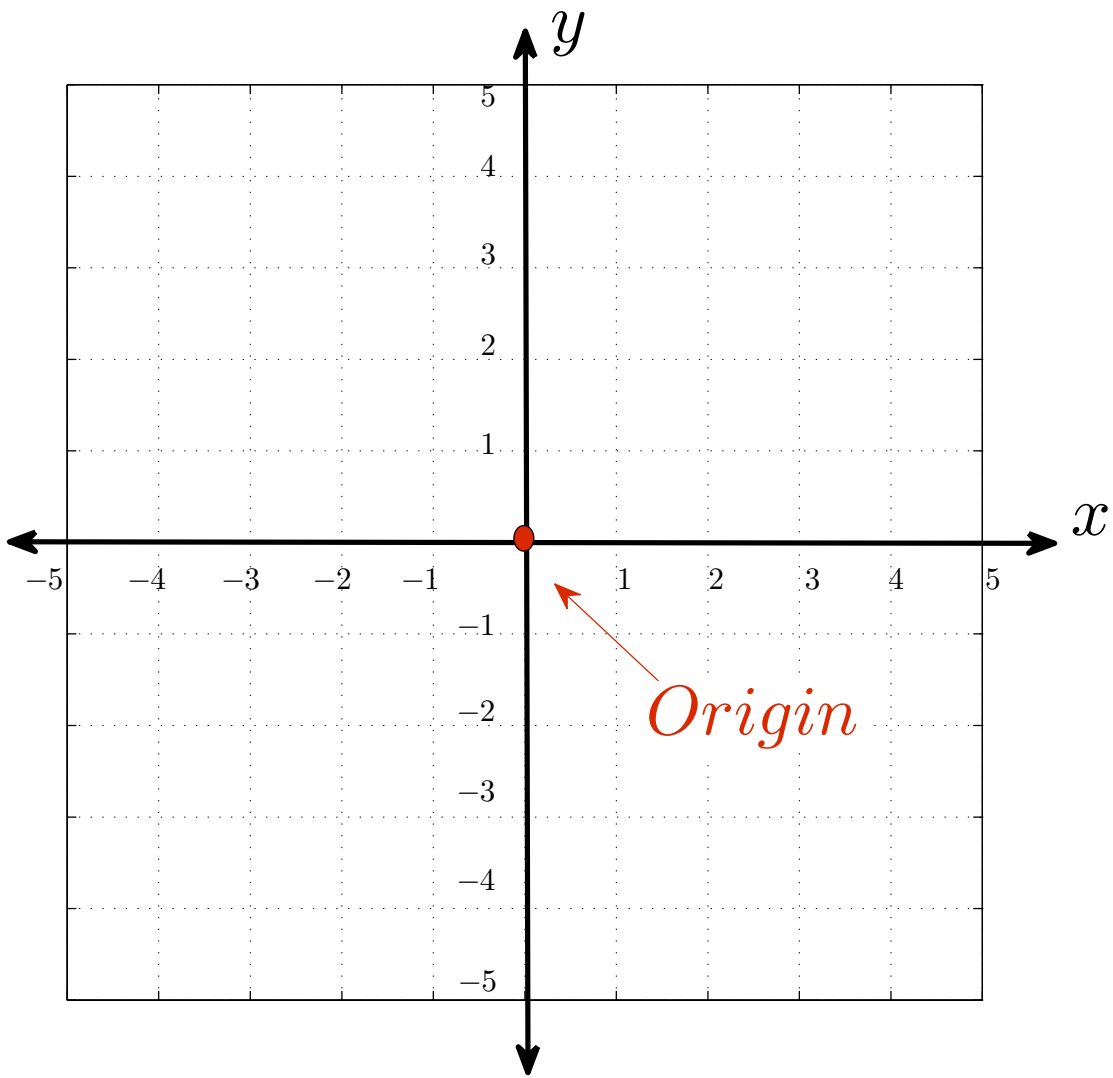
To graph the ordered pair (a, b) on the rectangular coordinate system, we:

1. begin at the origin and move along the x-axis a units right or a units left (right if a is positive and left if a is negative).
2. From that point we move b units up or down (up if b is positive and down if b is negative).
3. The point where we end up is the graph of the ordered pair.

Example 1: Plot (graph) the following ordered pairs: $(2,3)$, $(-2,3)$, $(-2,-3)$, $(2,-3)$,



To plot $(2,3)$, begin at the origin.



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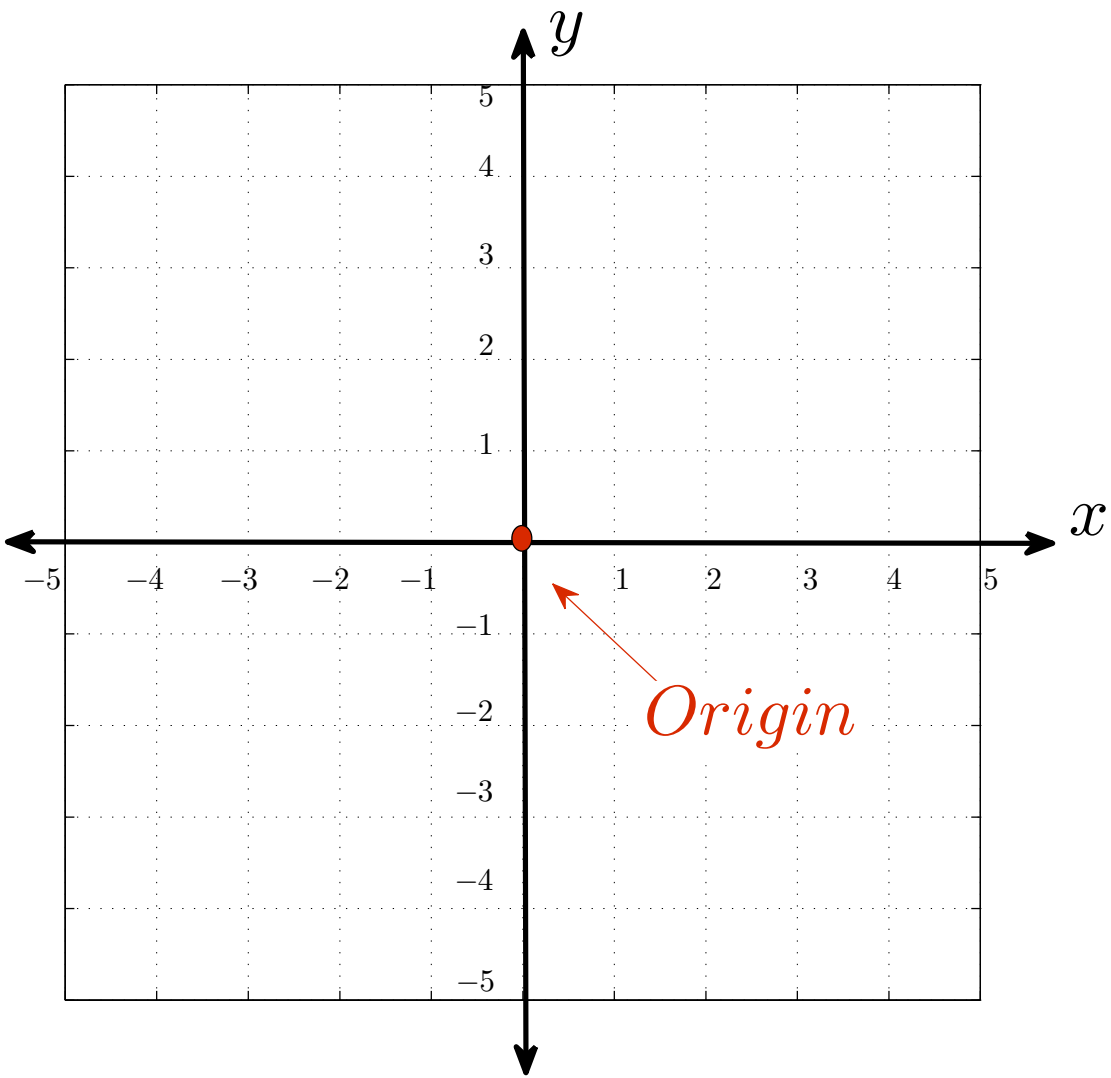

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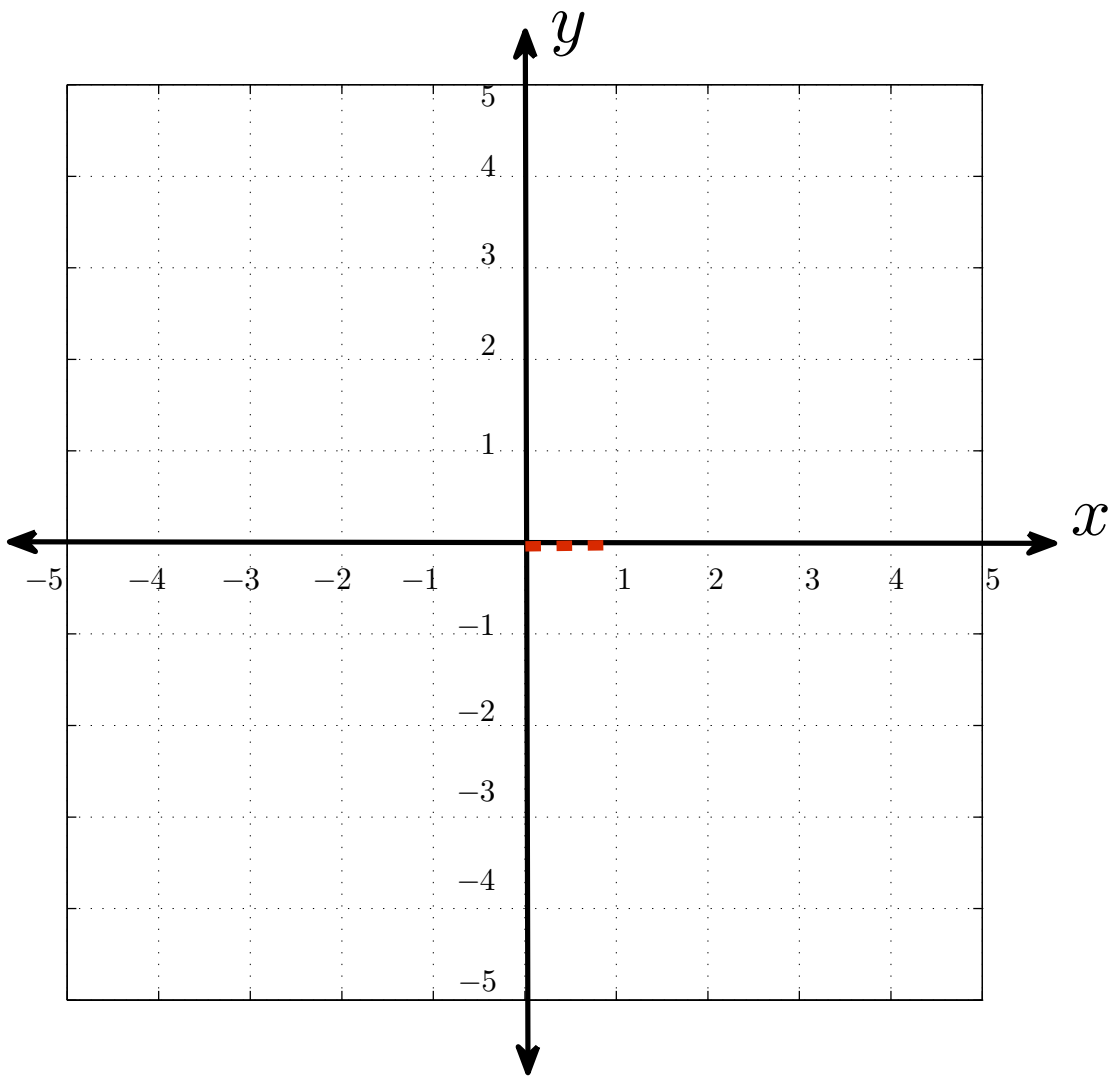

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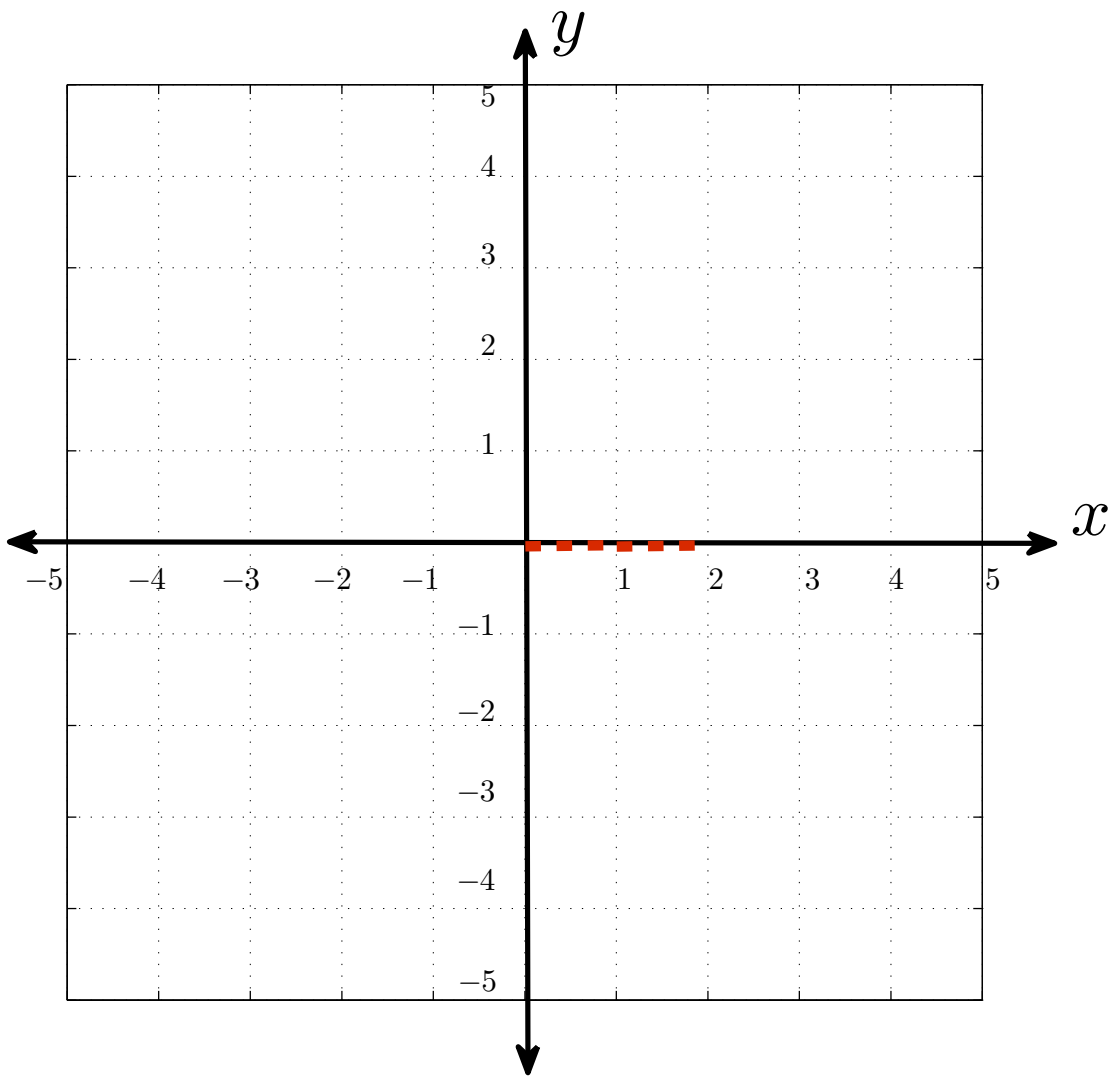
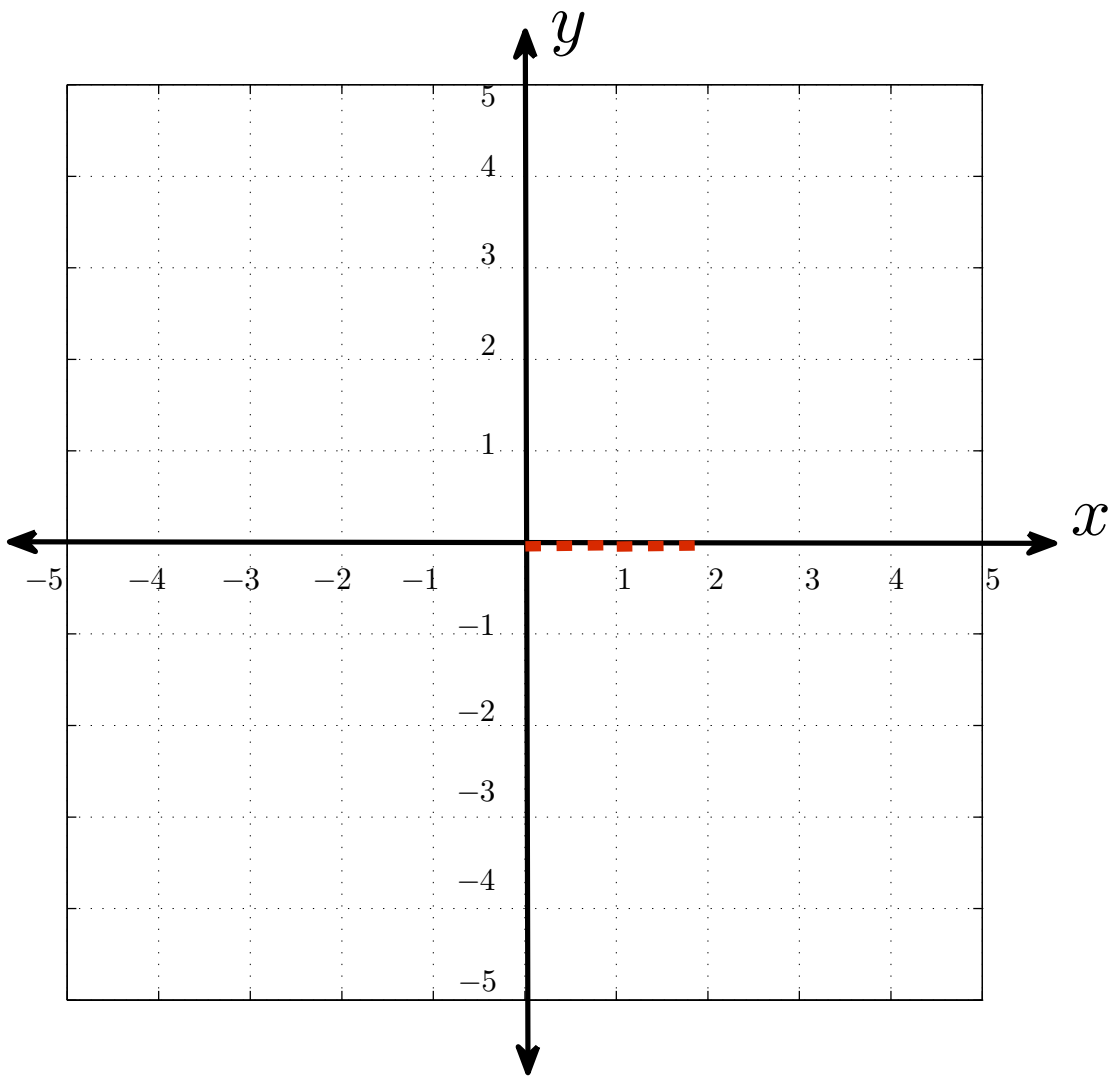


From that point, move in the upwards (positive y) direction 3 units.



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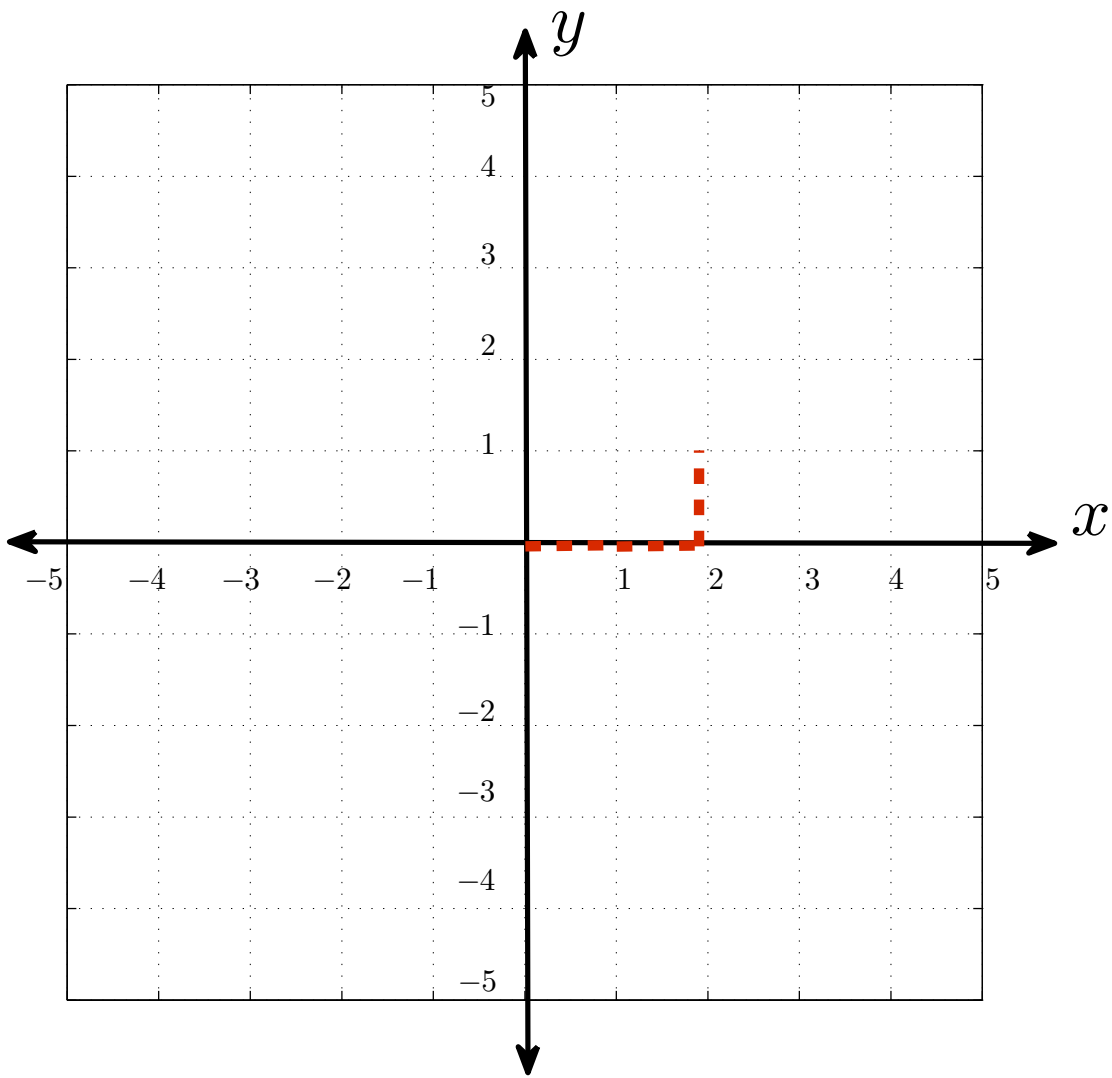

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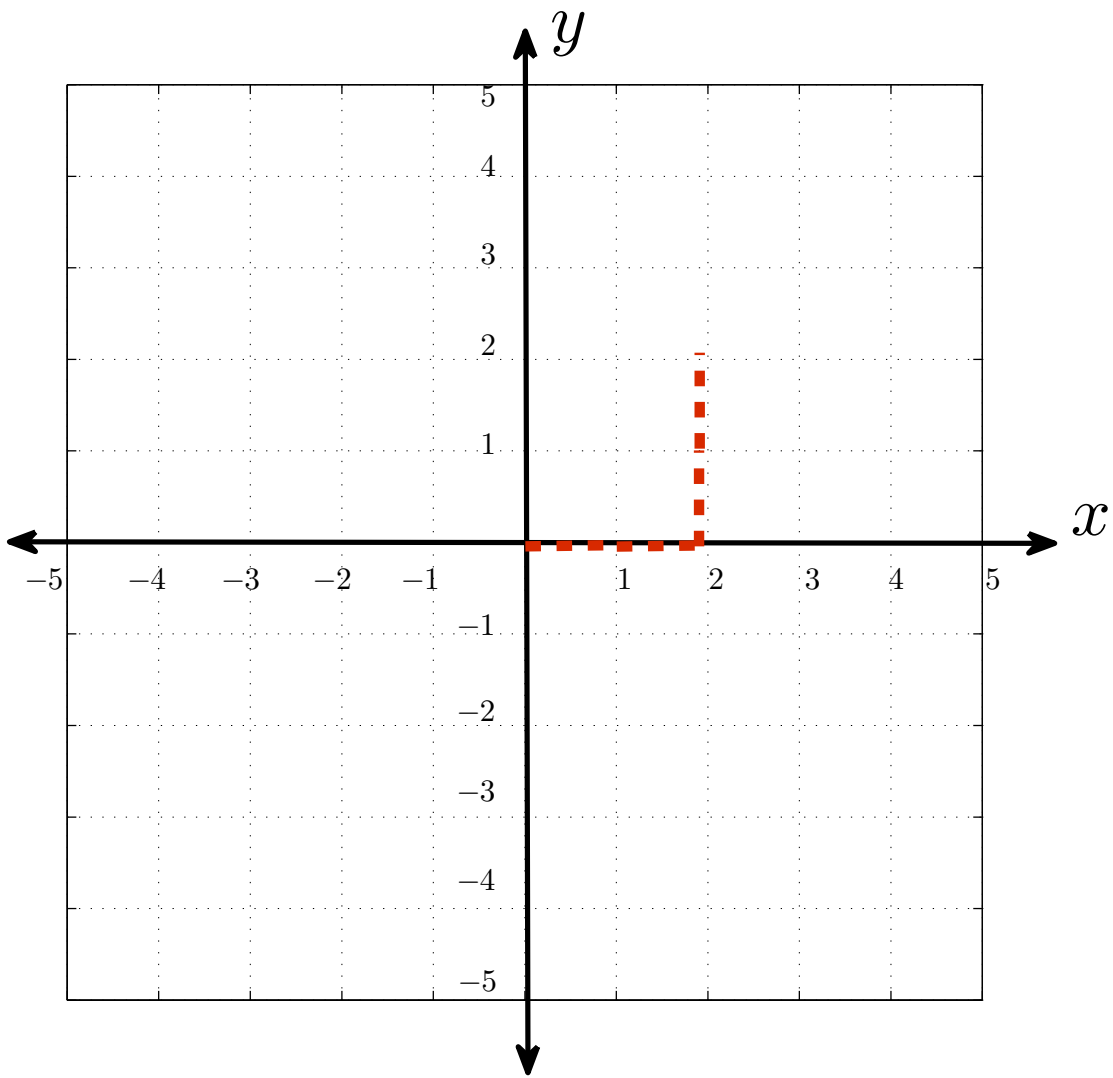

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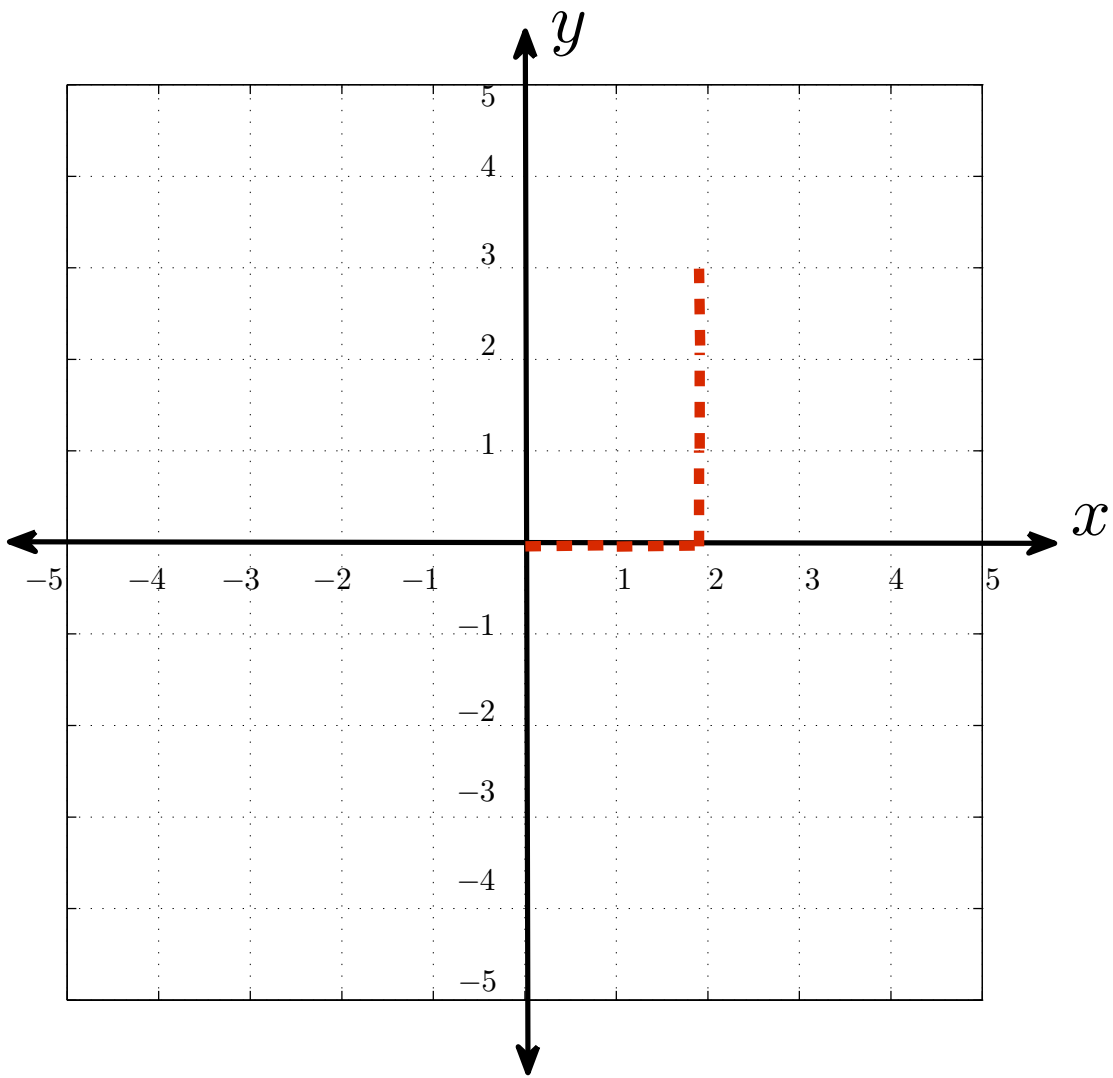

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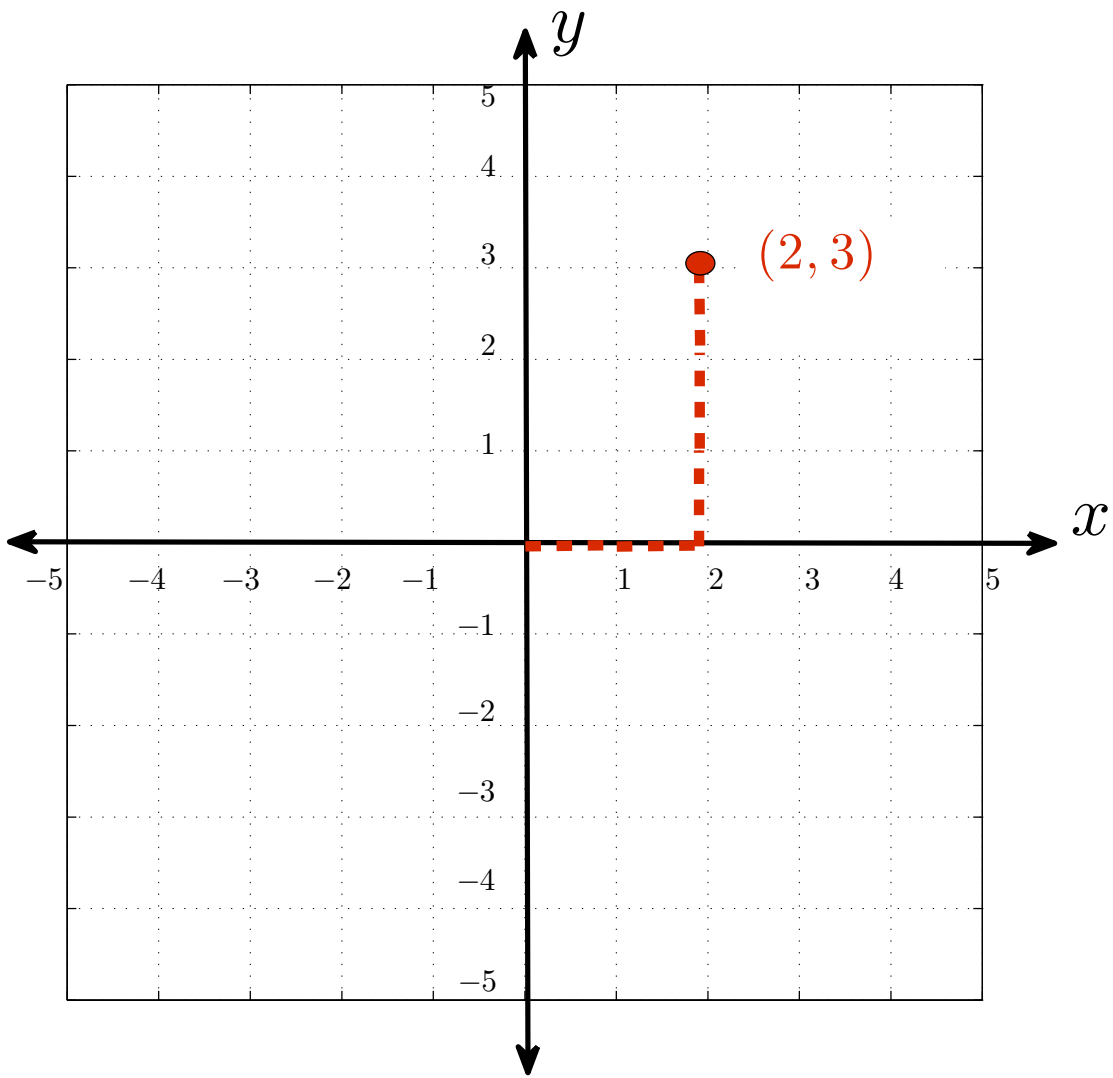

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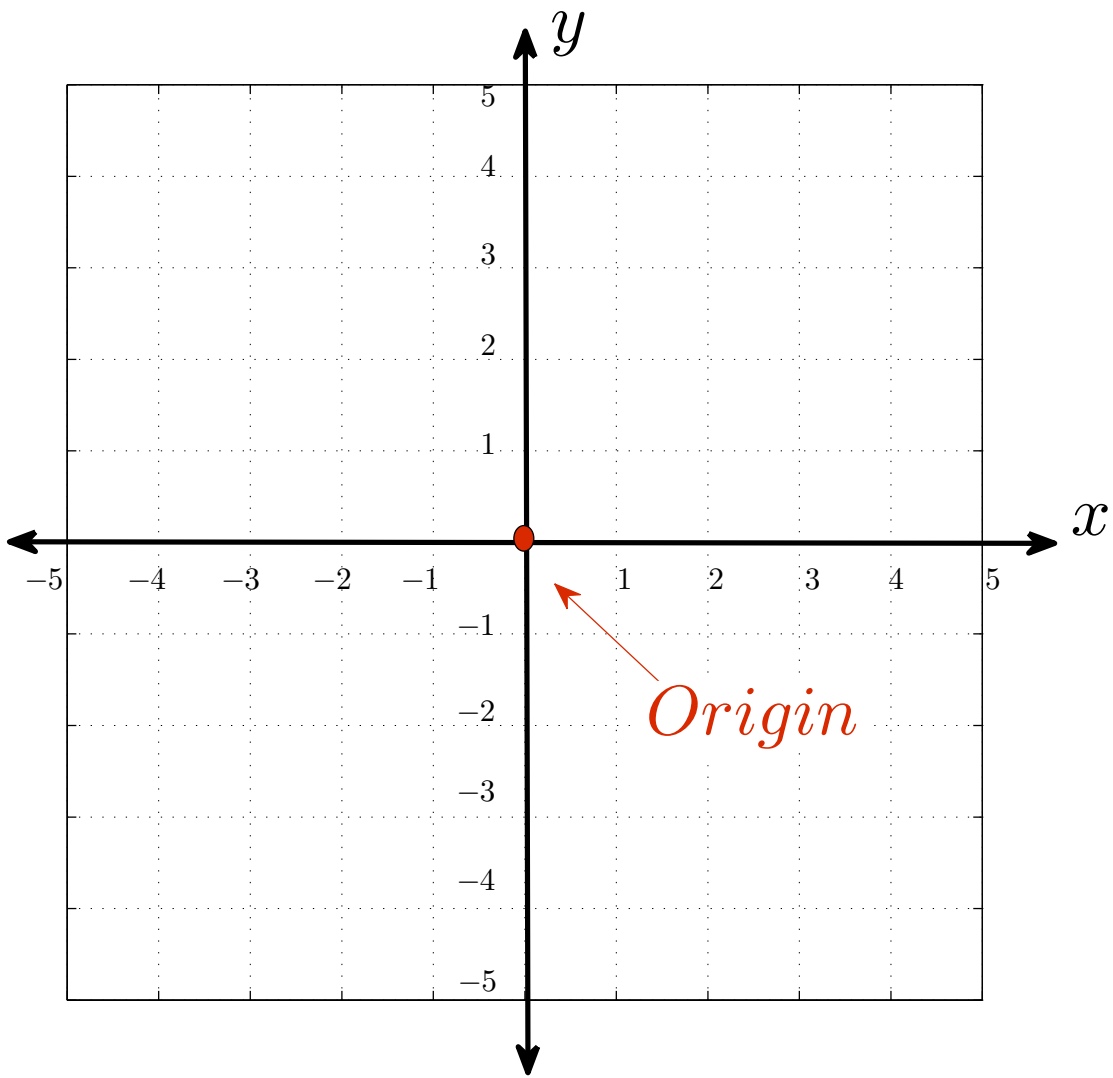

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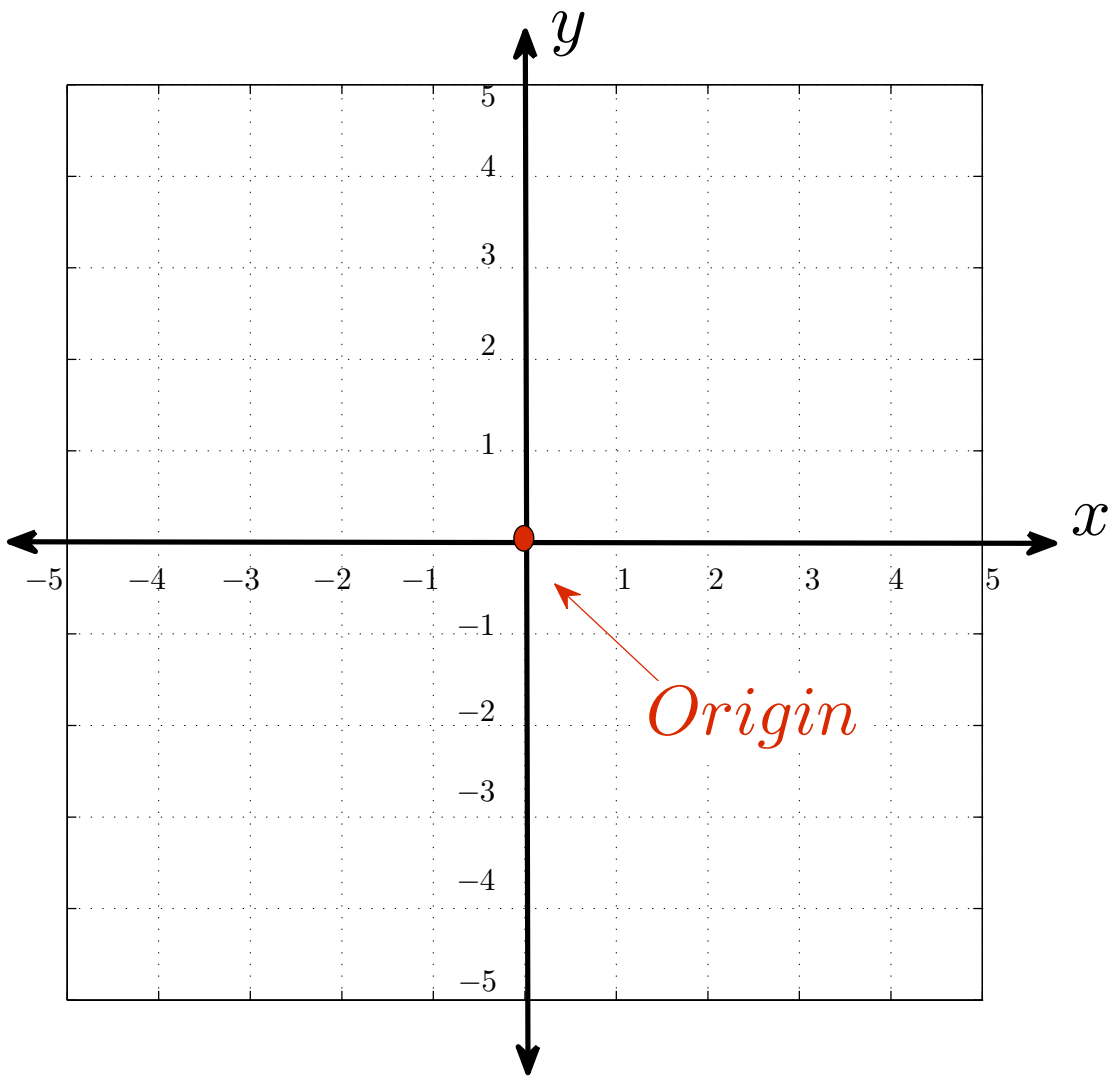

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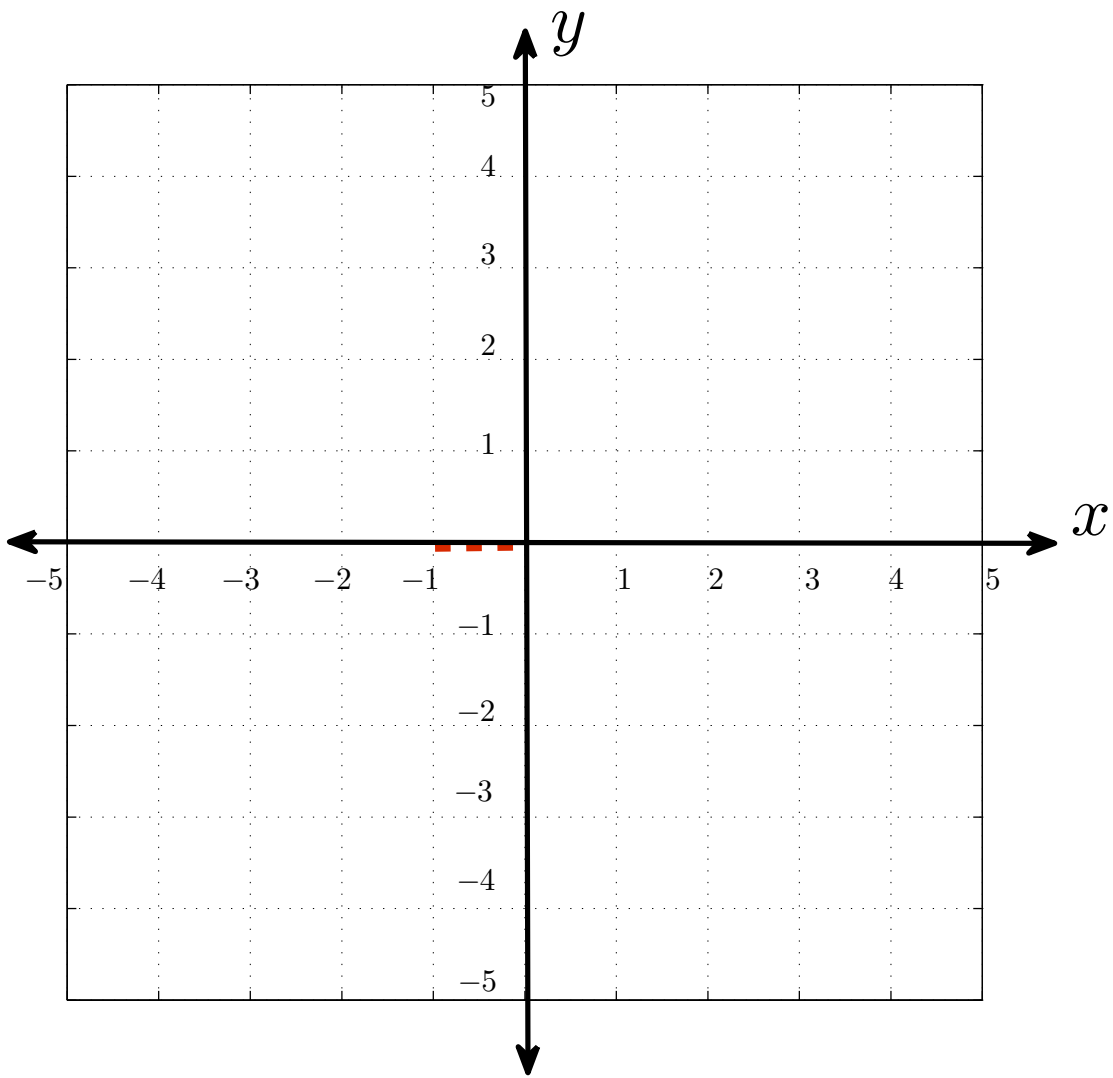

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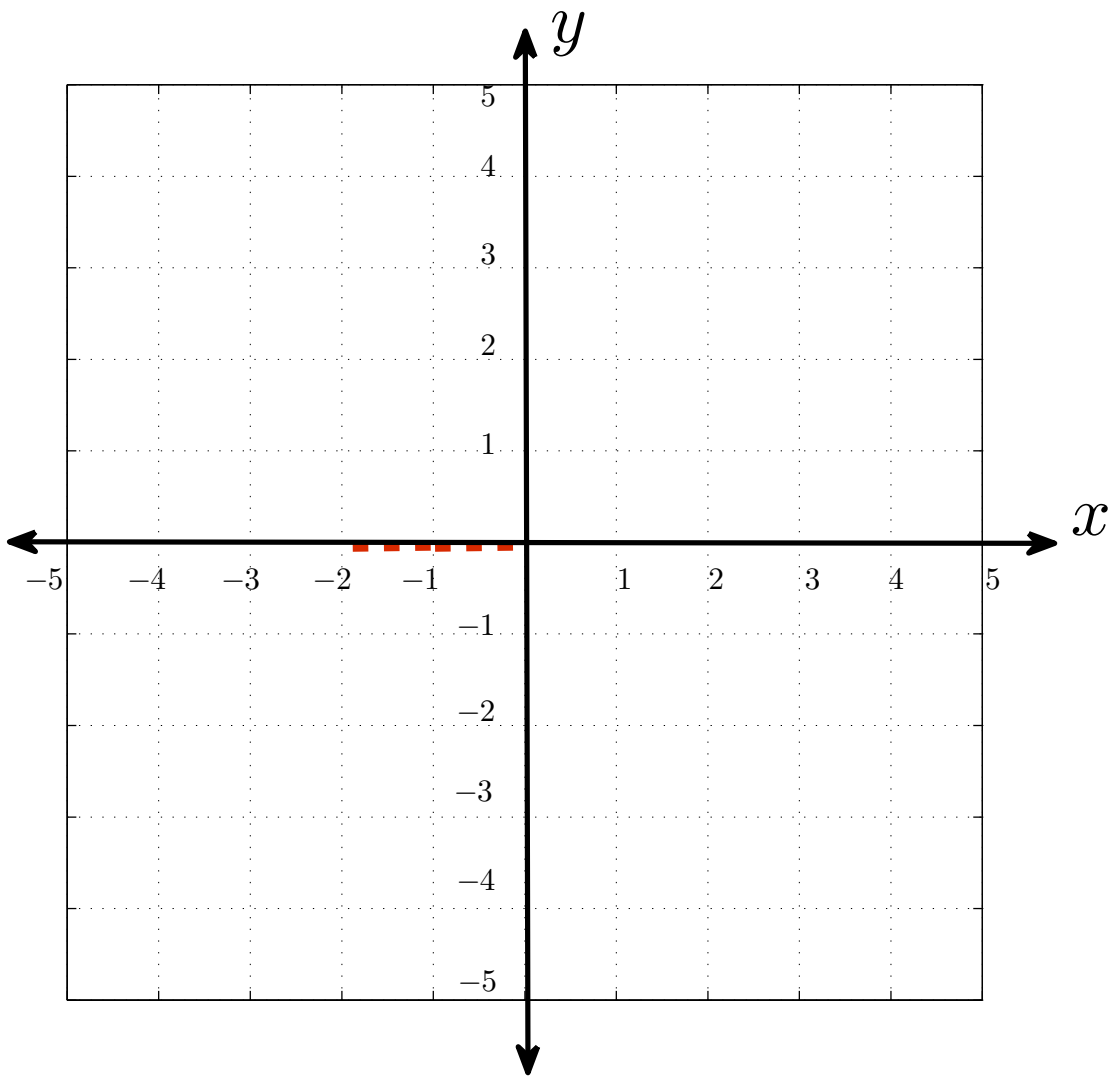

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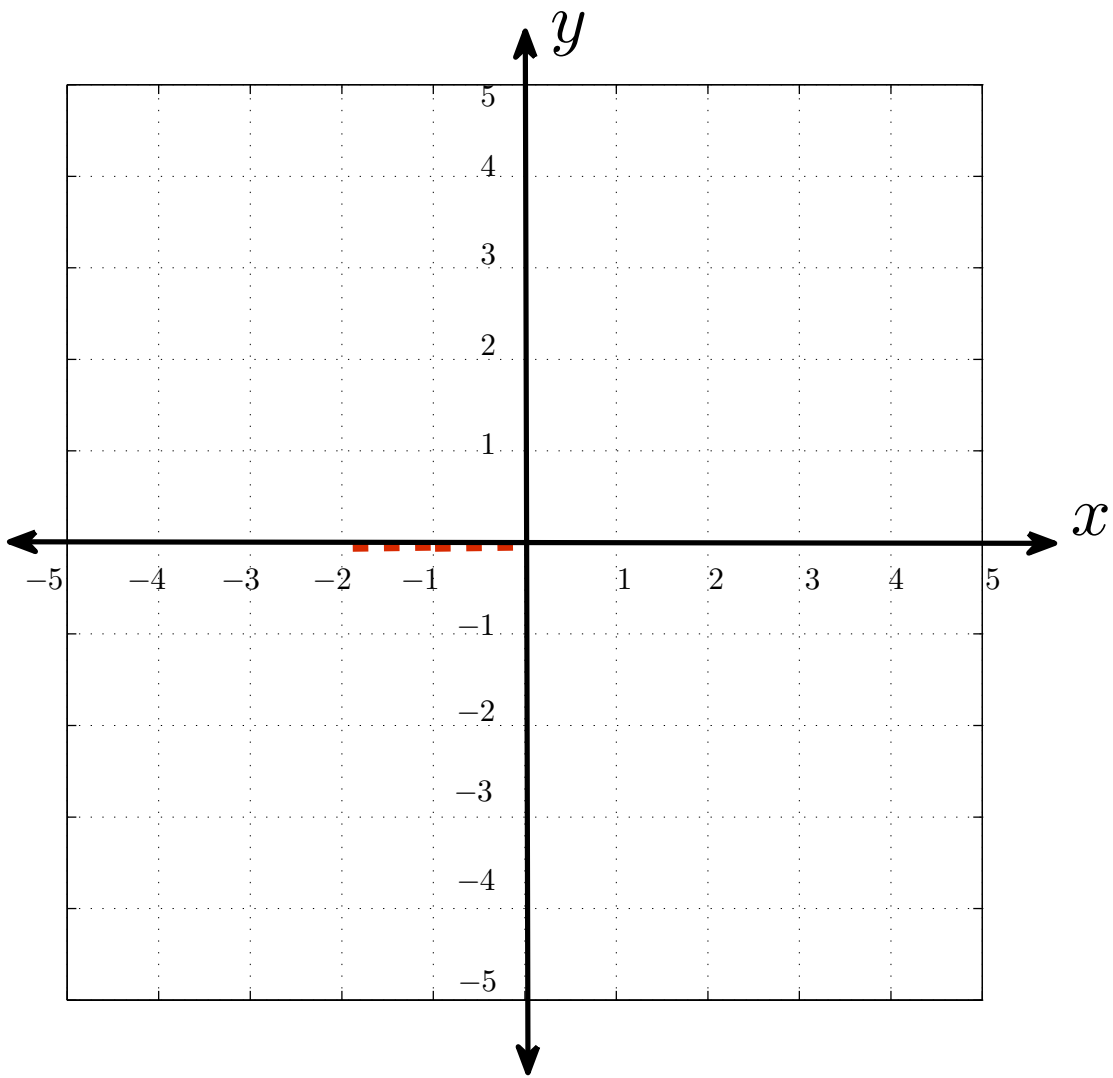

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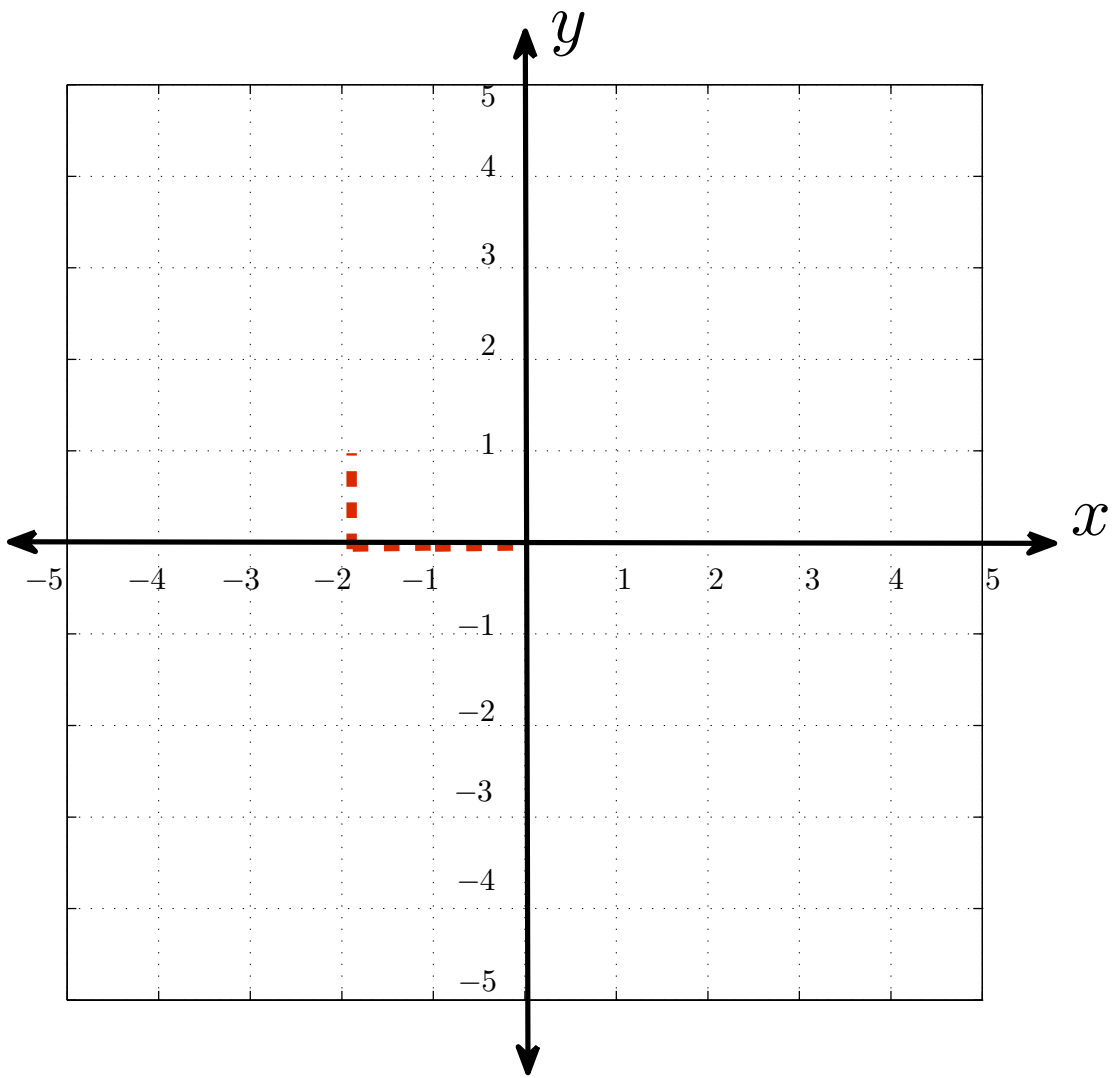

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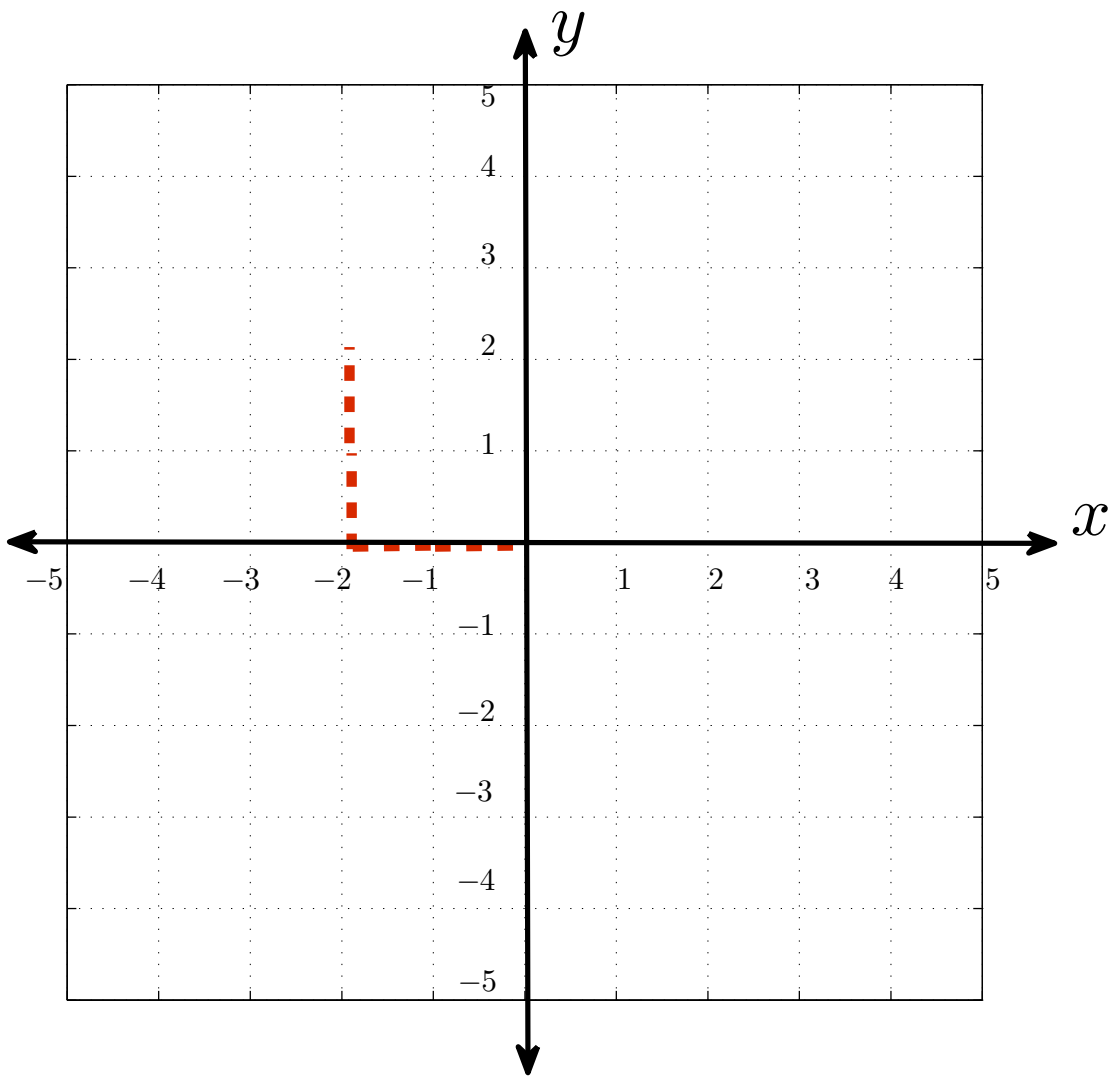

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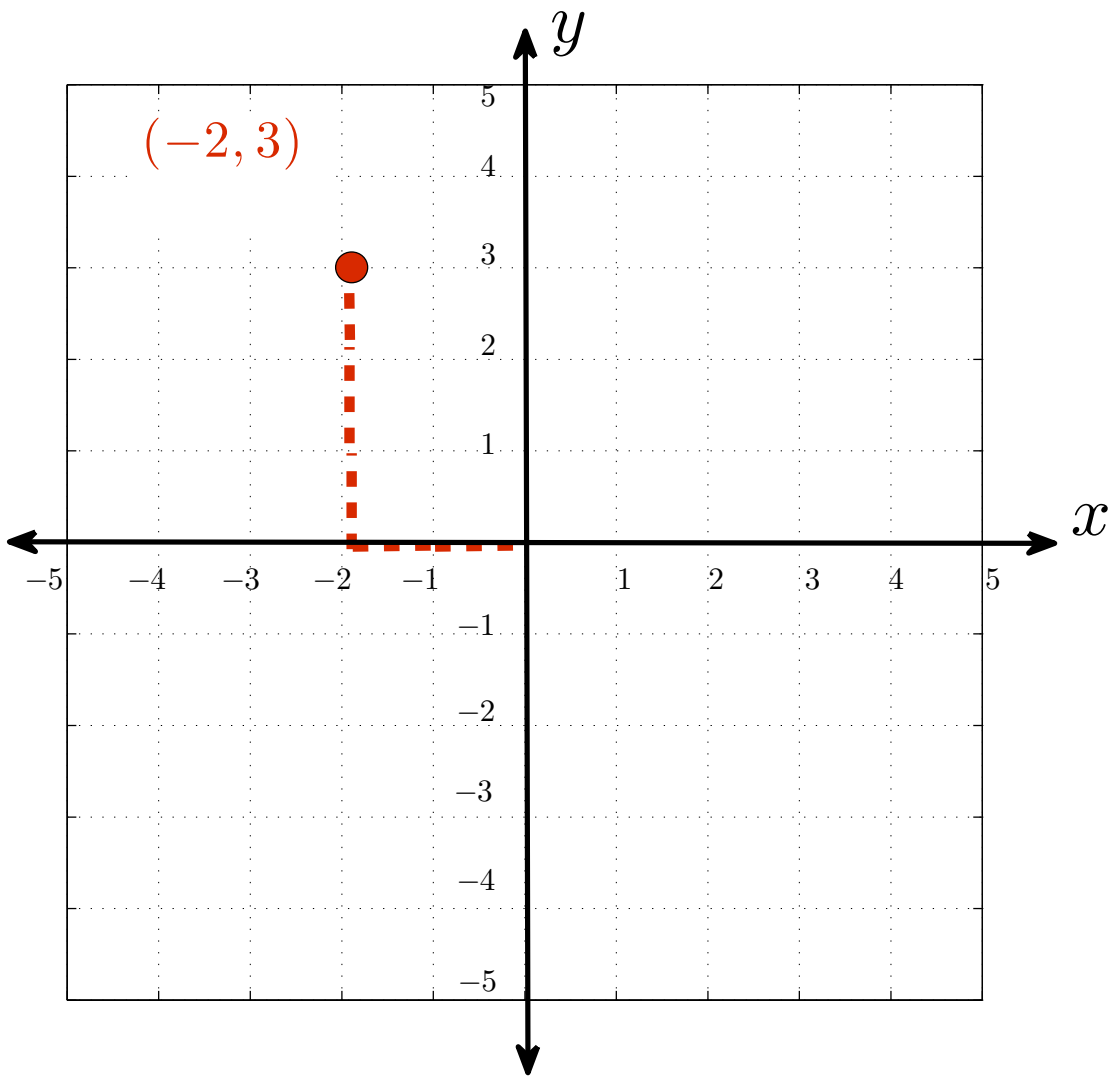

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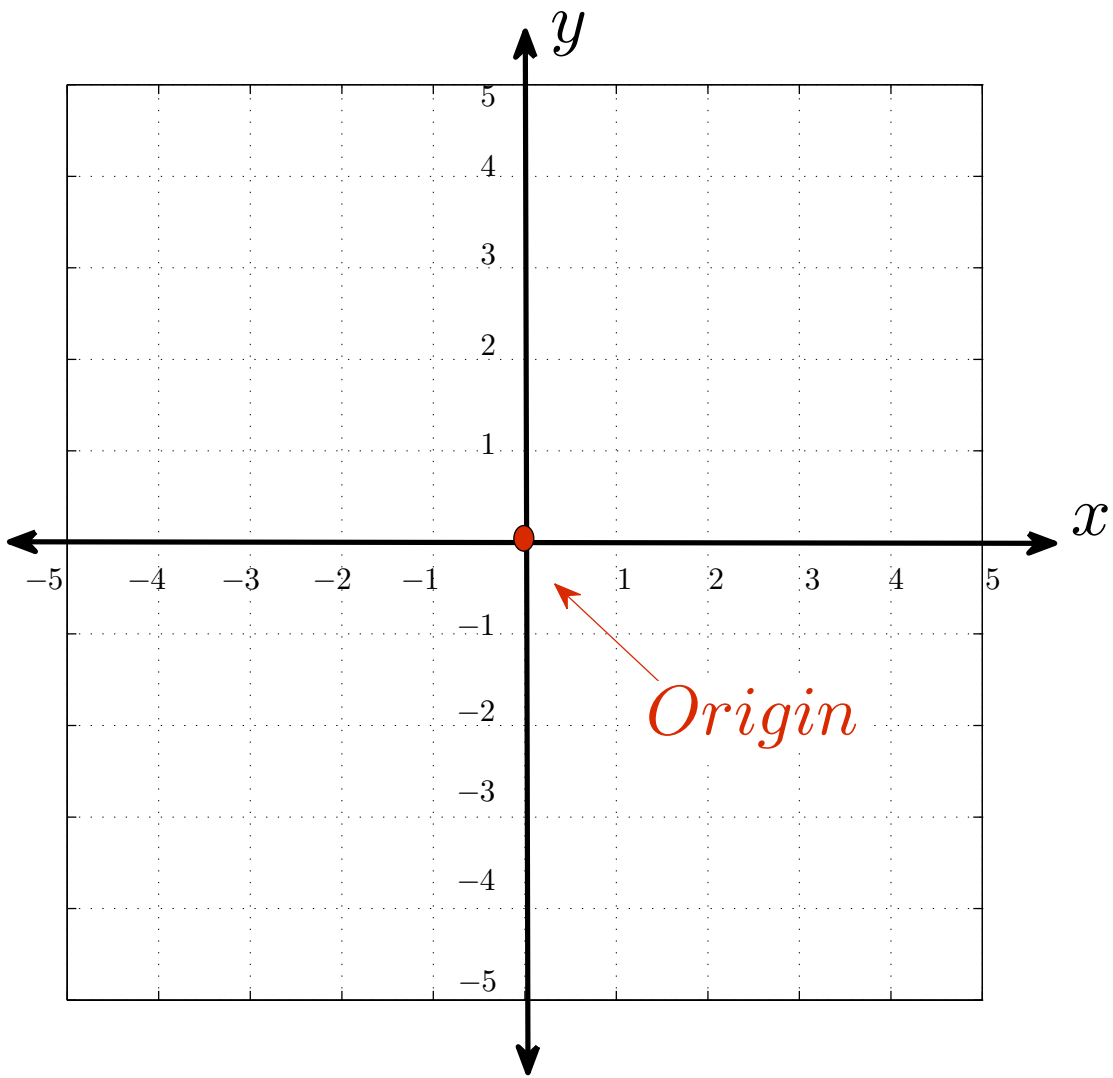

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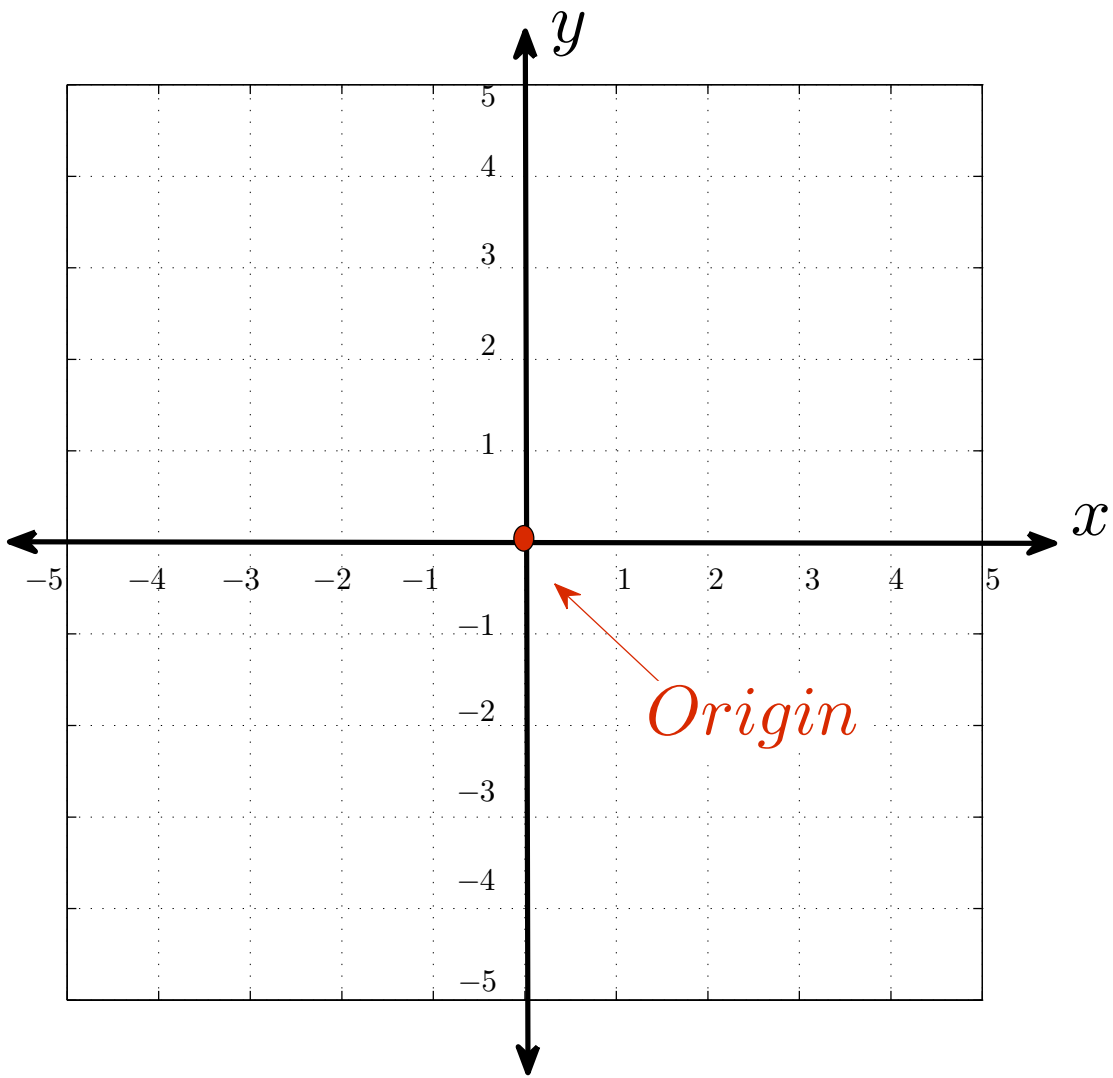

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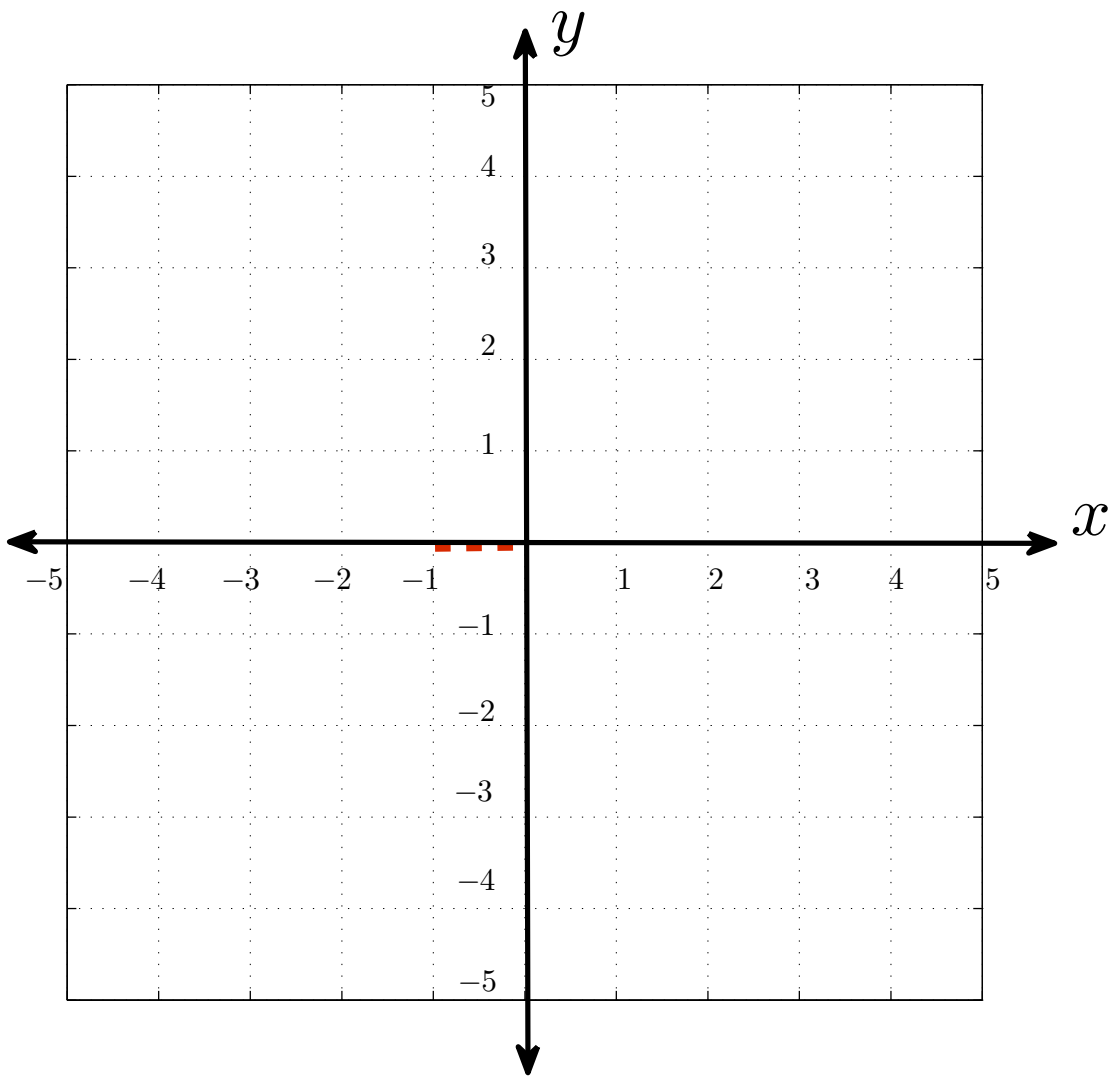

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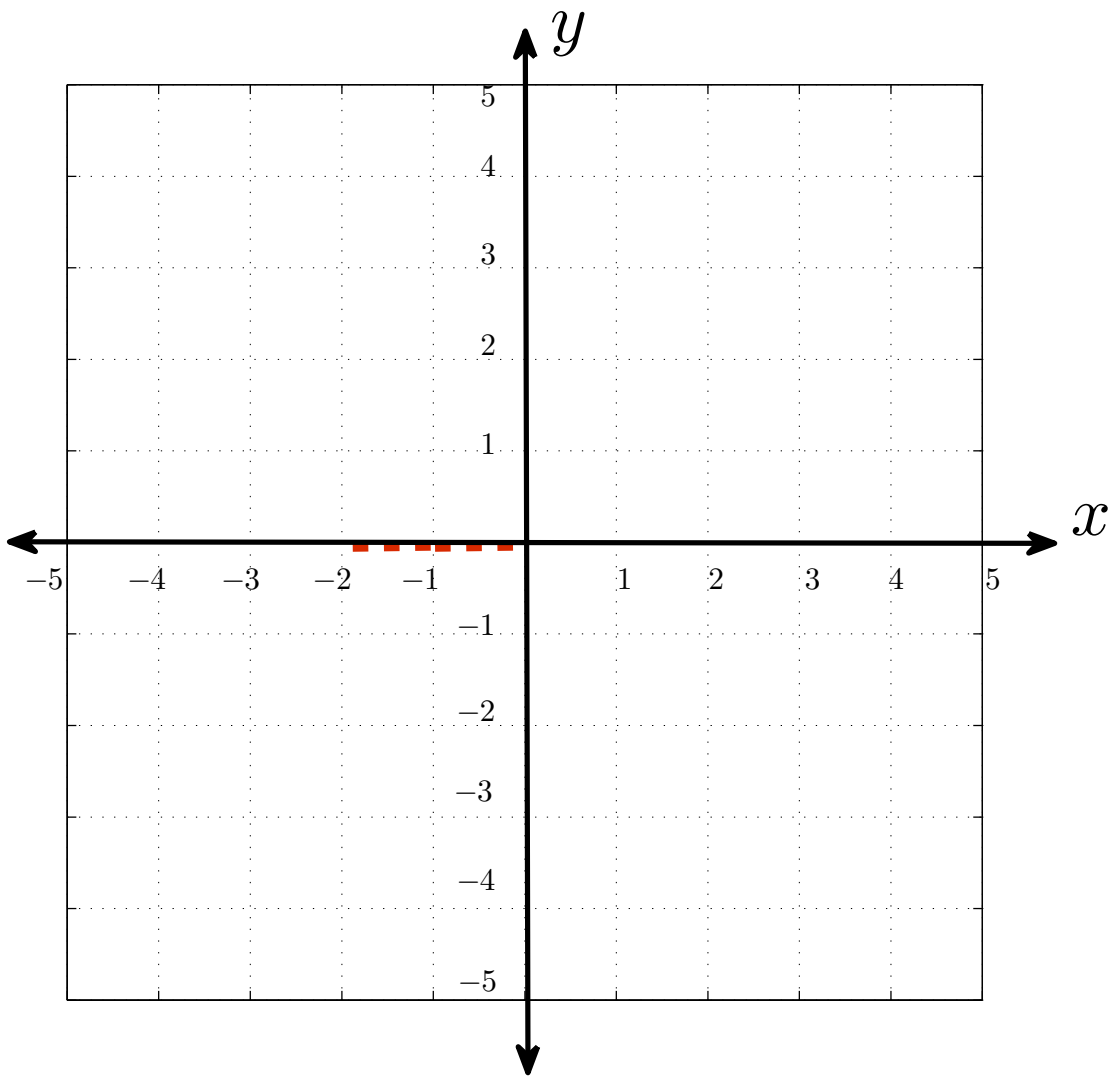

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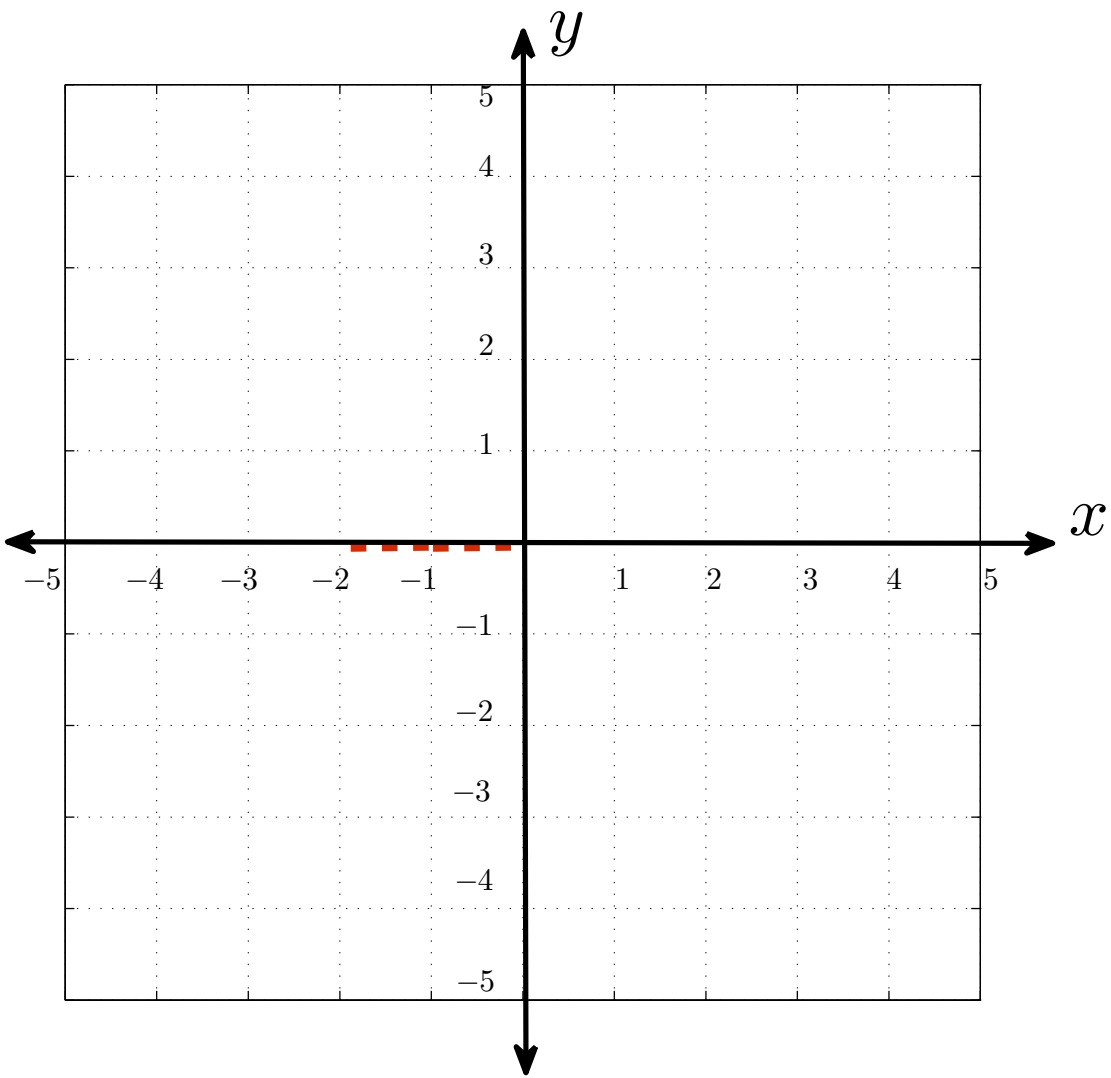

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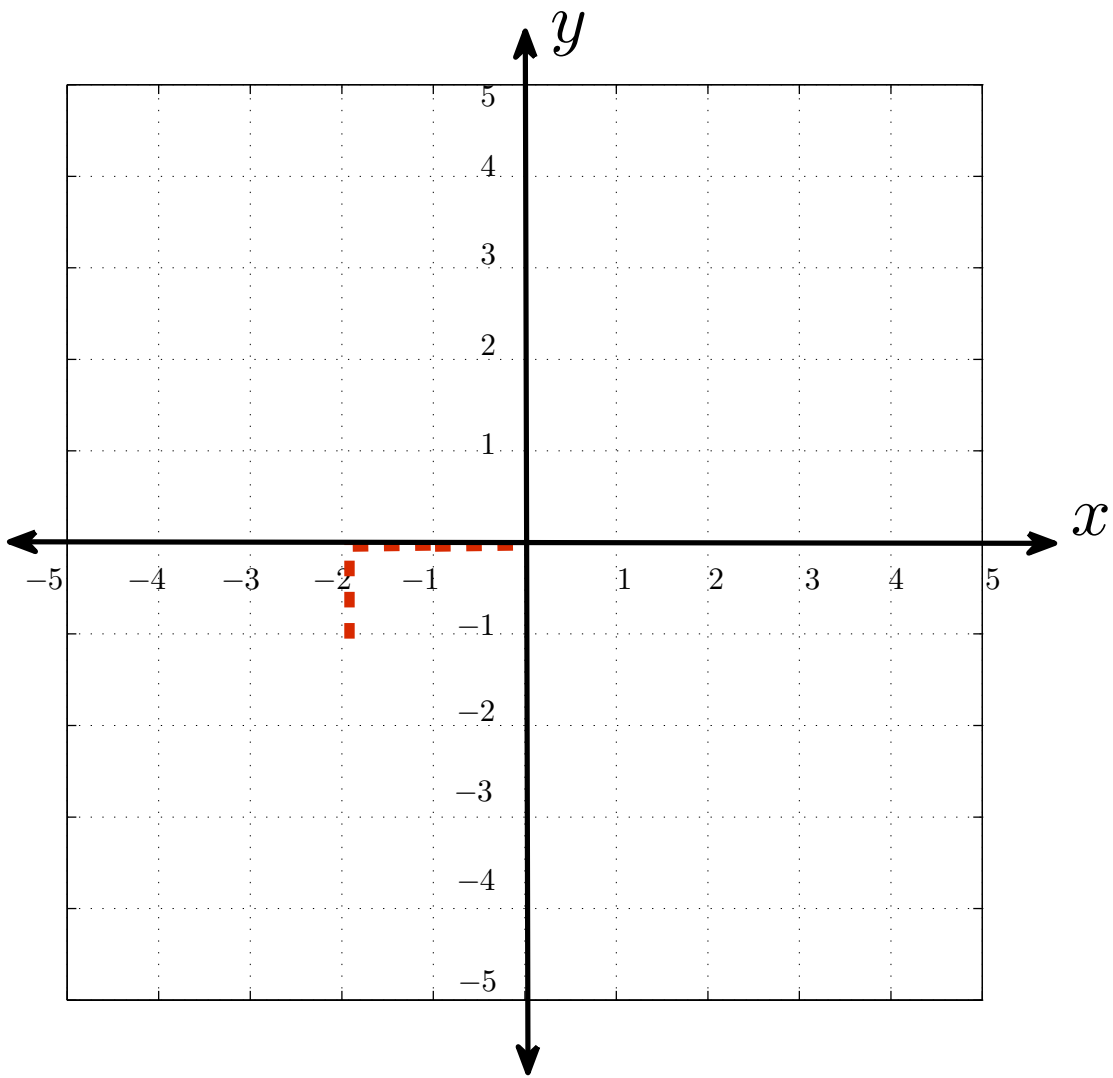

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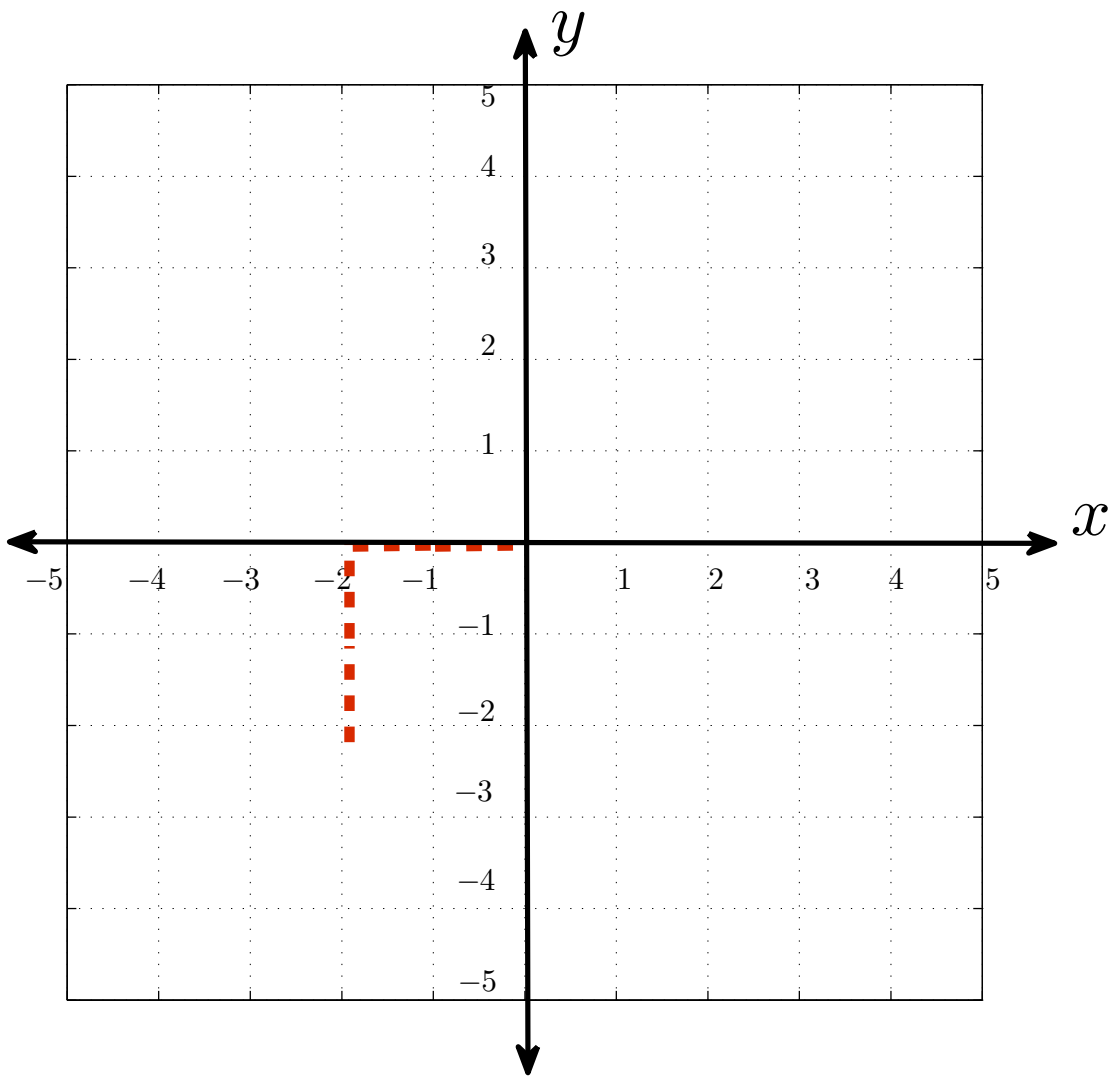

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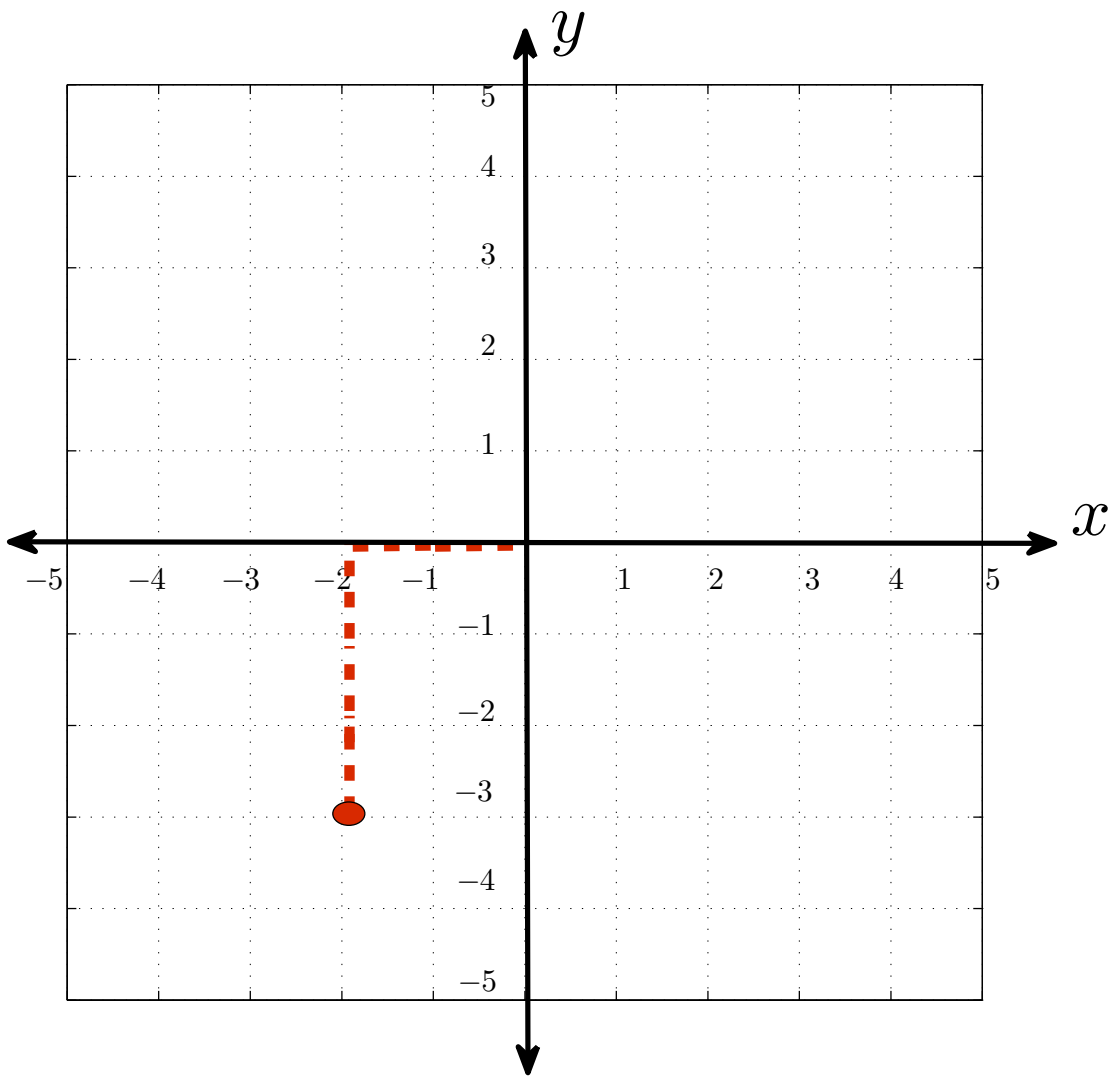

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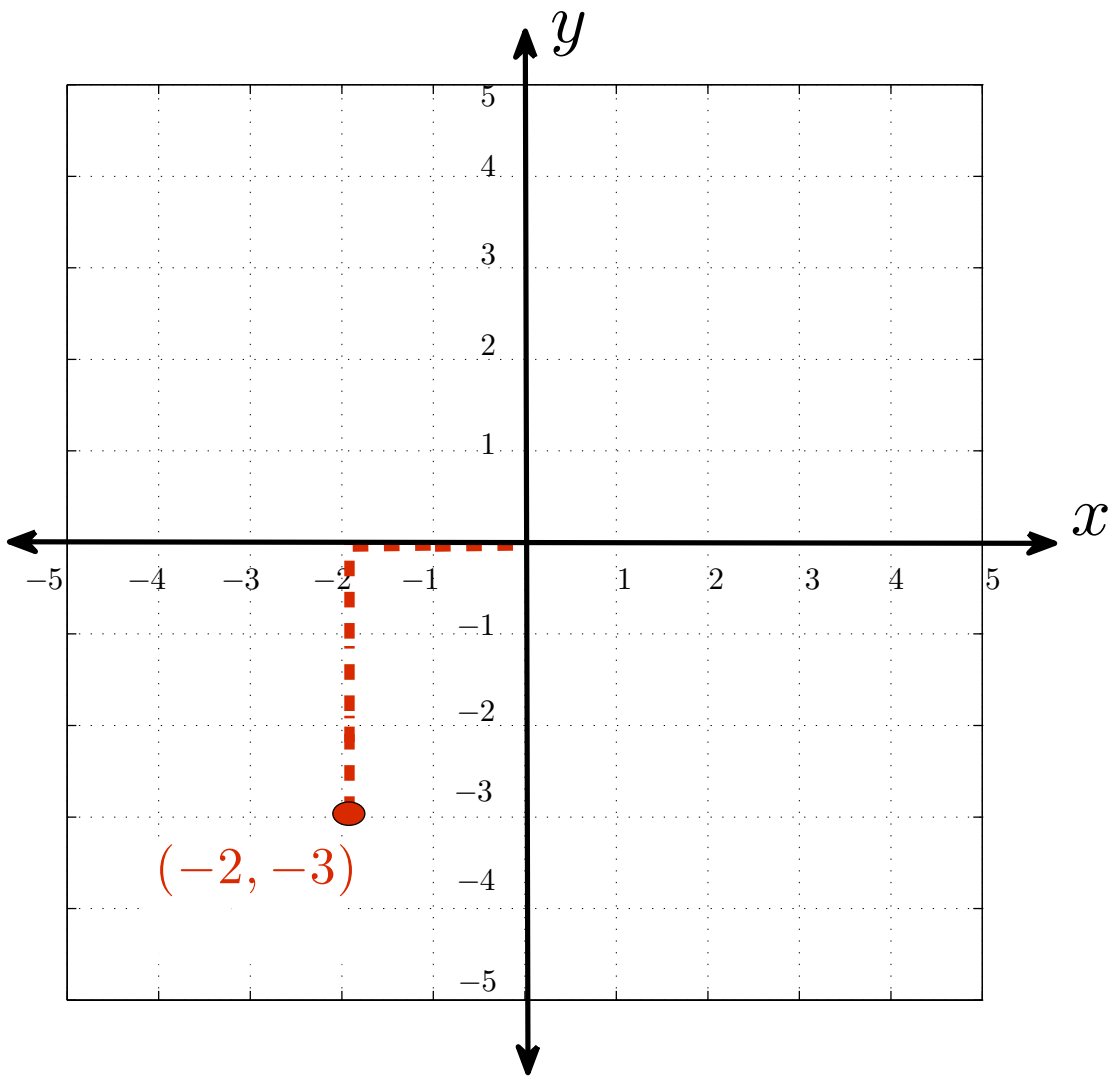

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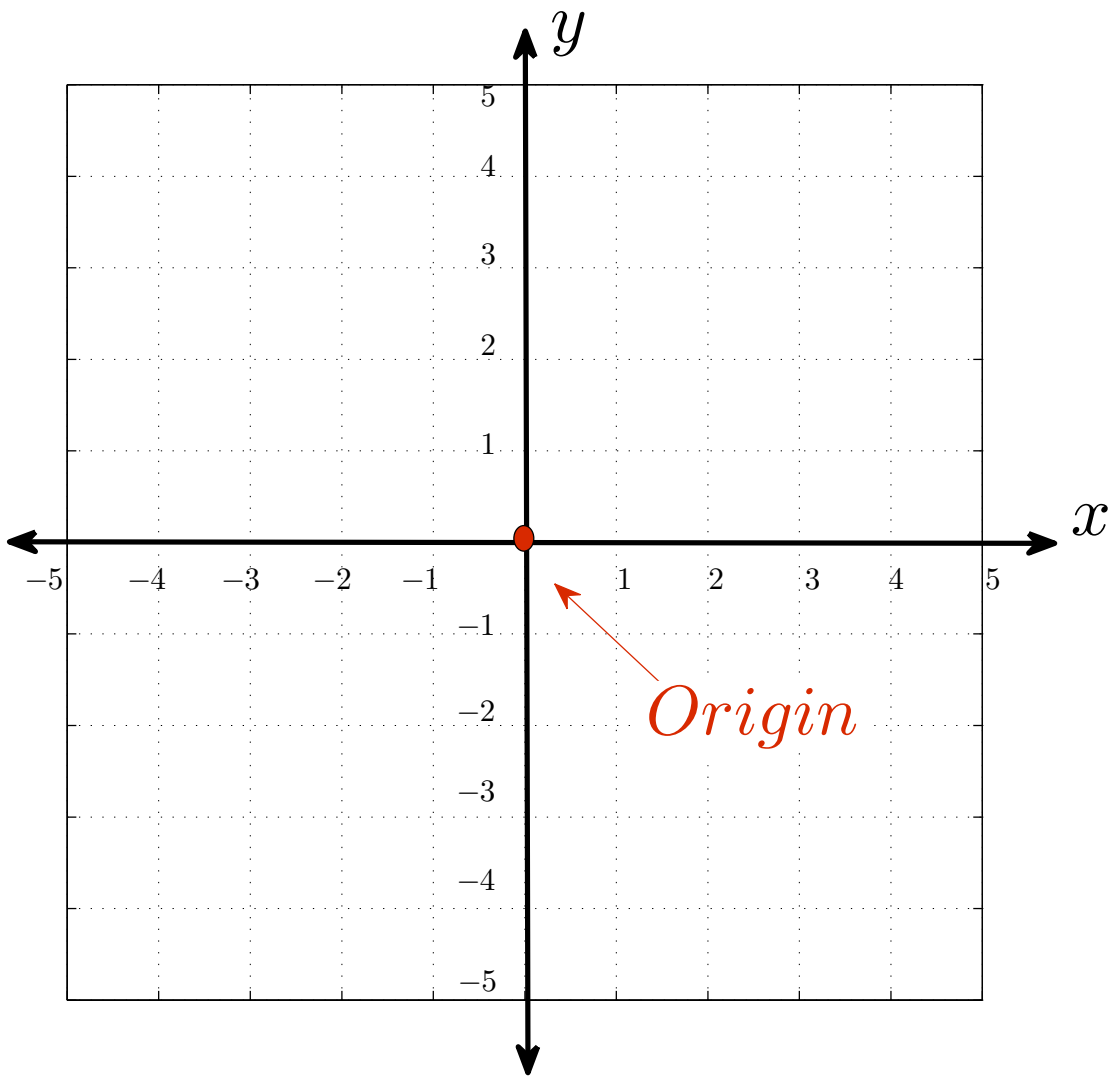

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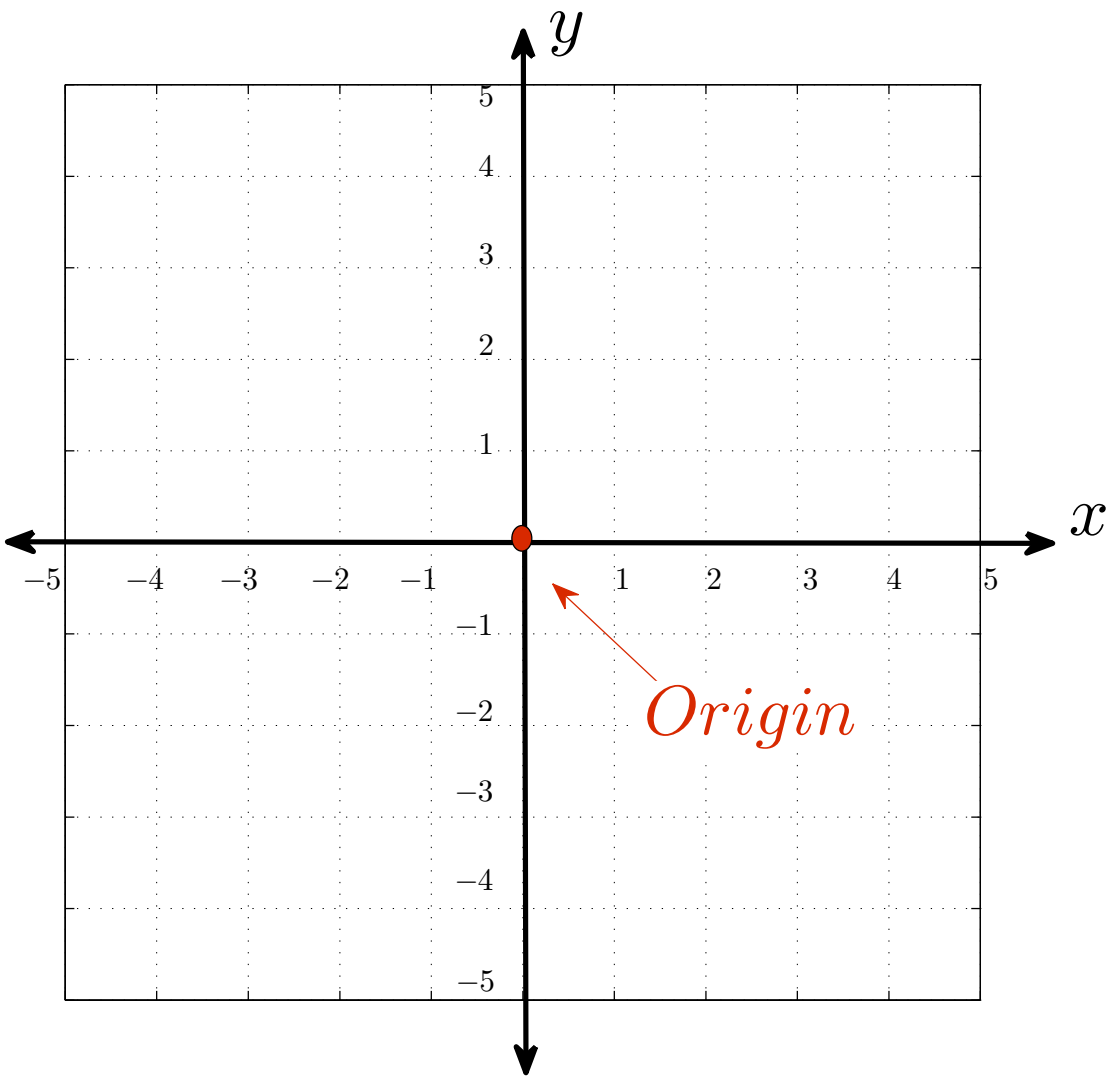

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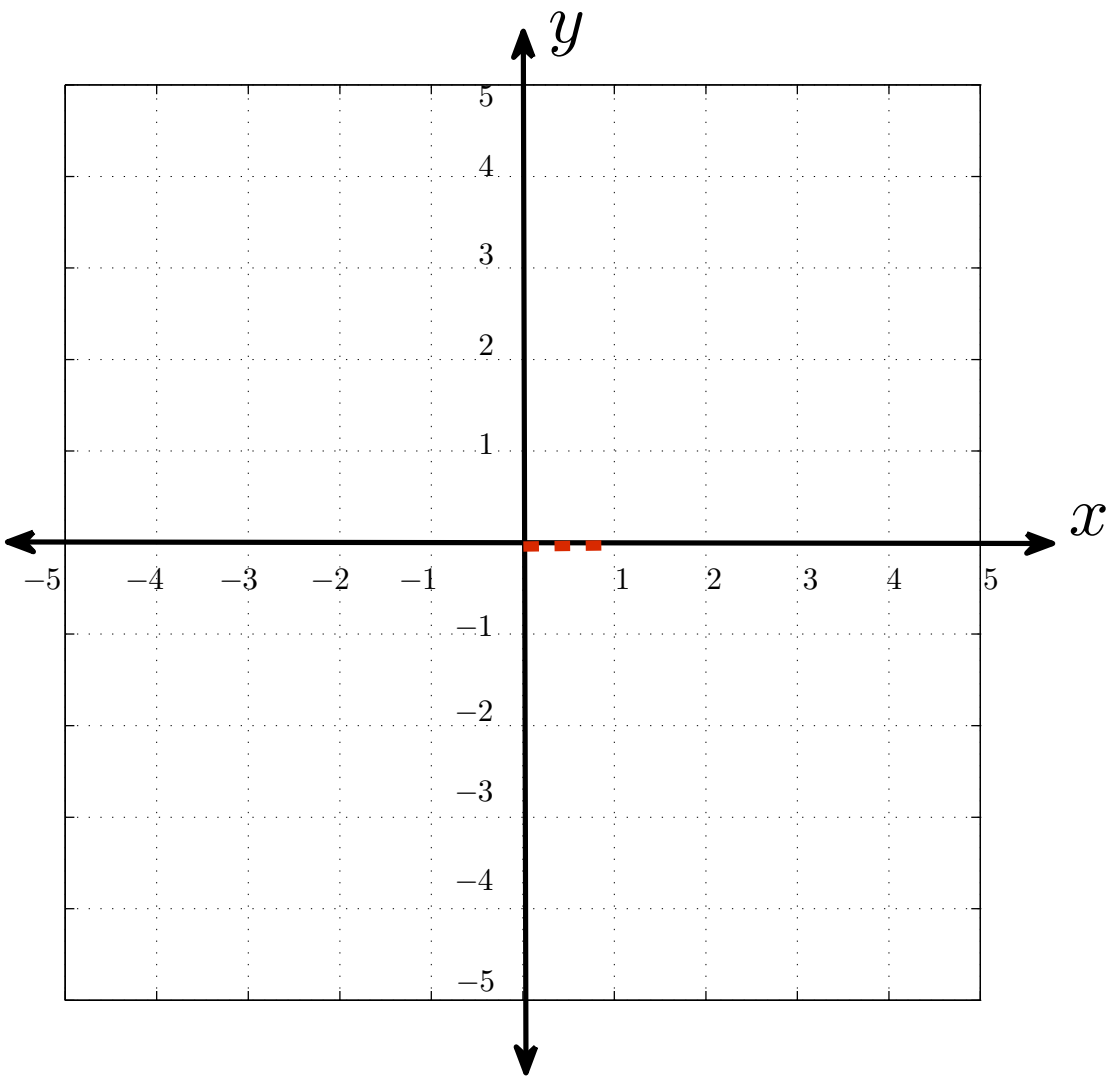

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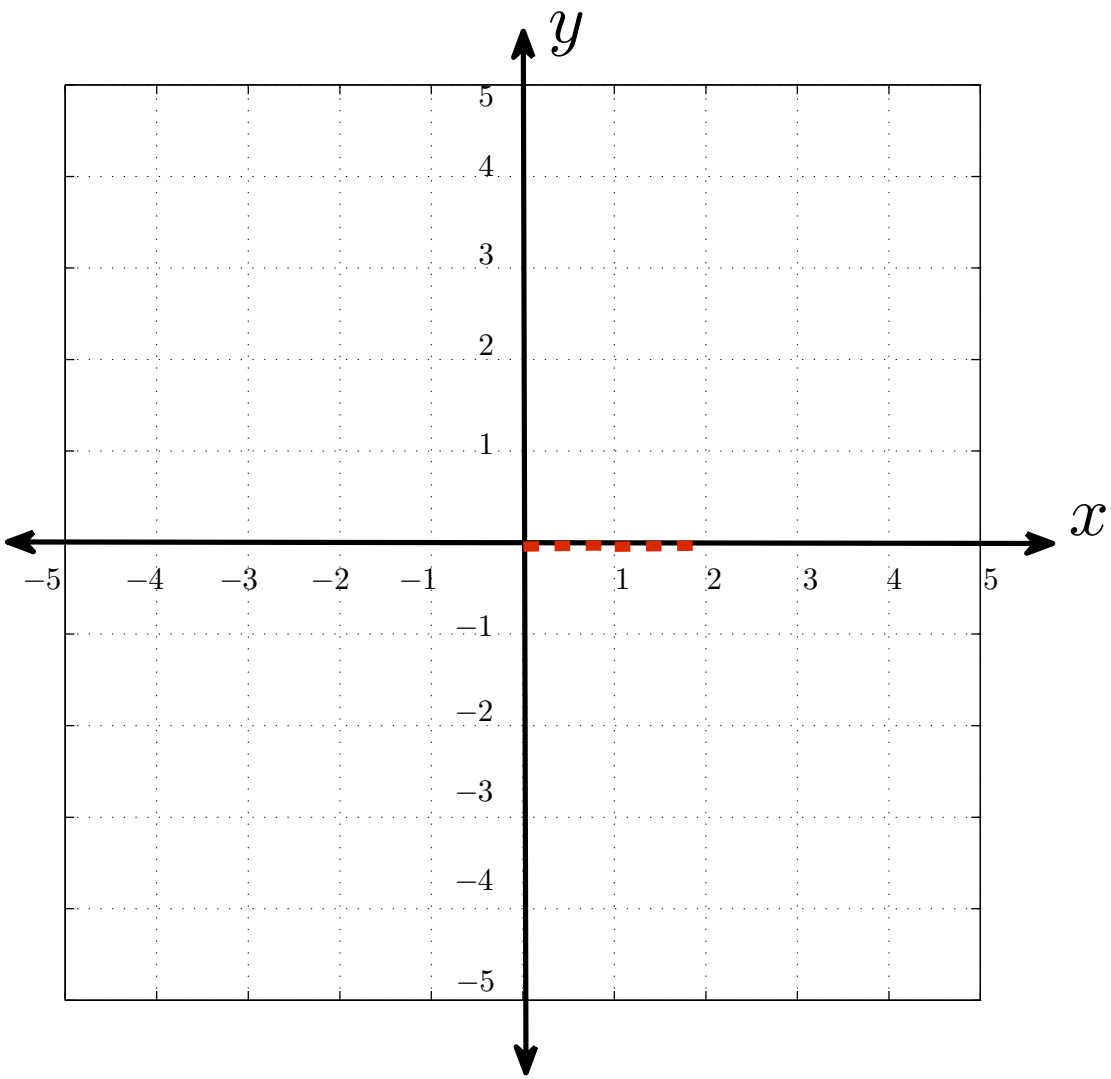

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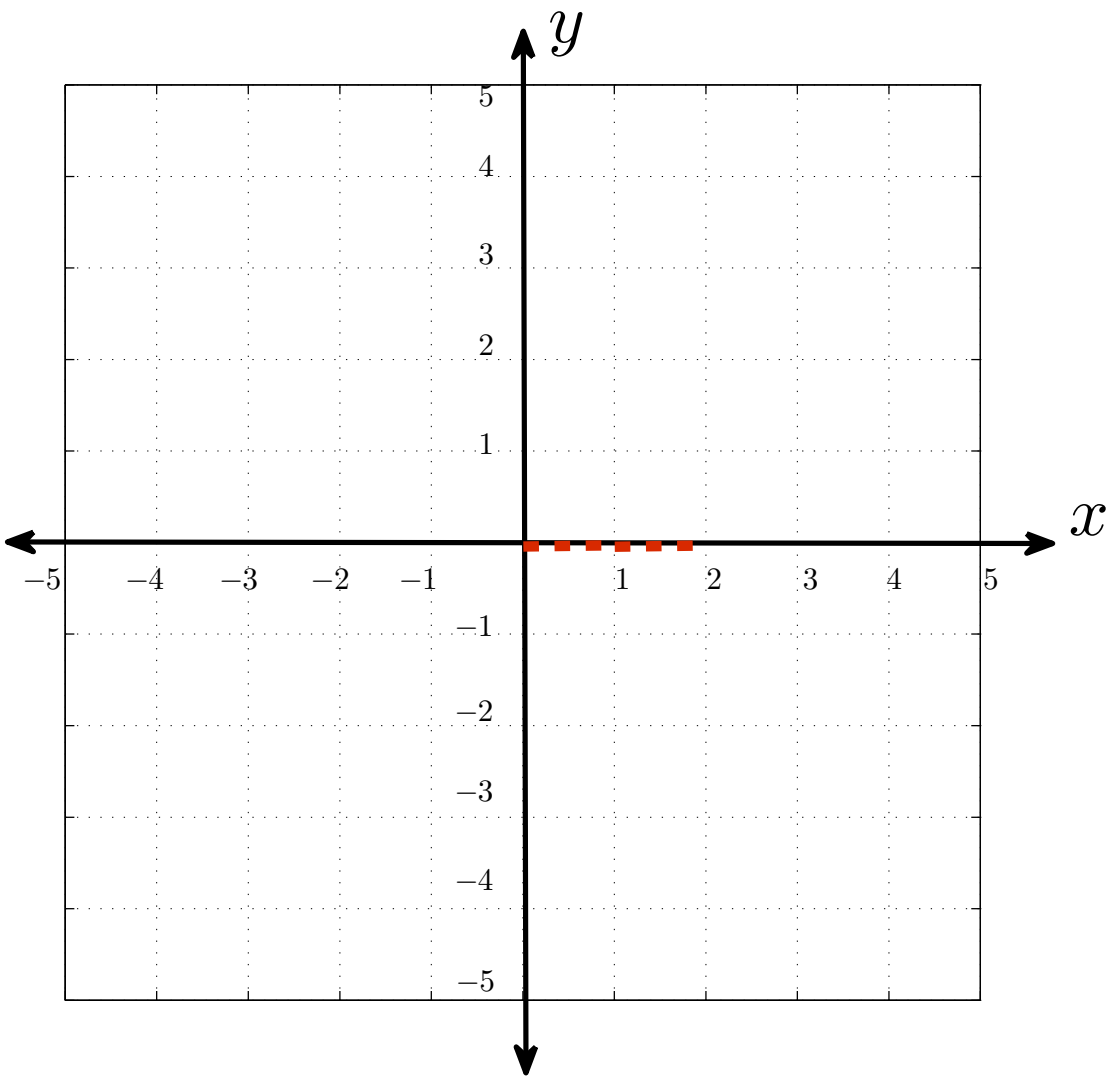

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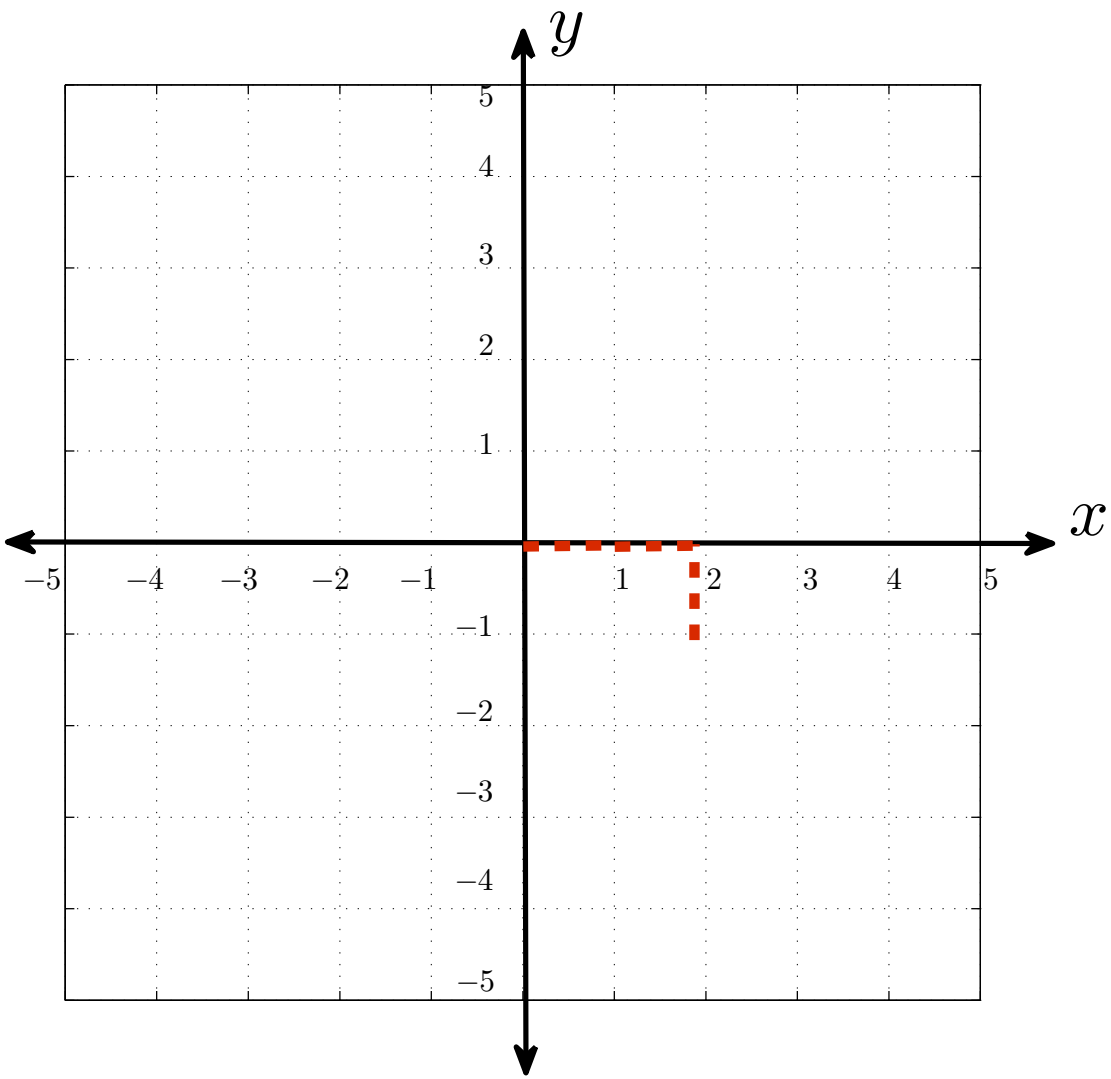

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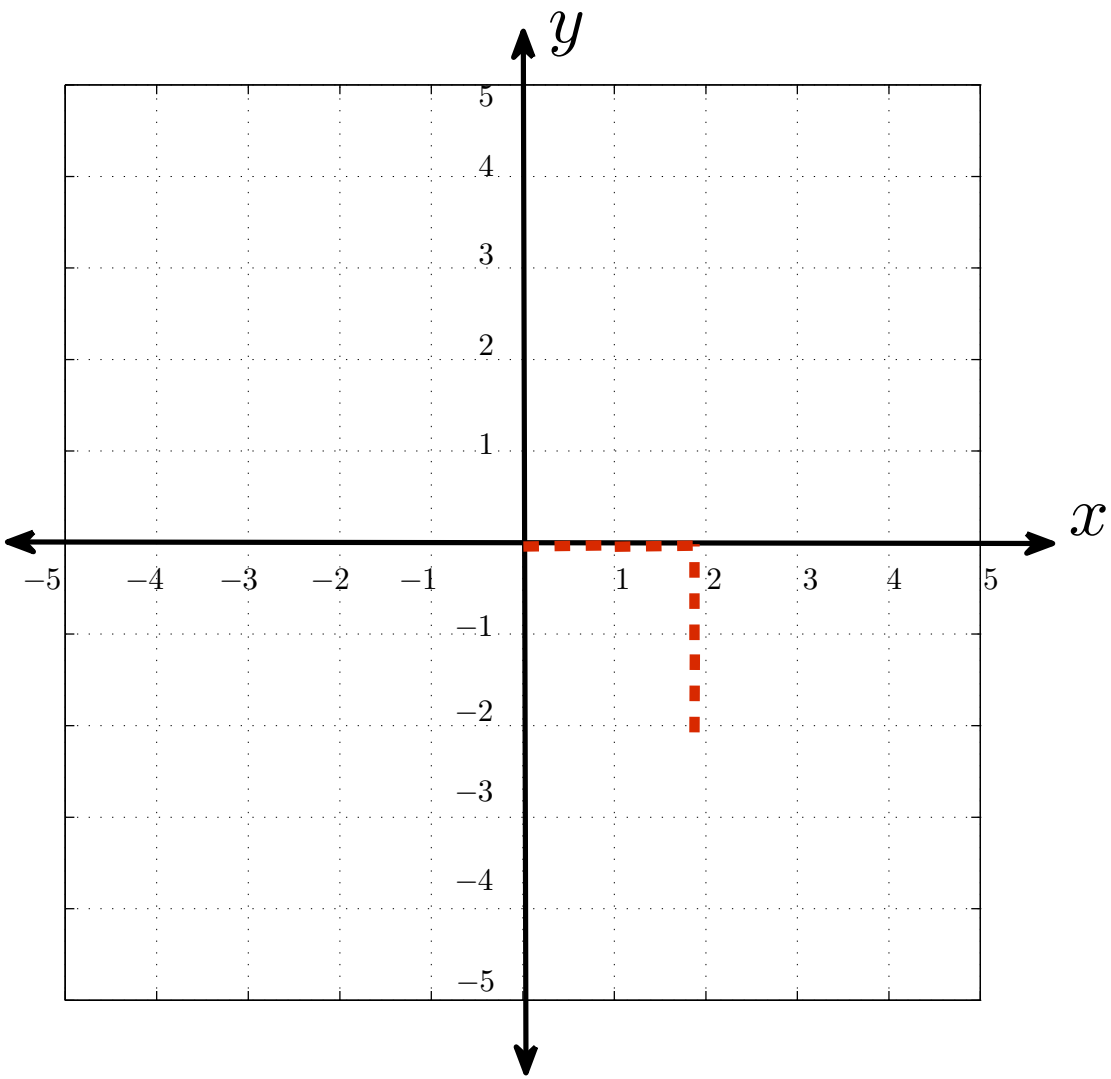

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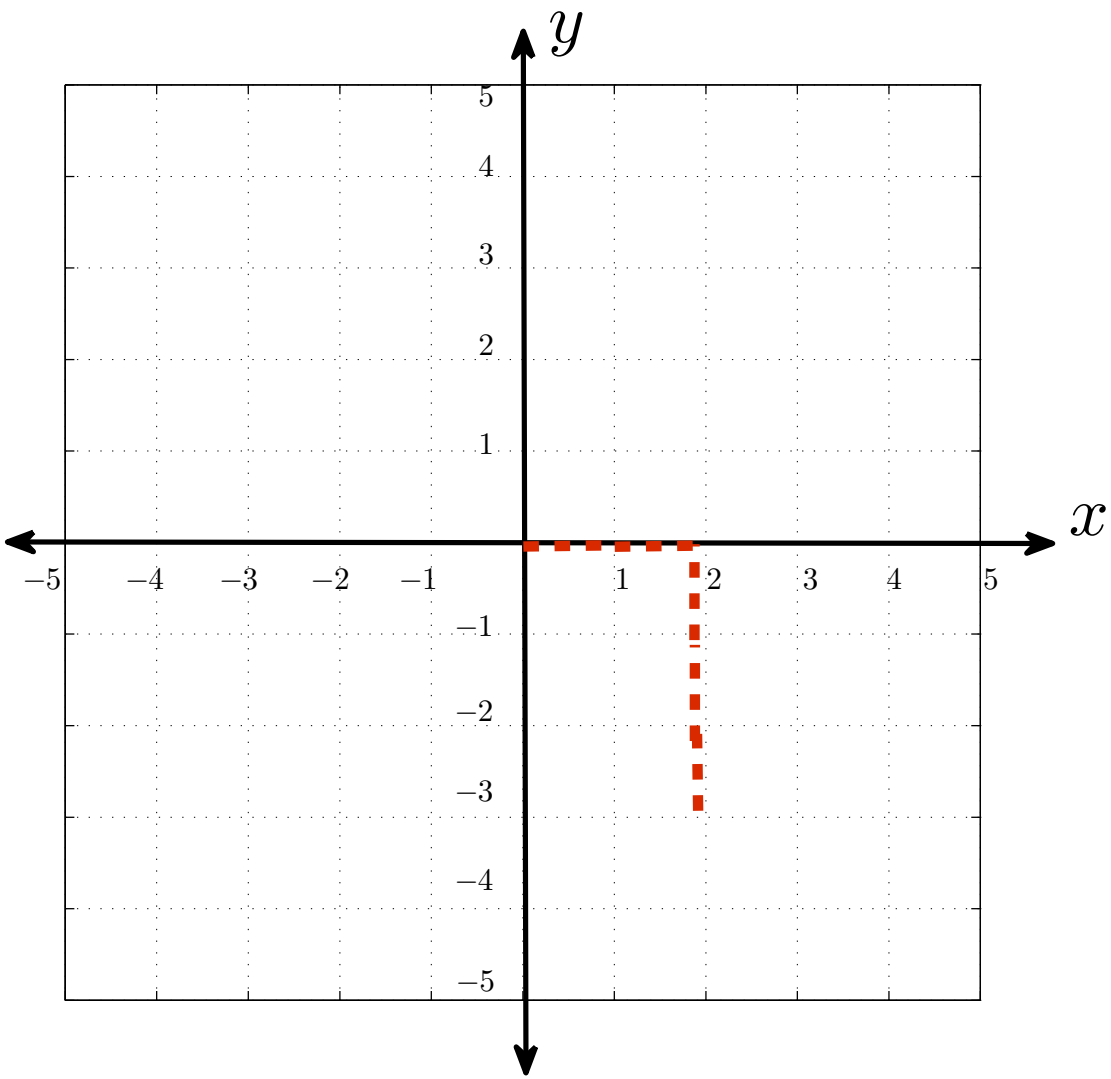

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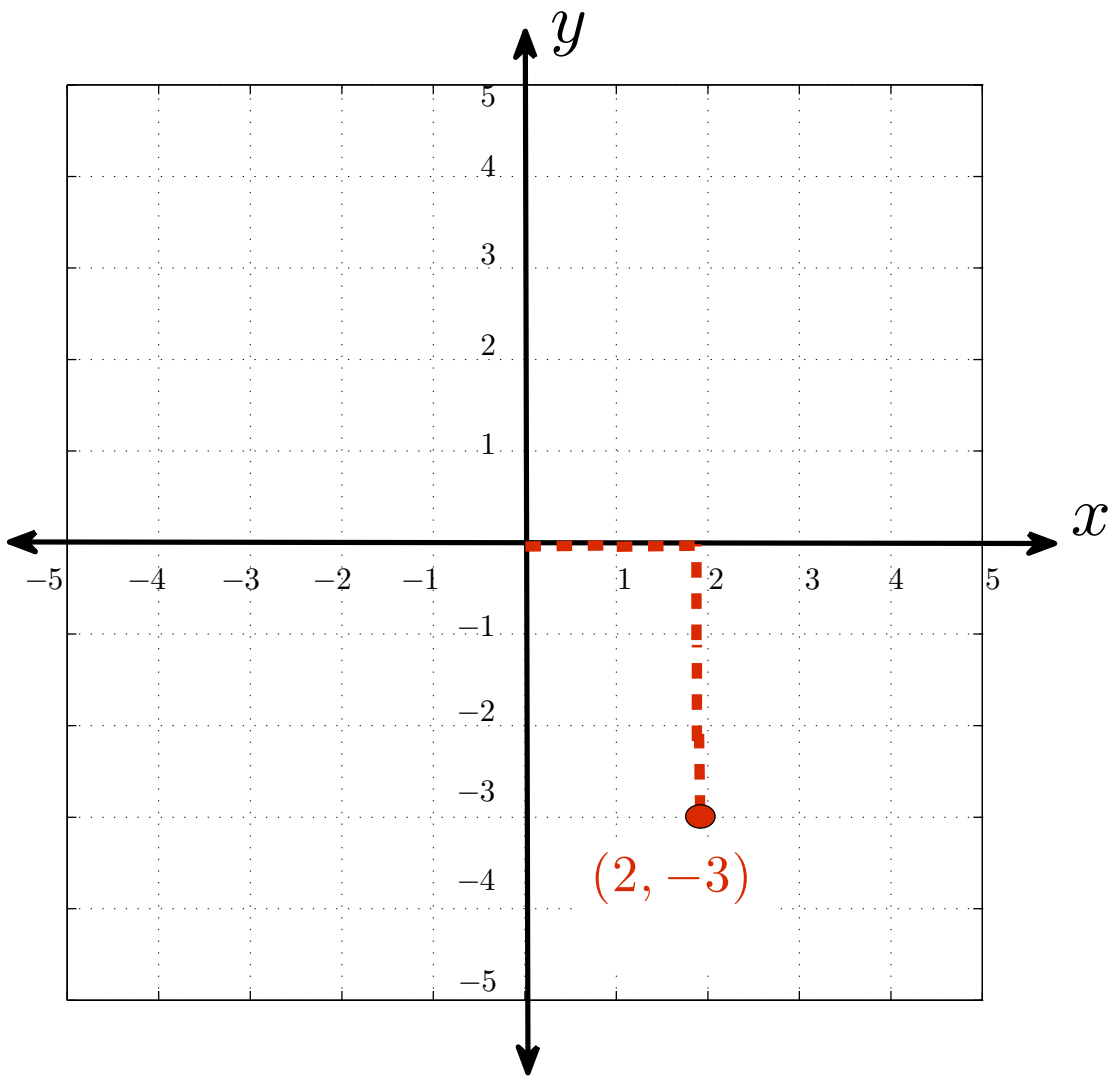
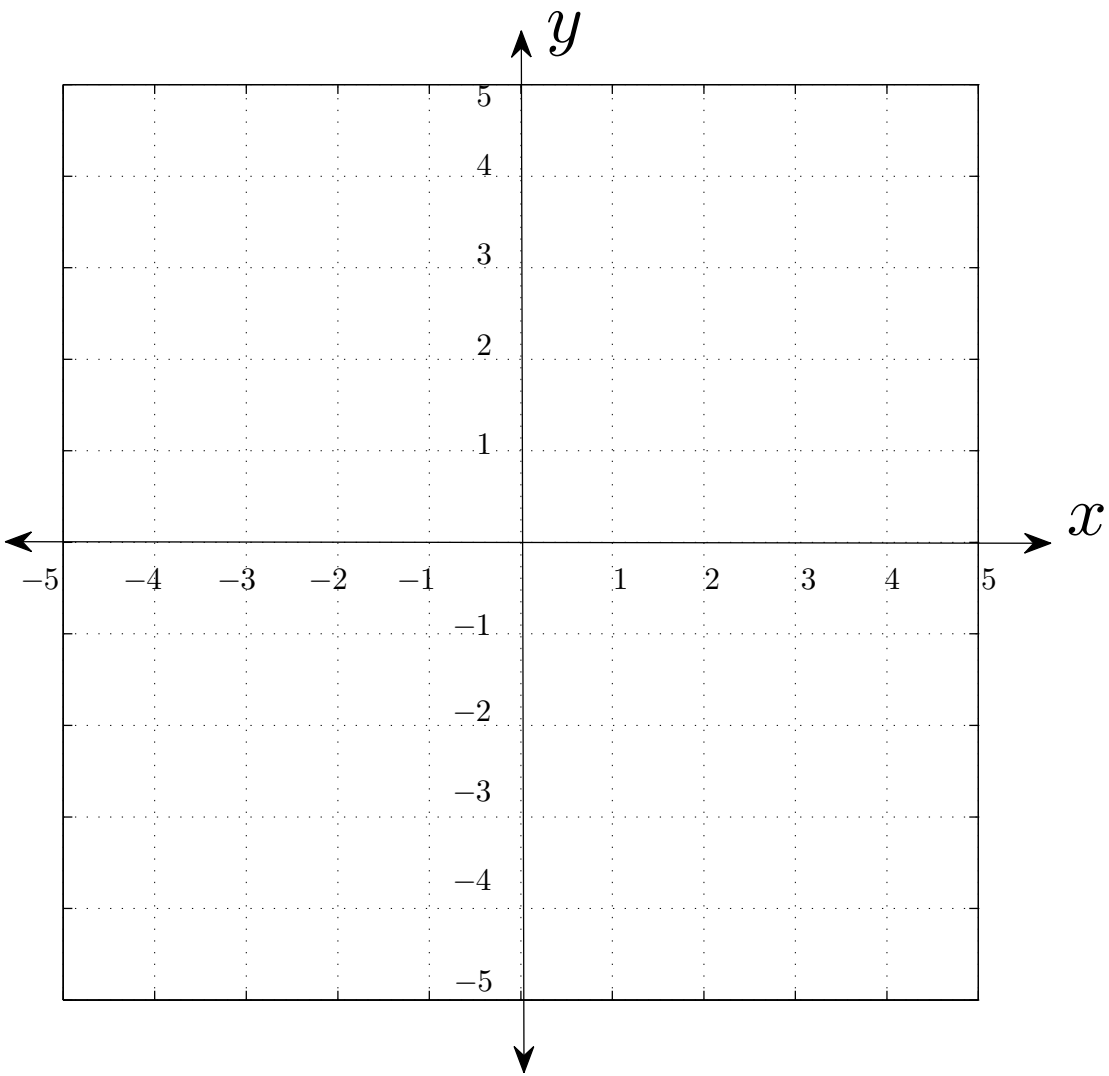
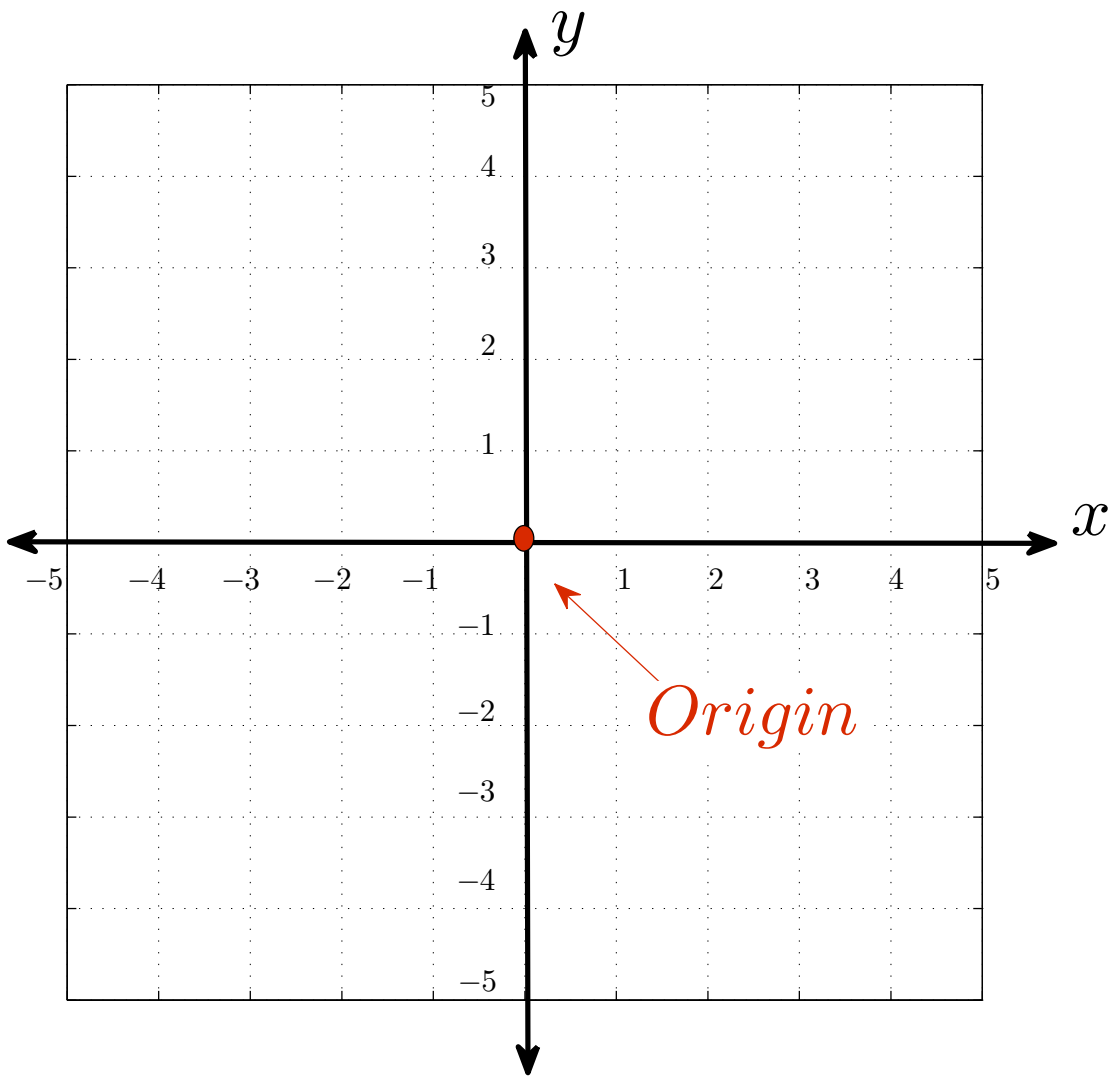


Graphing Ordered Pairs

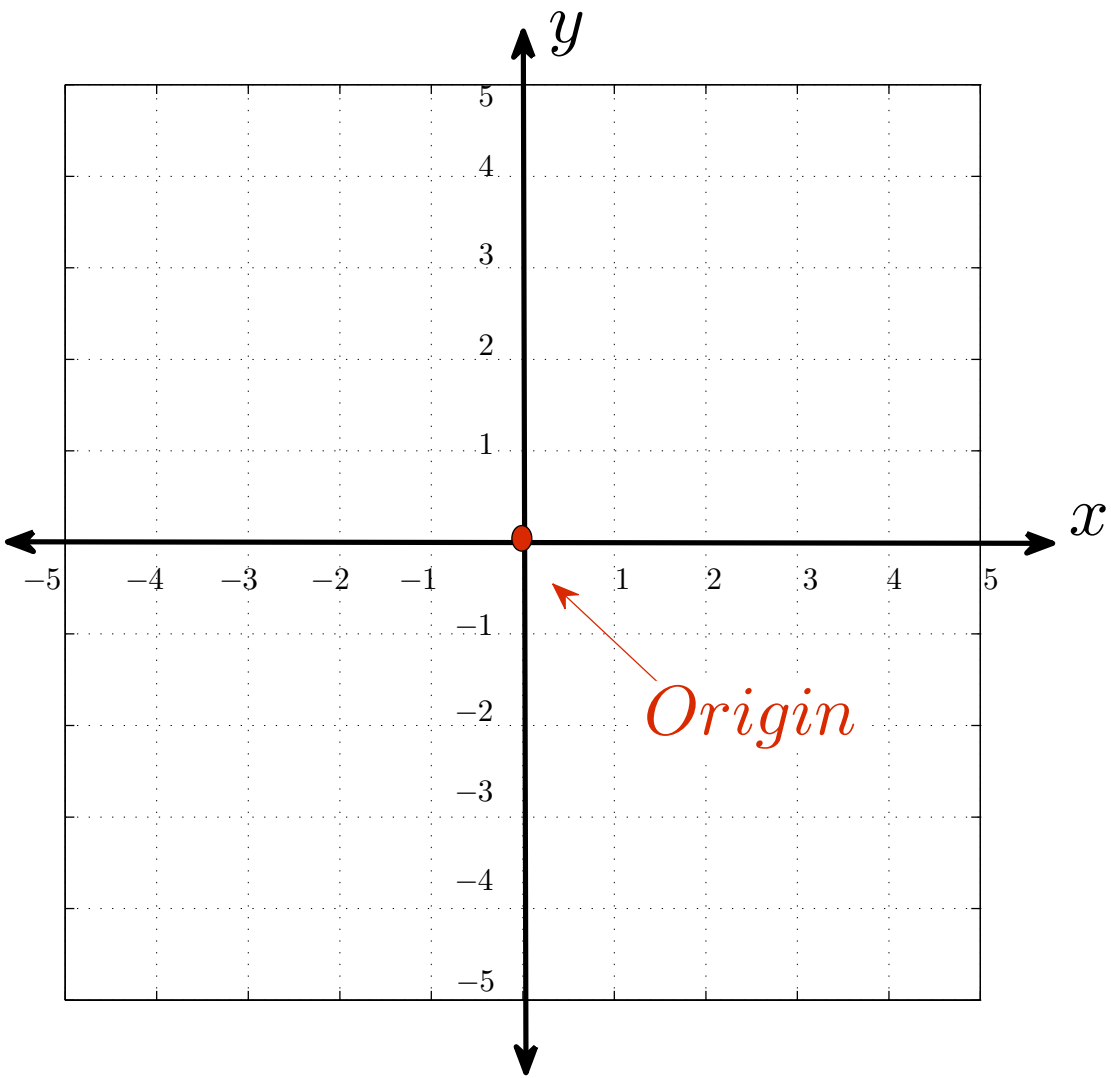
Example 2: Plot (graph) the following ordered pairs: $(3,0),\ (0,2),\ (-3,0),\ (0,-2),$



To plot $(3,0)$, begin at the origin.



To plot $(3,0)$, begin at the origin. Travel along the x-axis 3 units right, in the positive direction.



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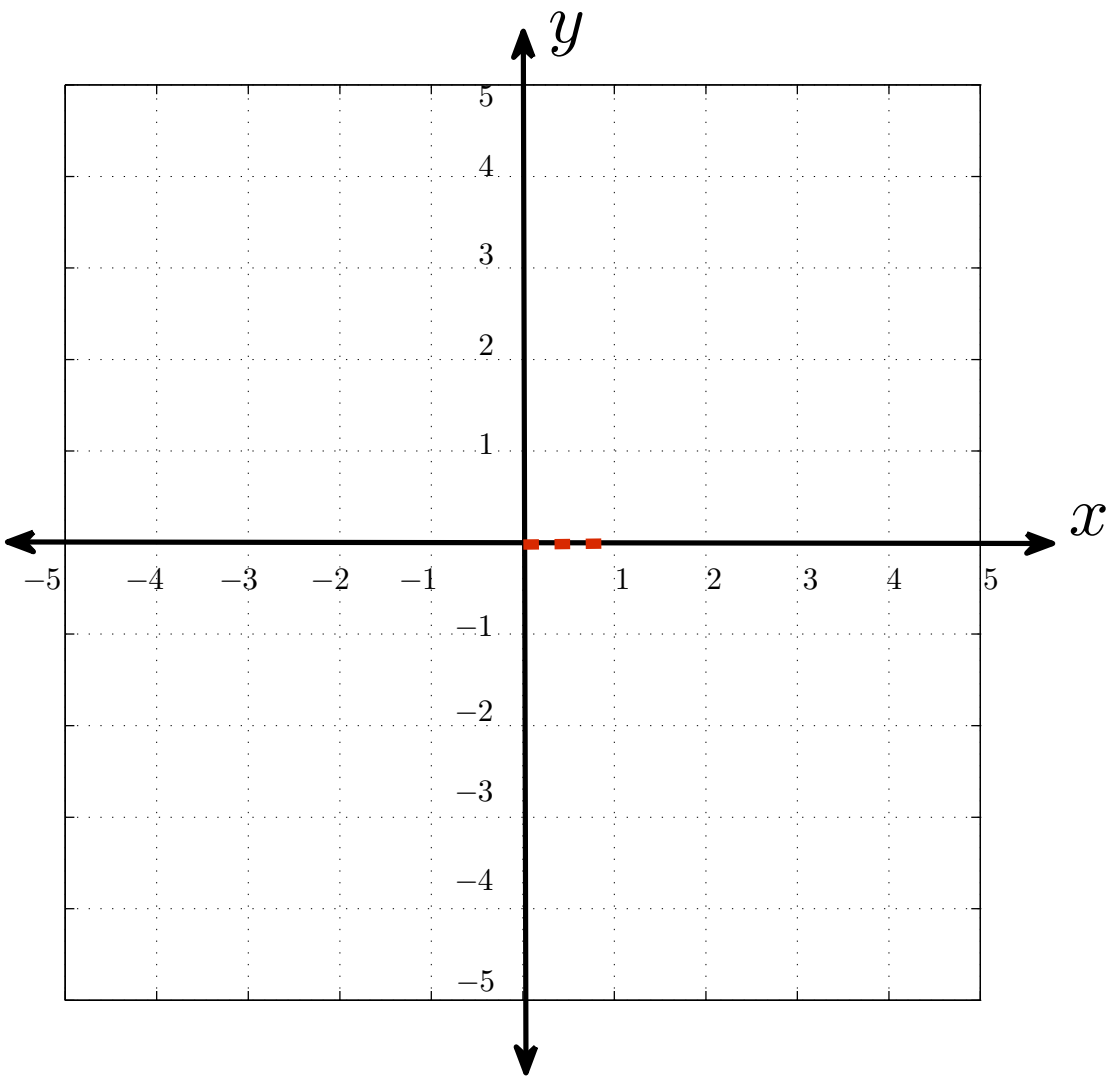

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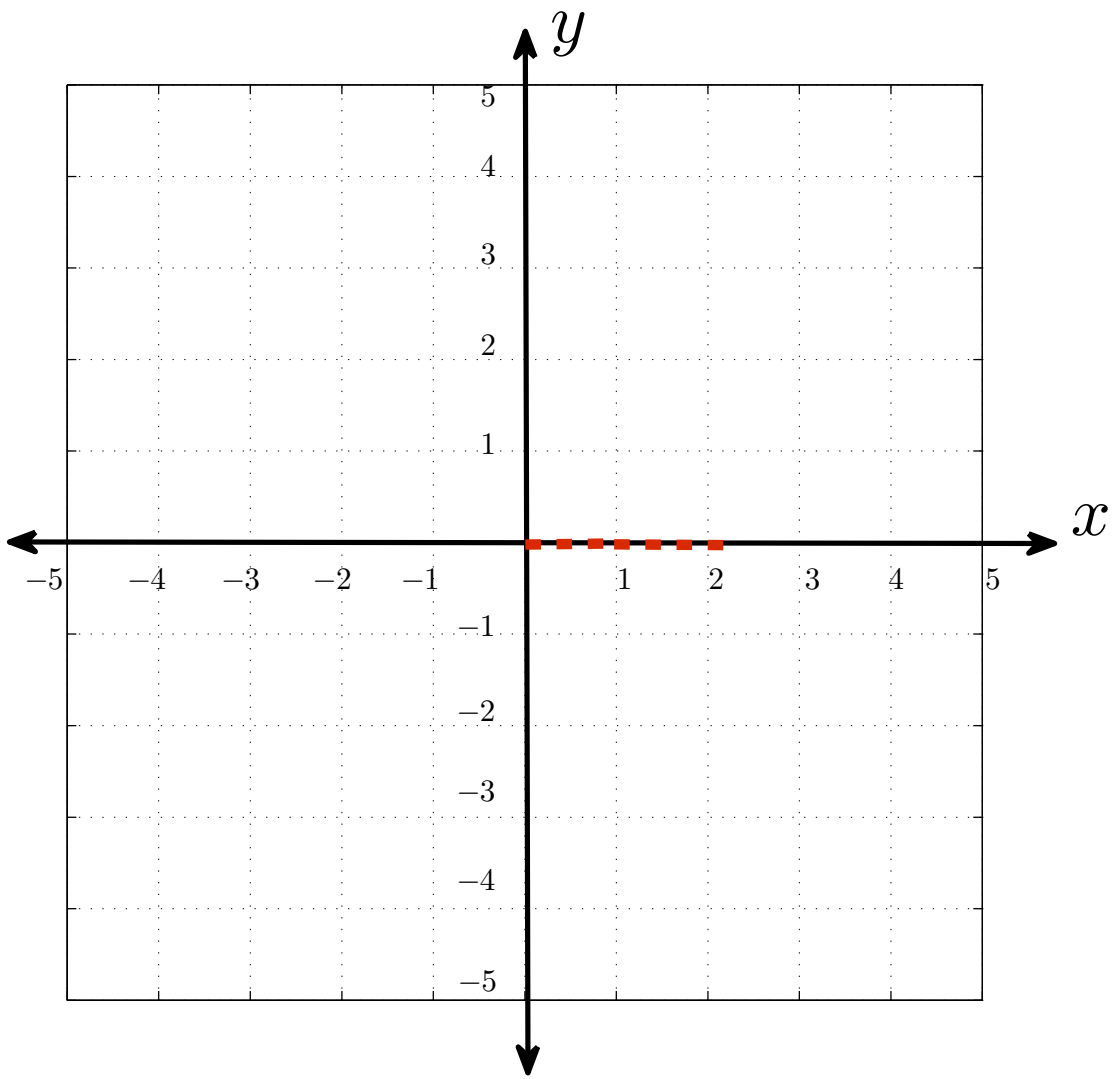

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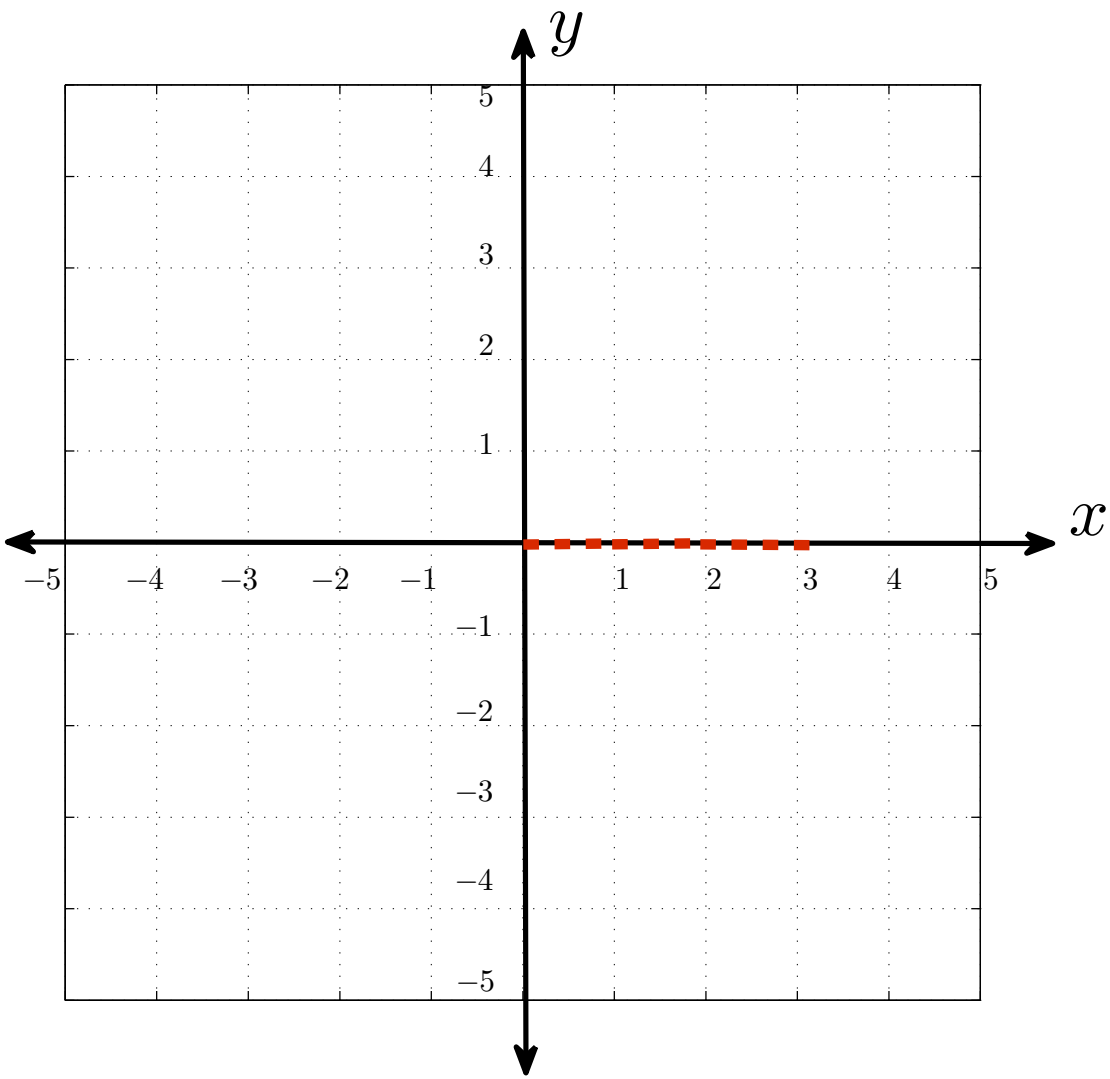

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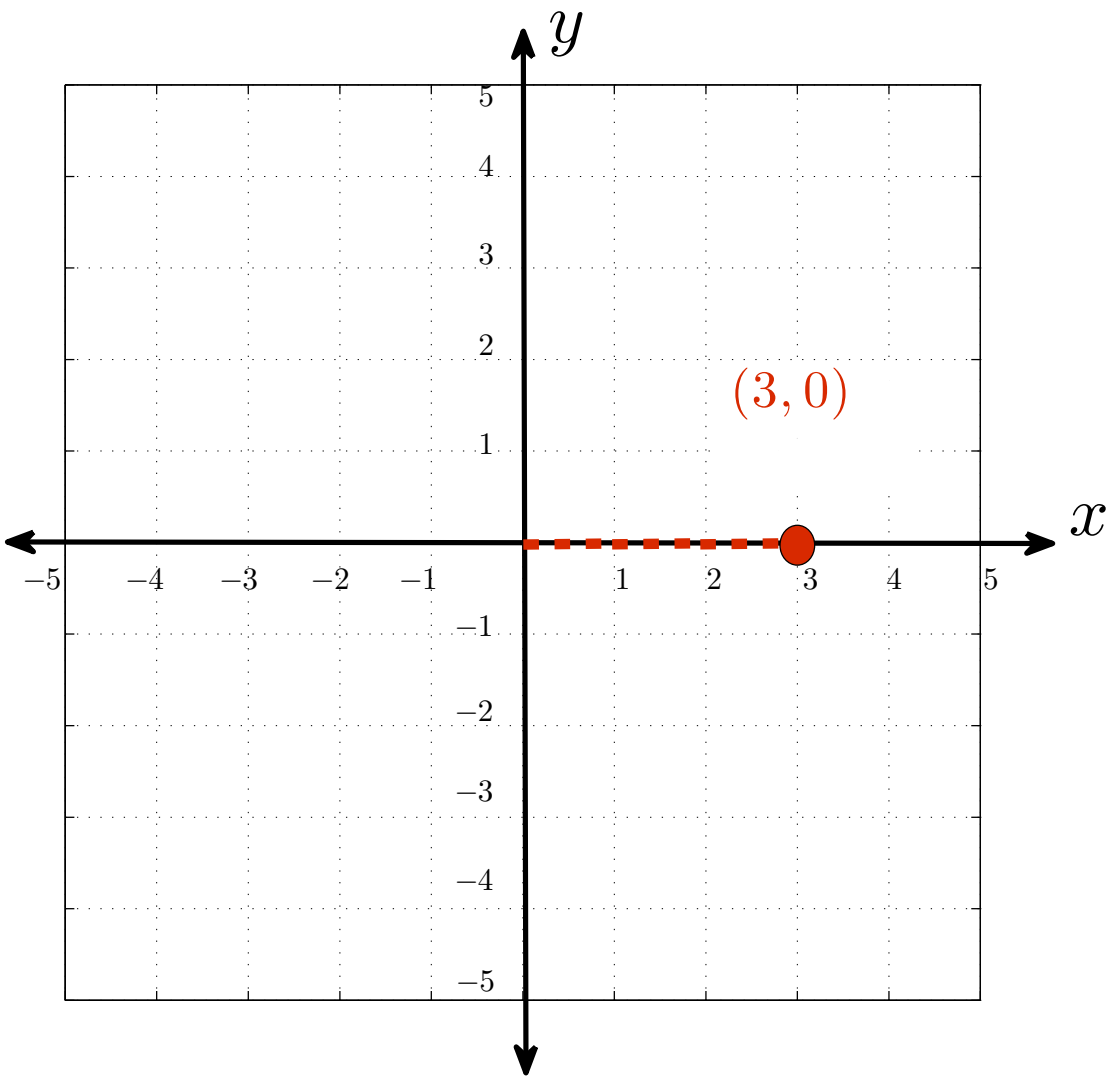

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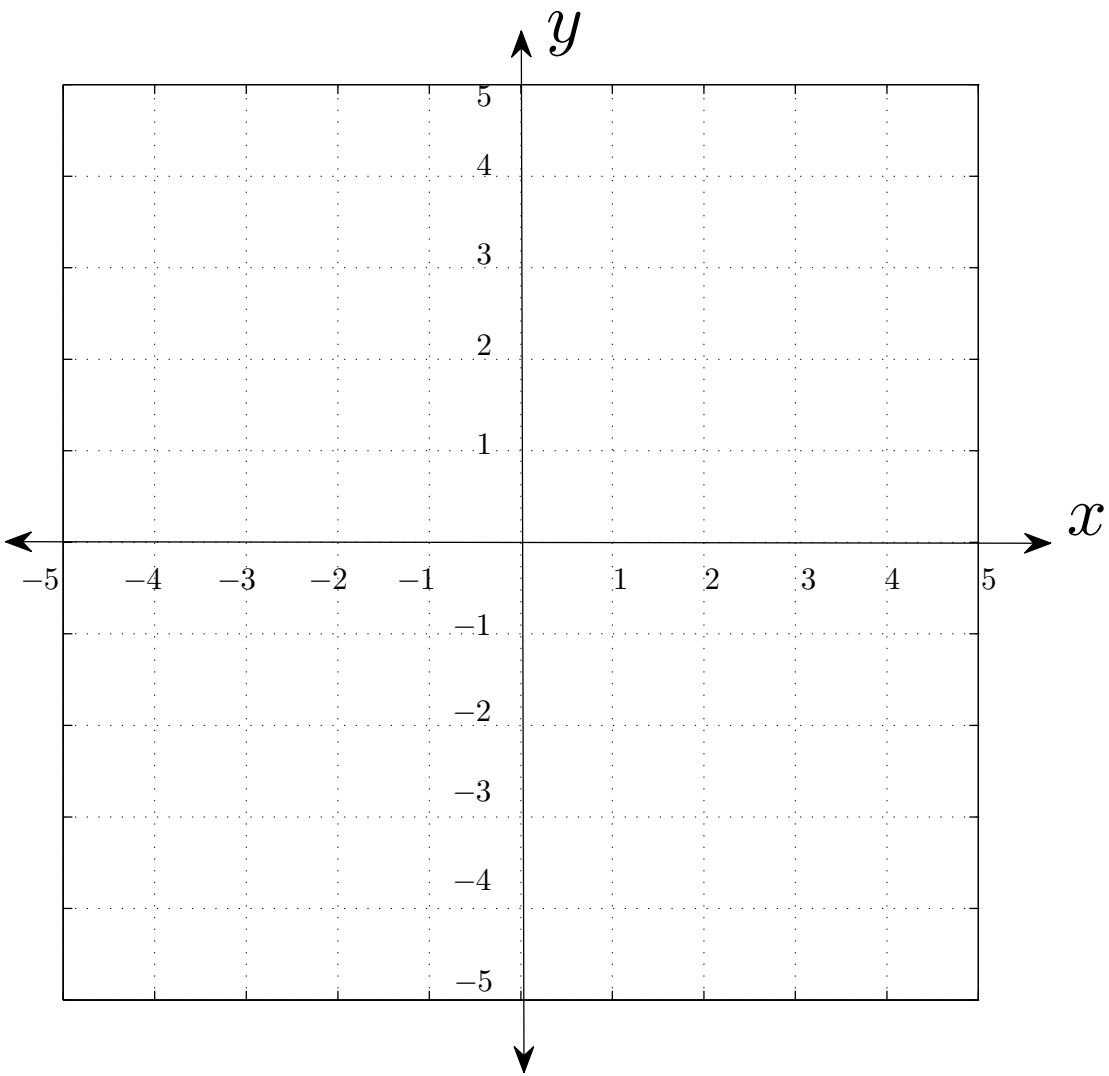
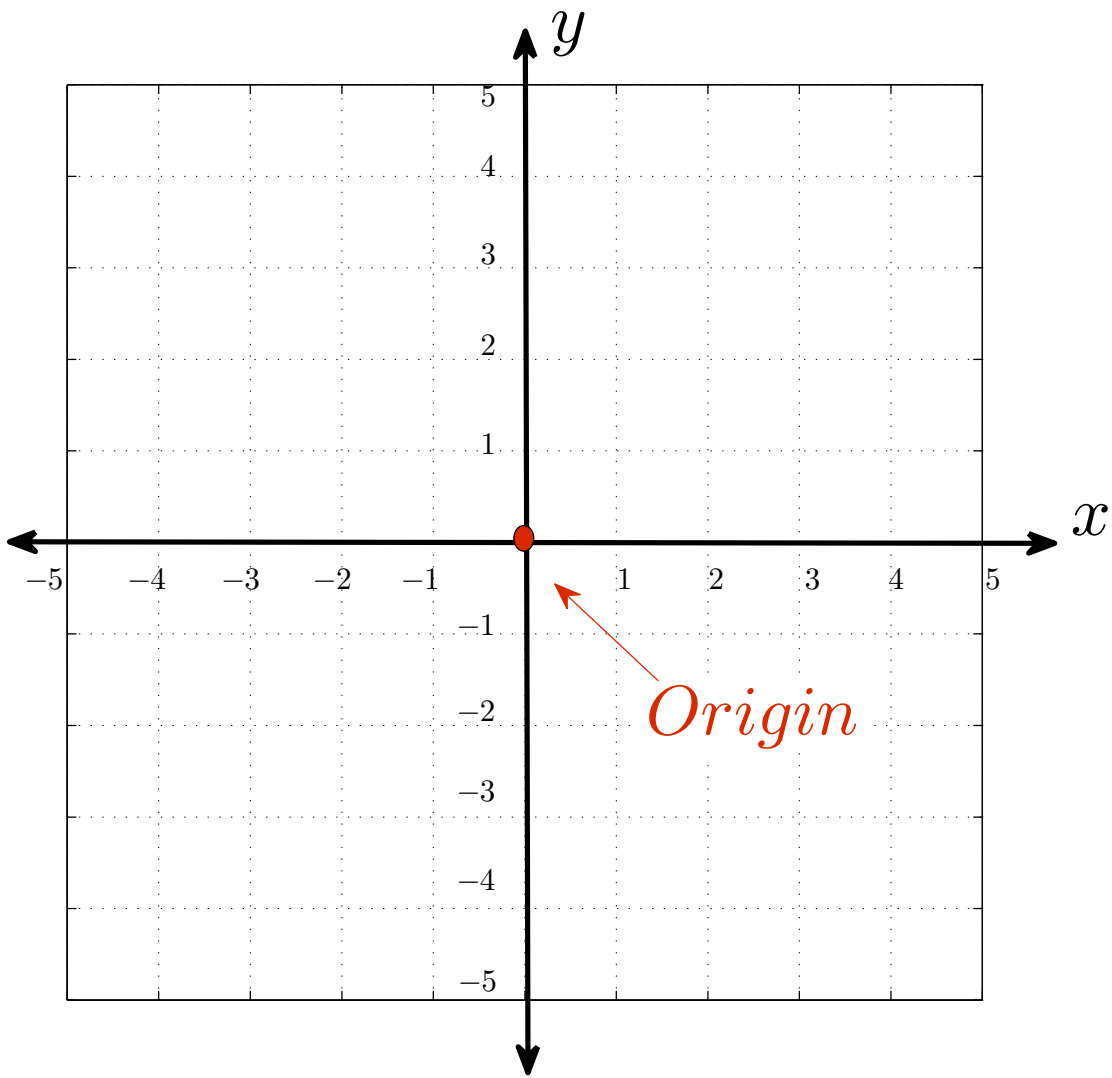
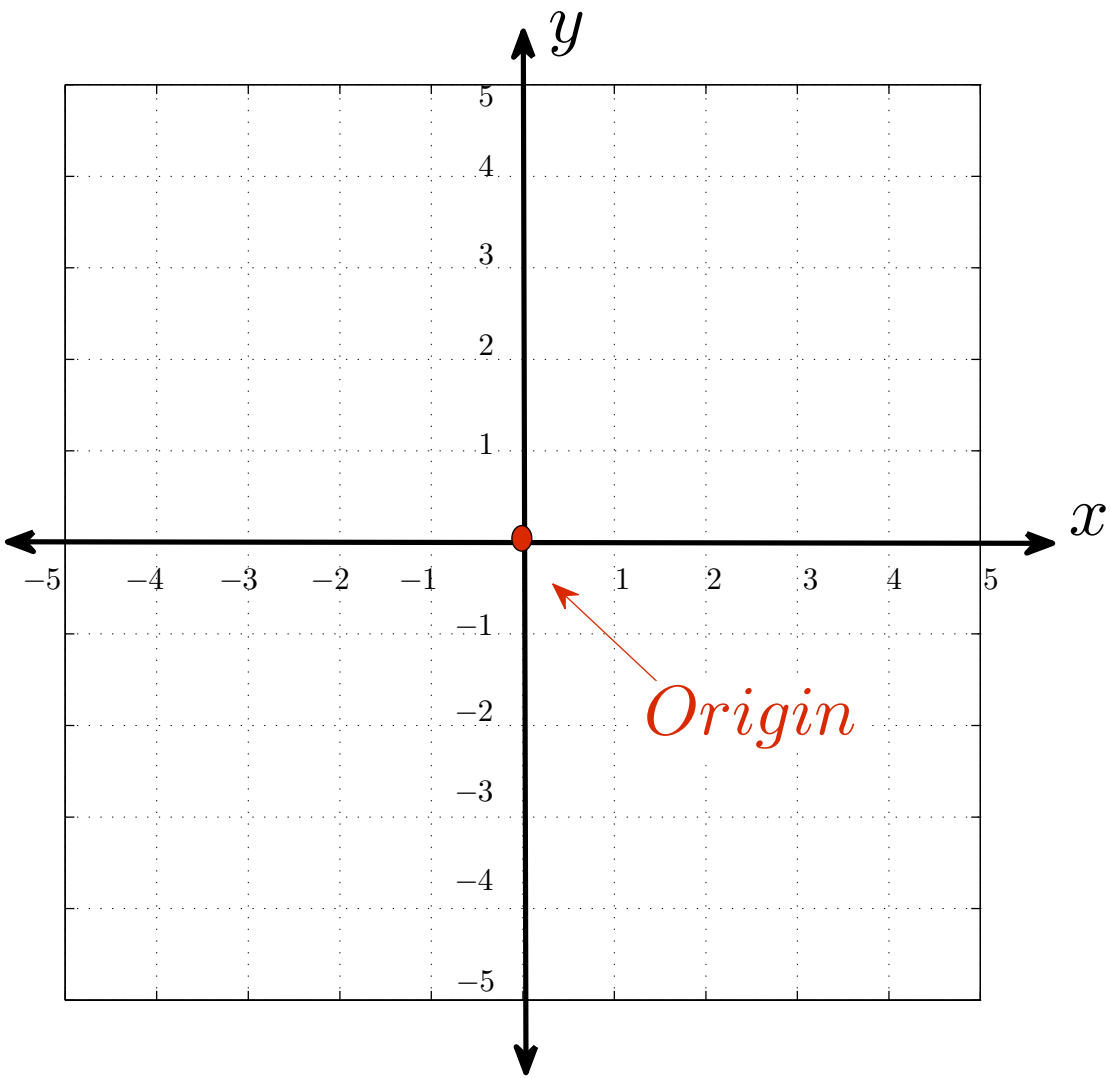


To plot $(0,2)$, begin at the origin. Travel along the x-axis 0 units.



From that point (the origin), move up 2 spaces in the positive y direction.



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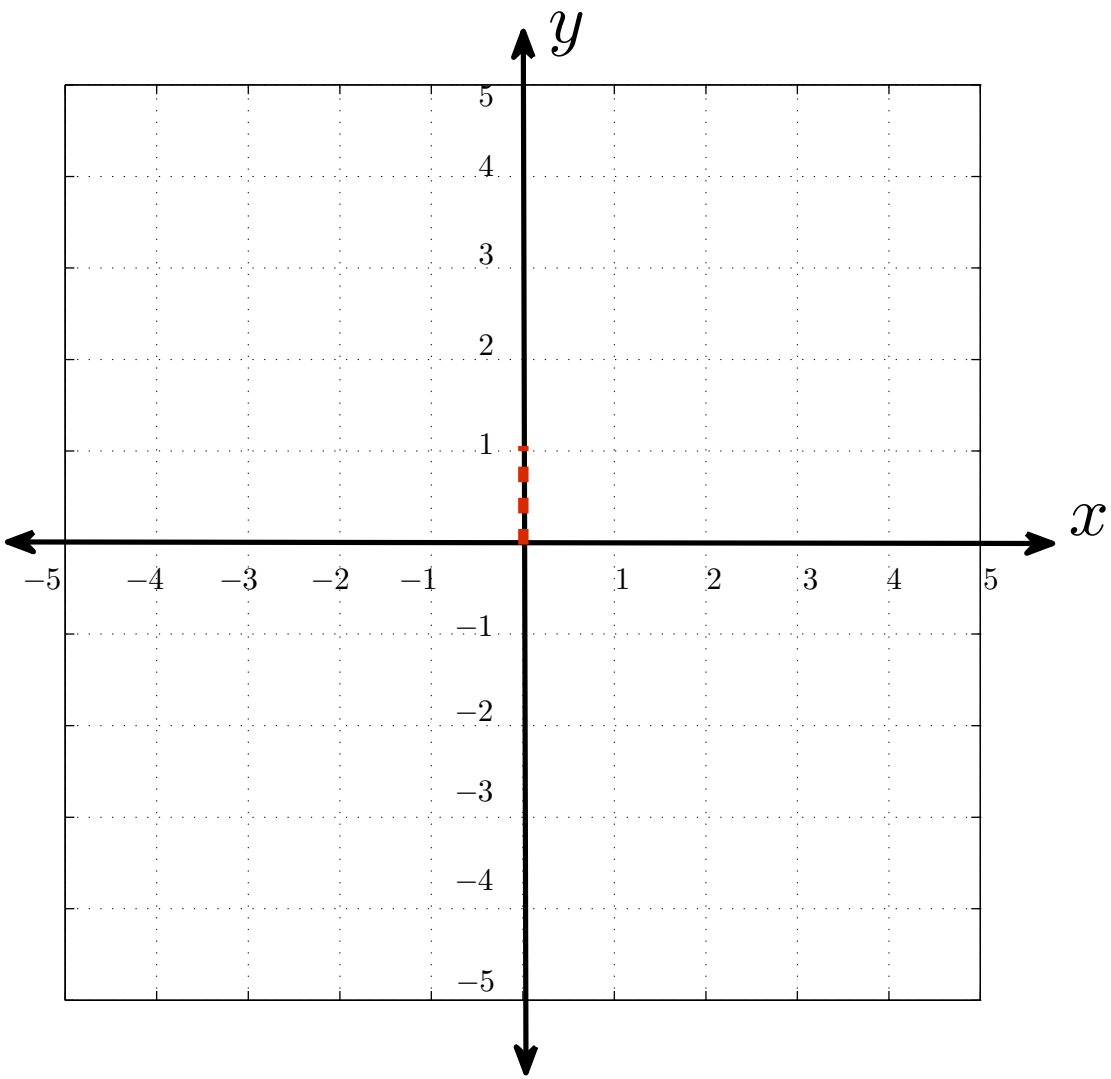

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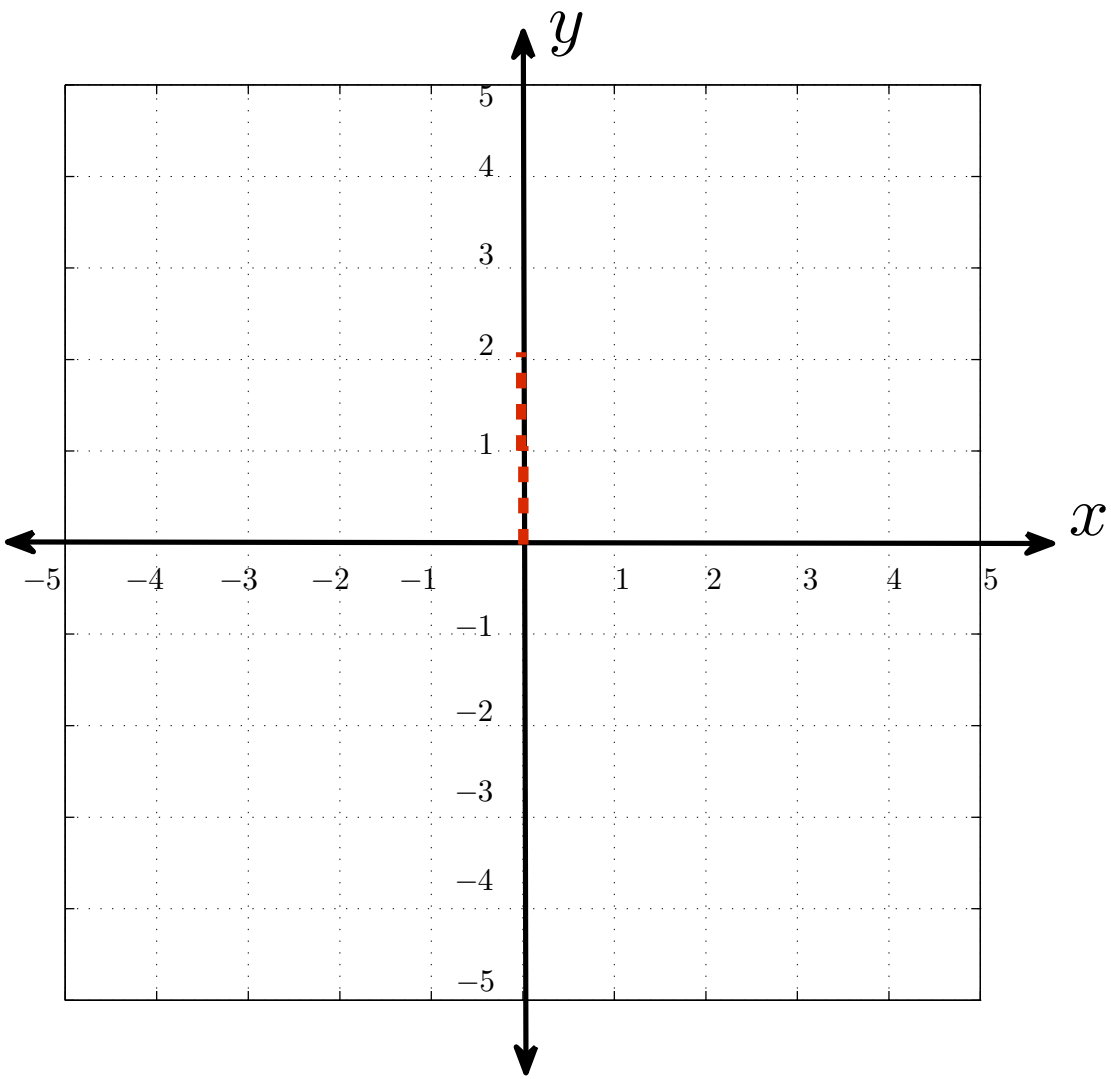

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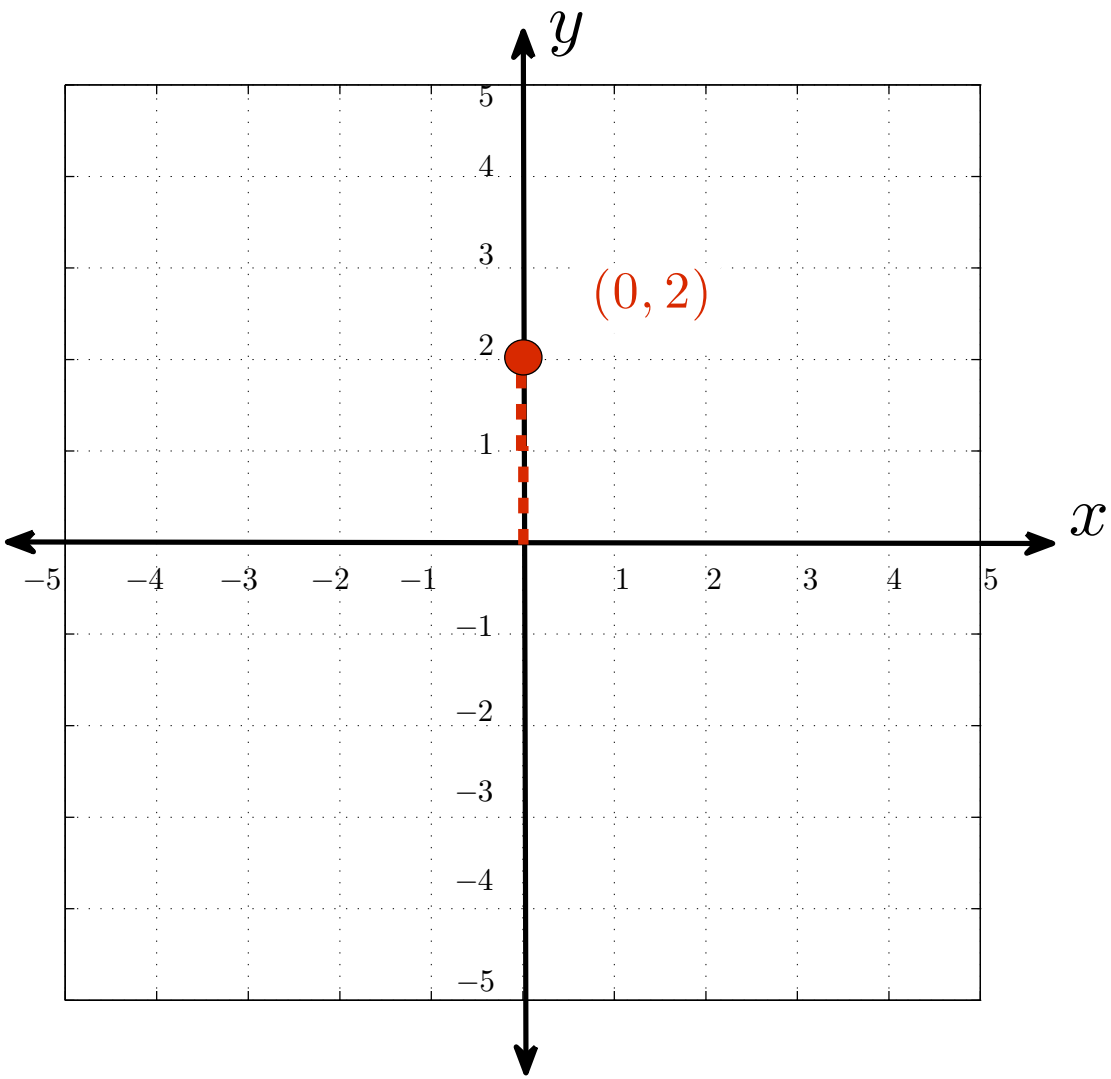

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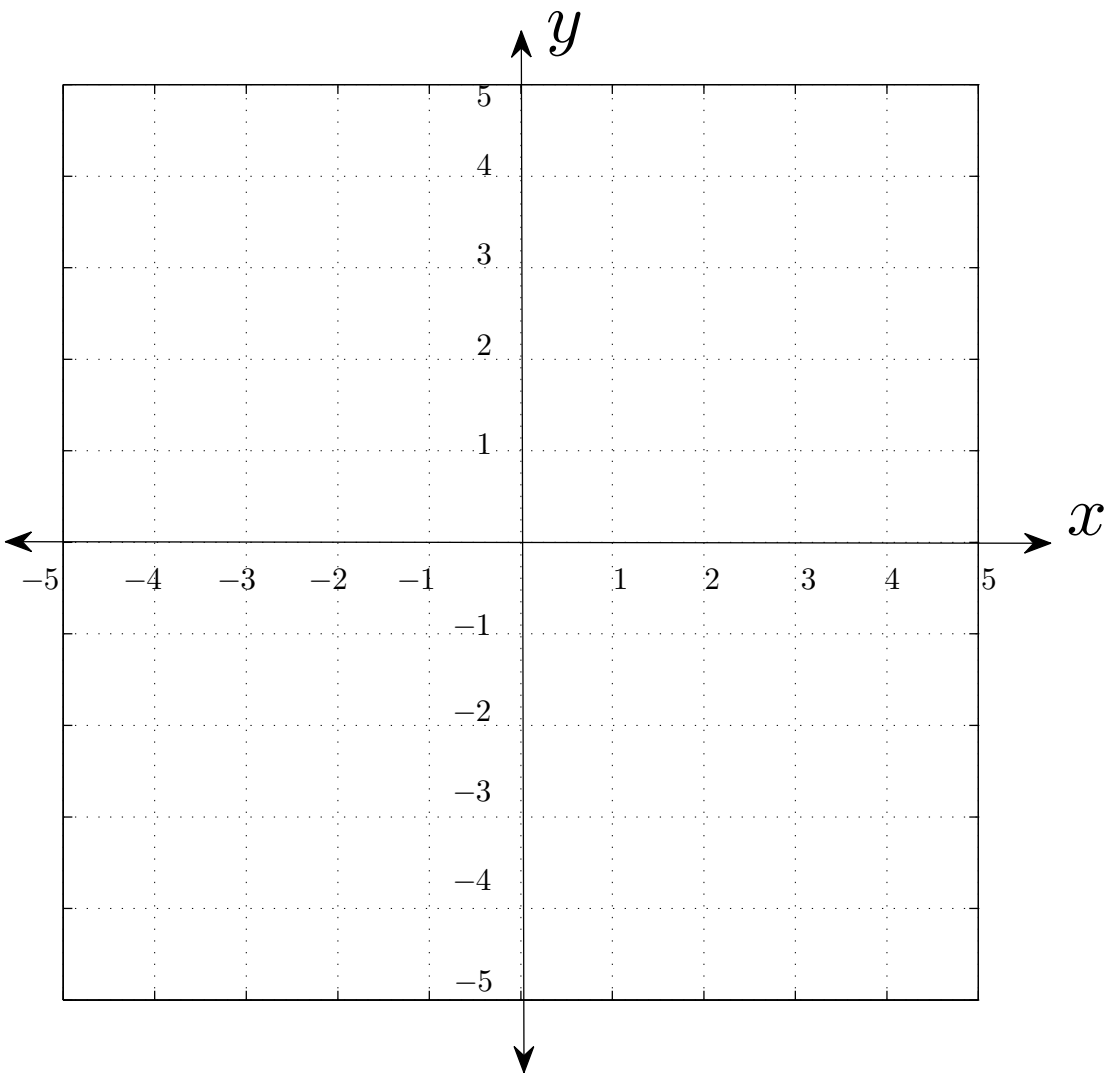
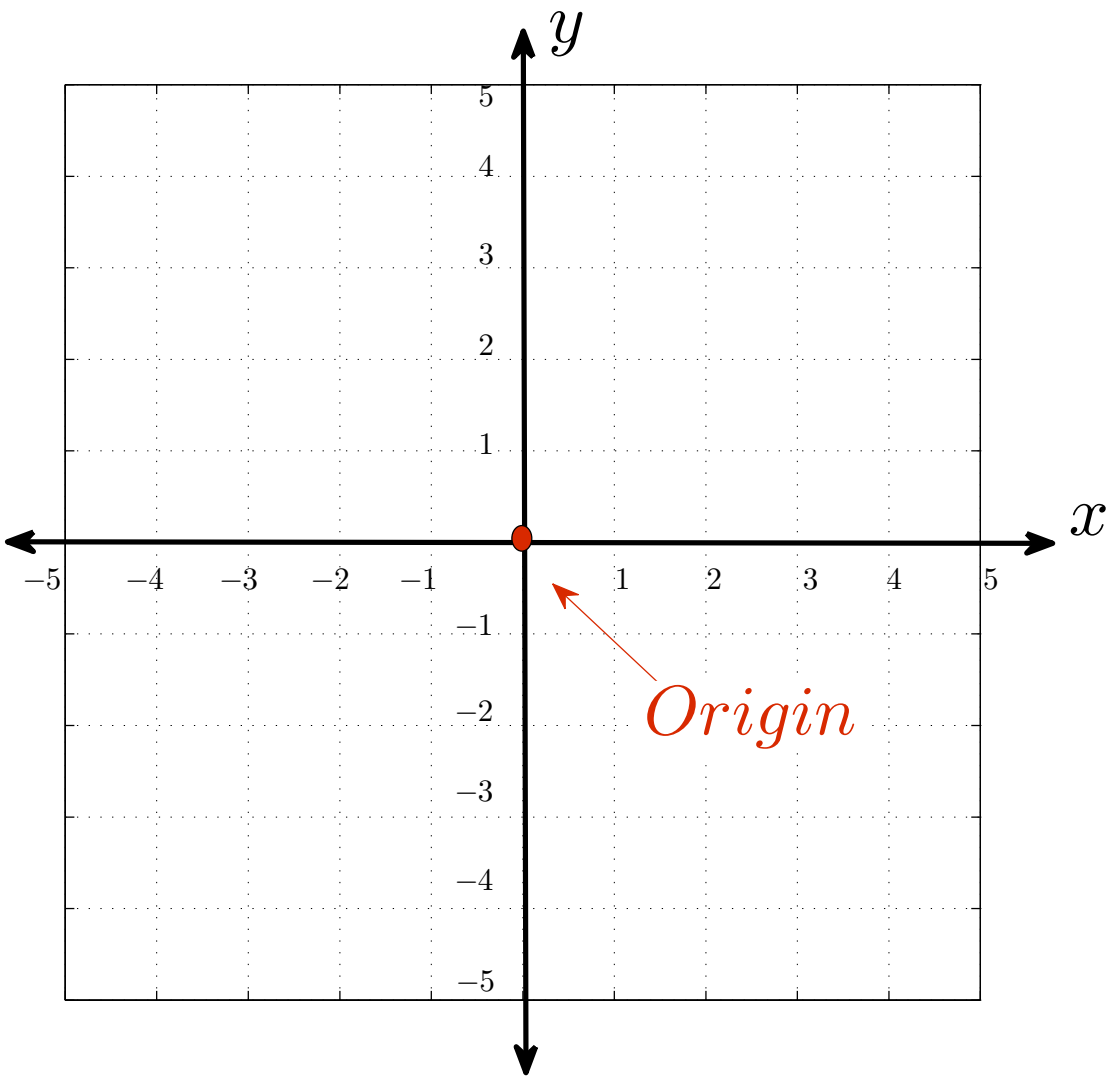
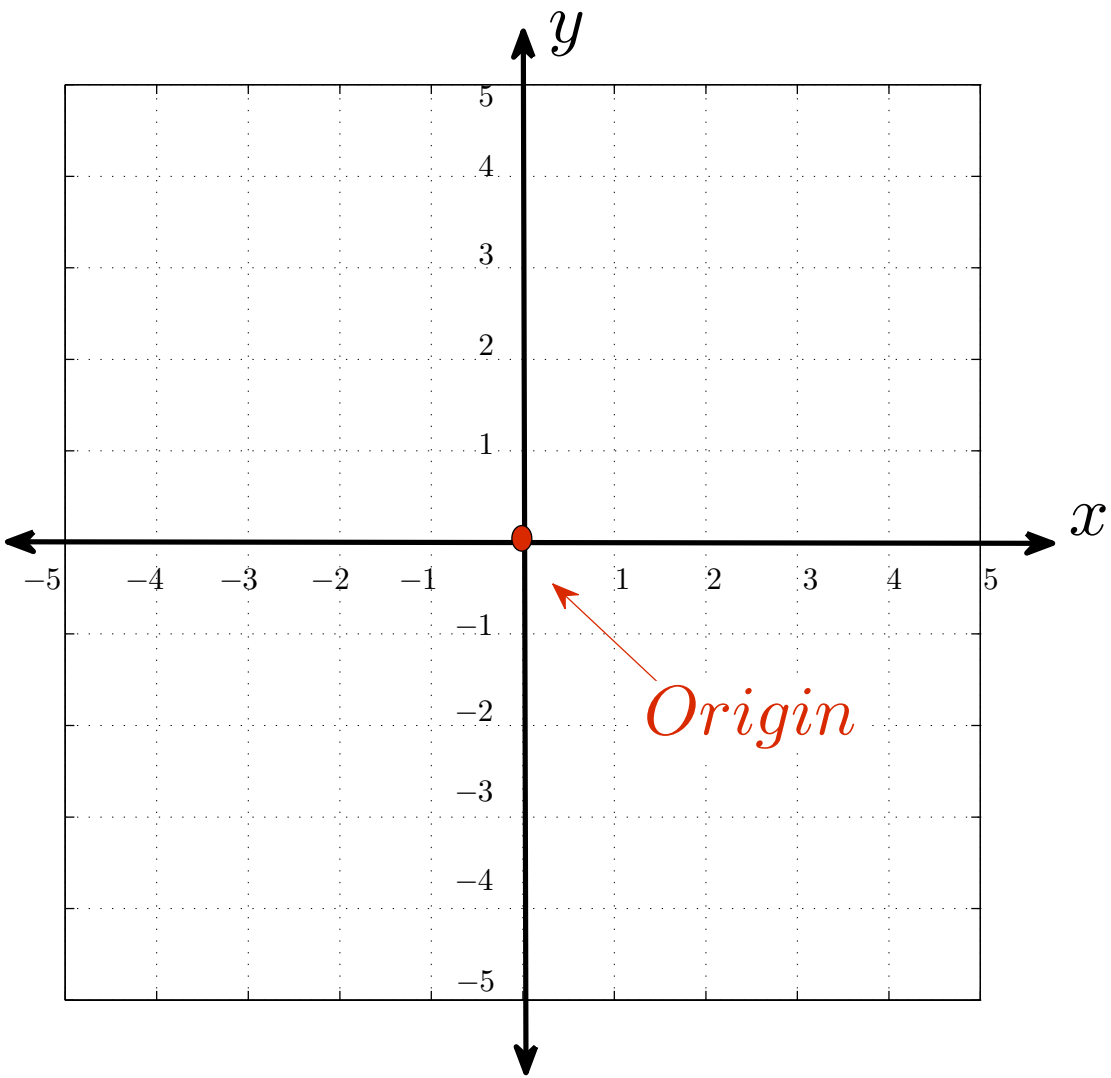


To plot $(-3,0)$, begin at the origin.



To plot $(-3,0)$, begin at the origin. Travel along the x-axis 3 units left (the negative x direction).



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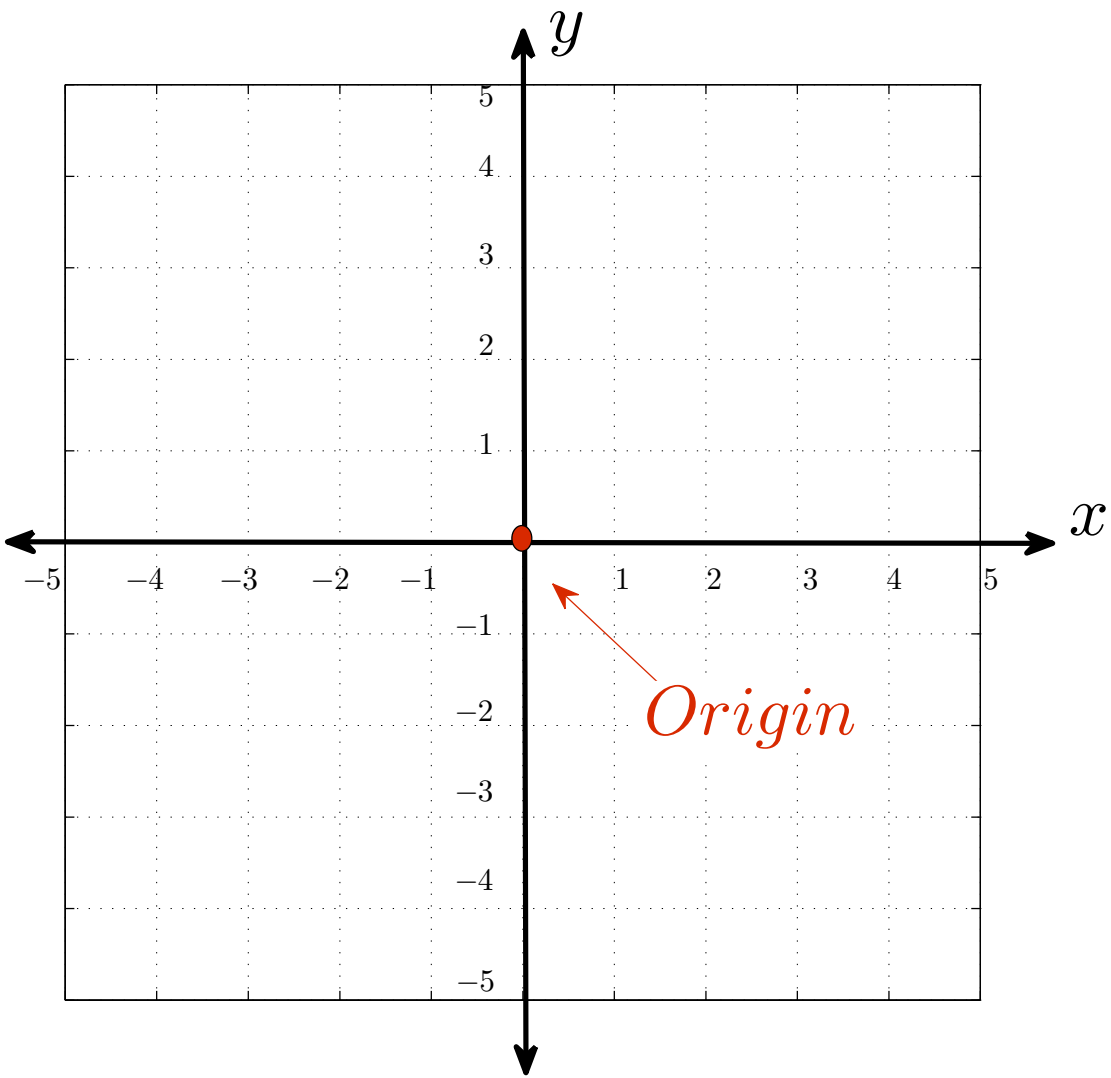

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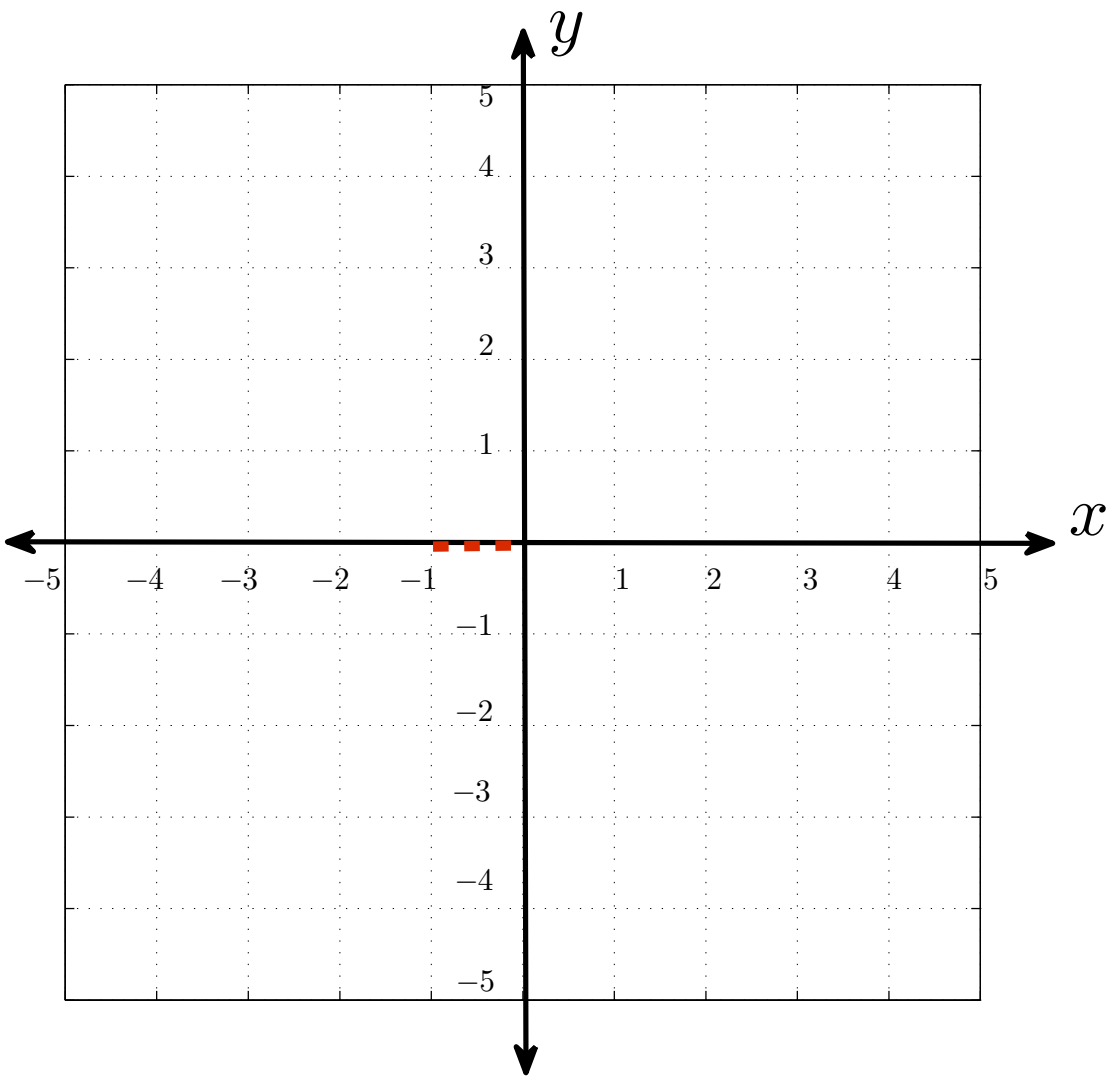

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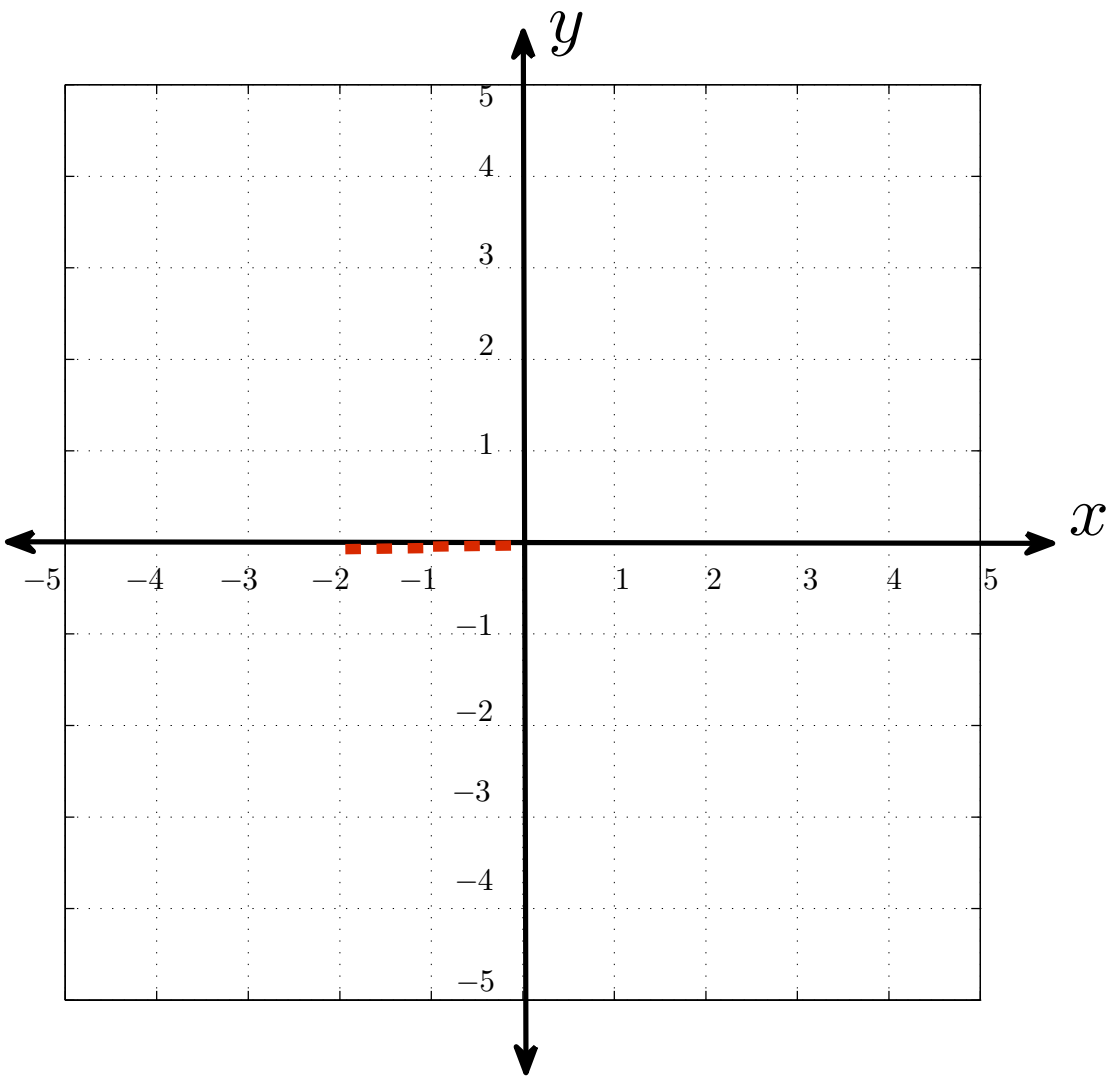

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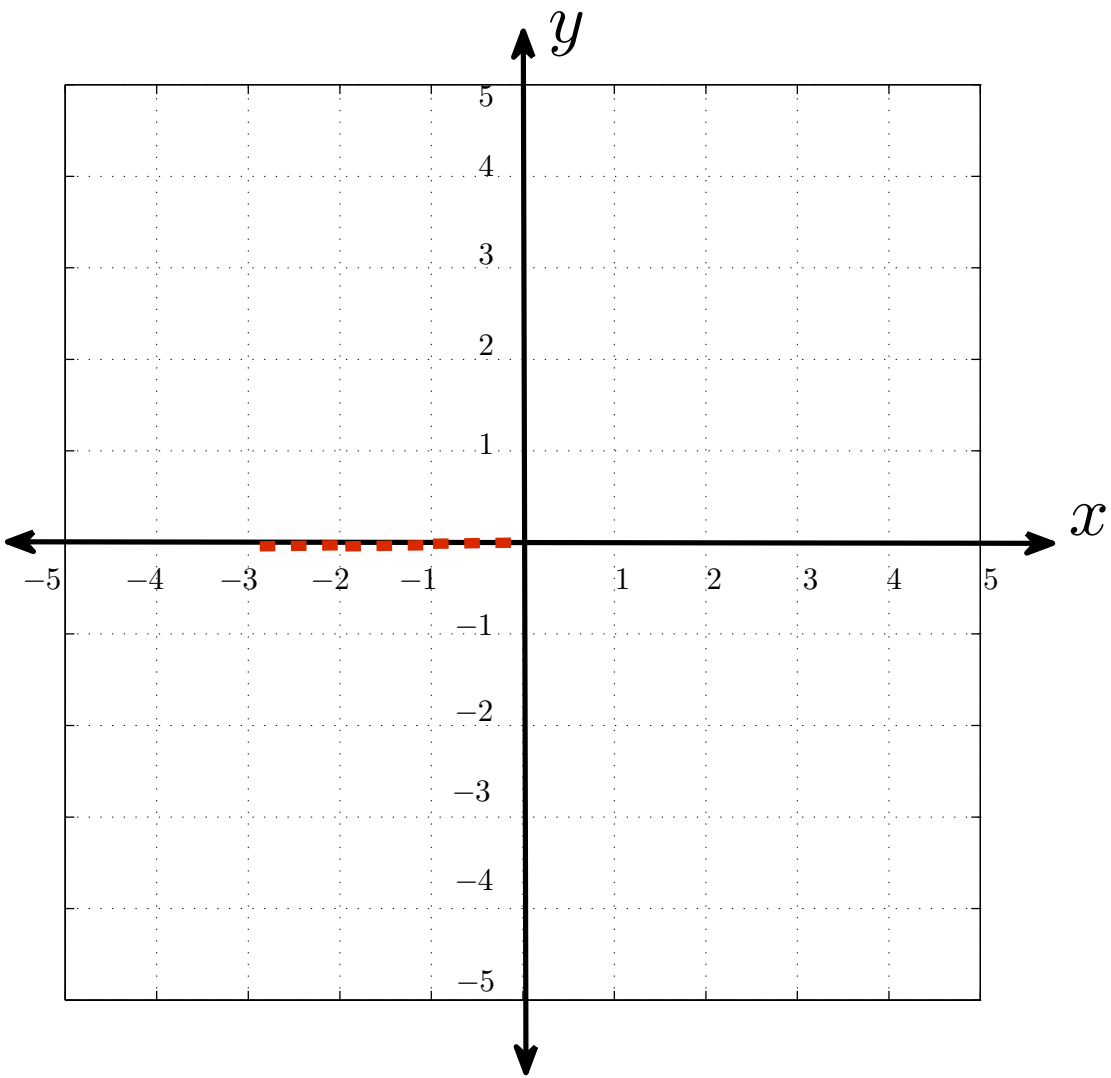
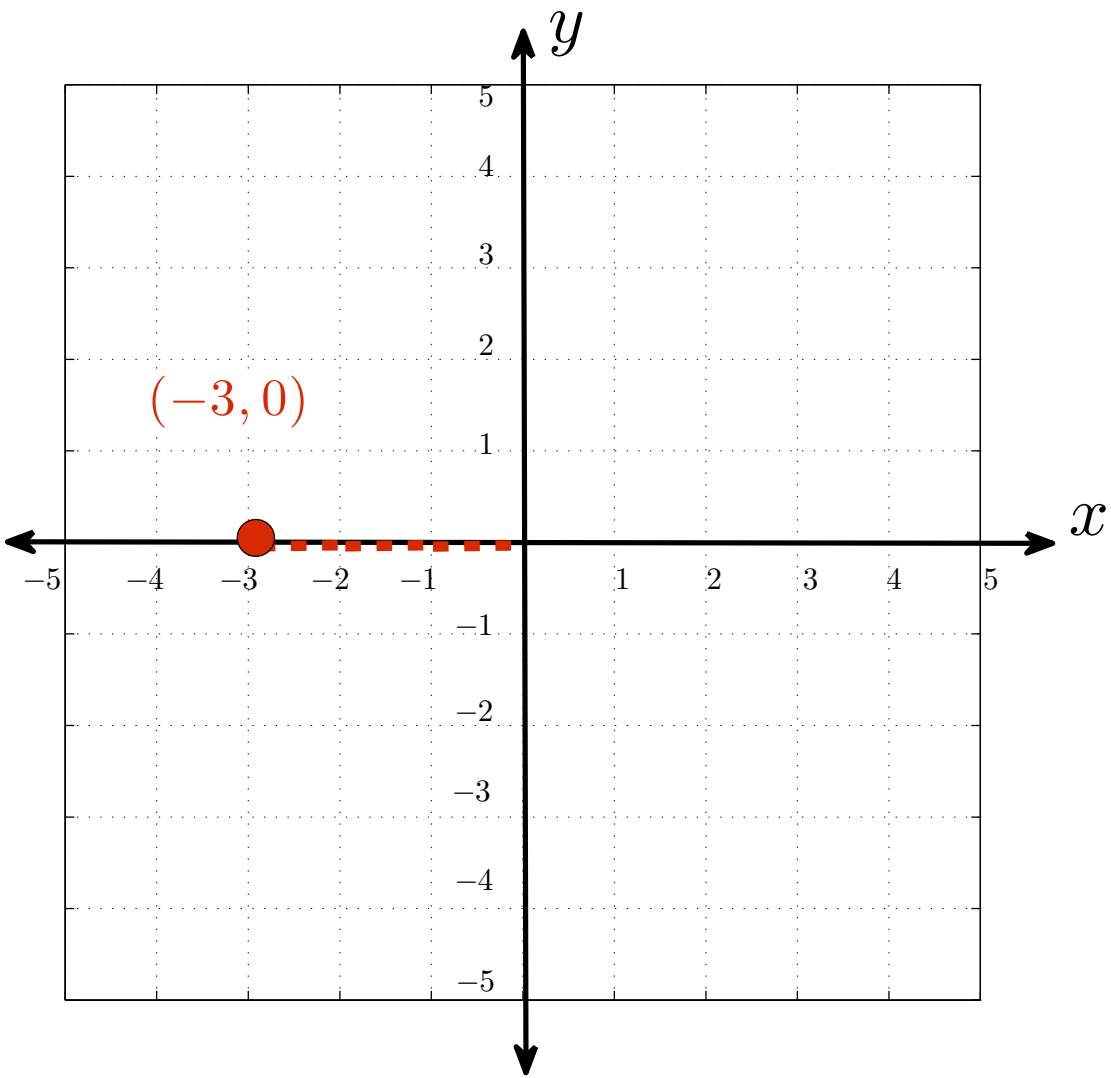


From that point (the origin), move up 0 spaces in the y direction.



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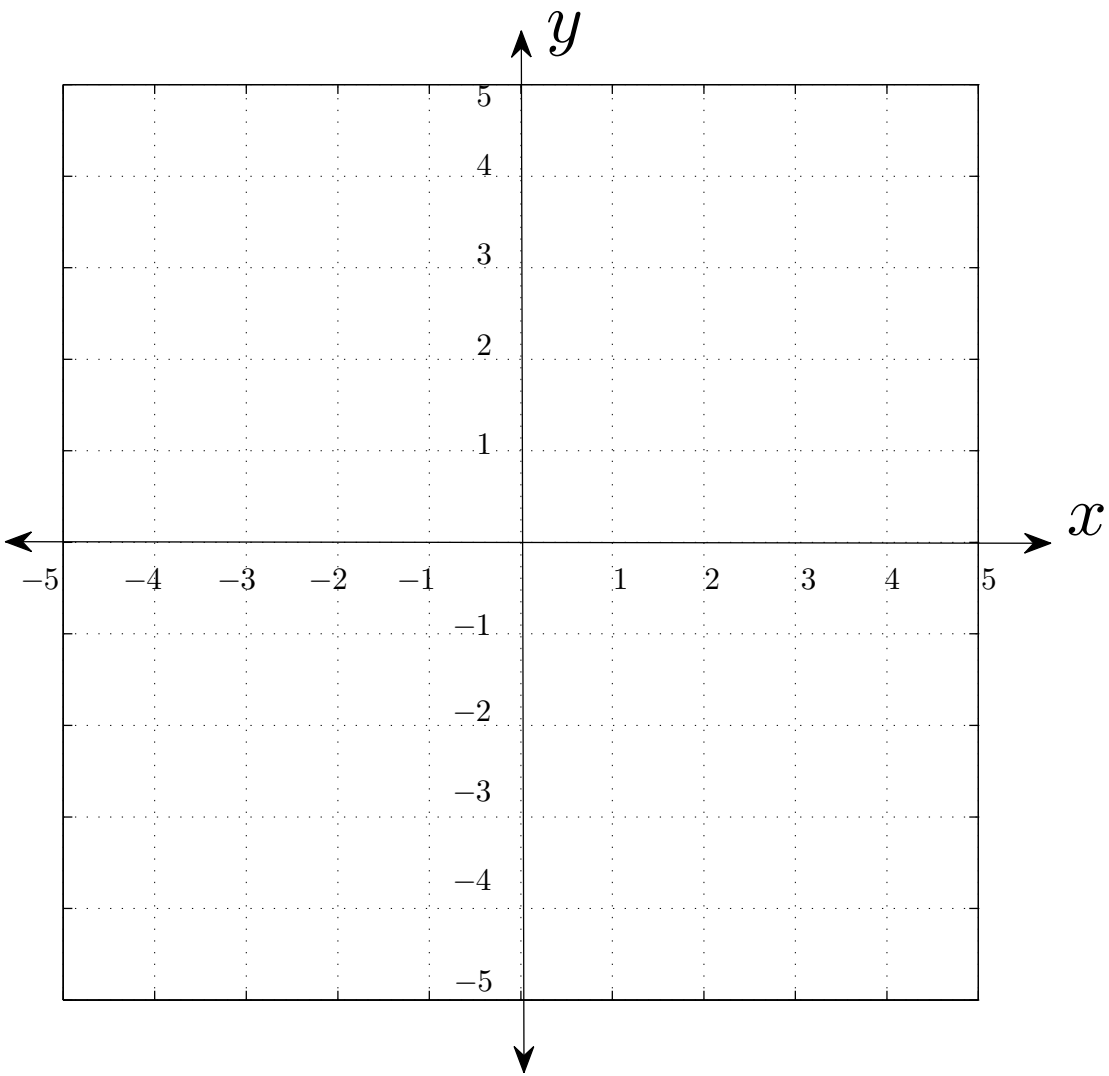
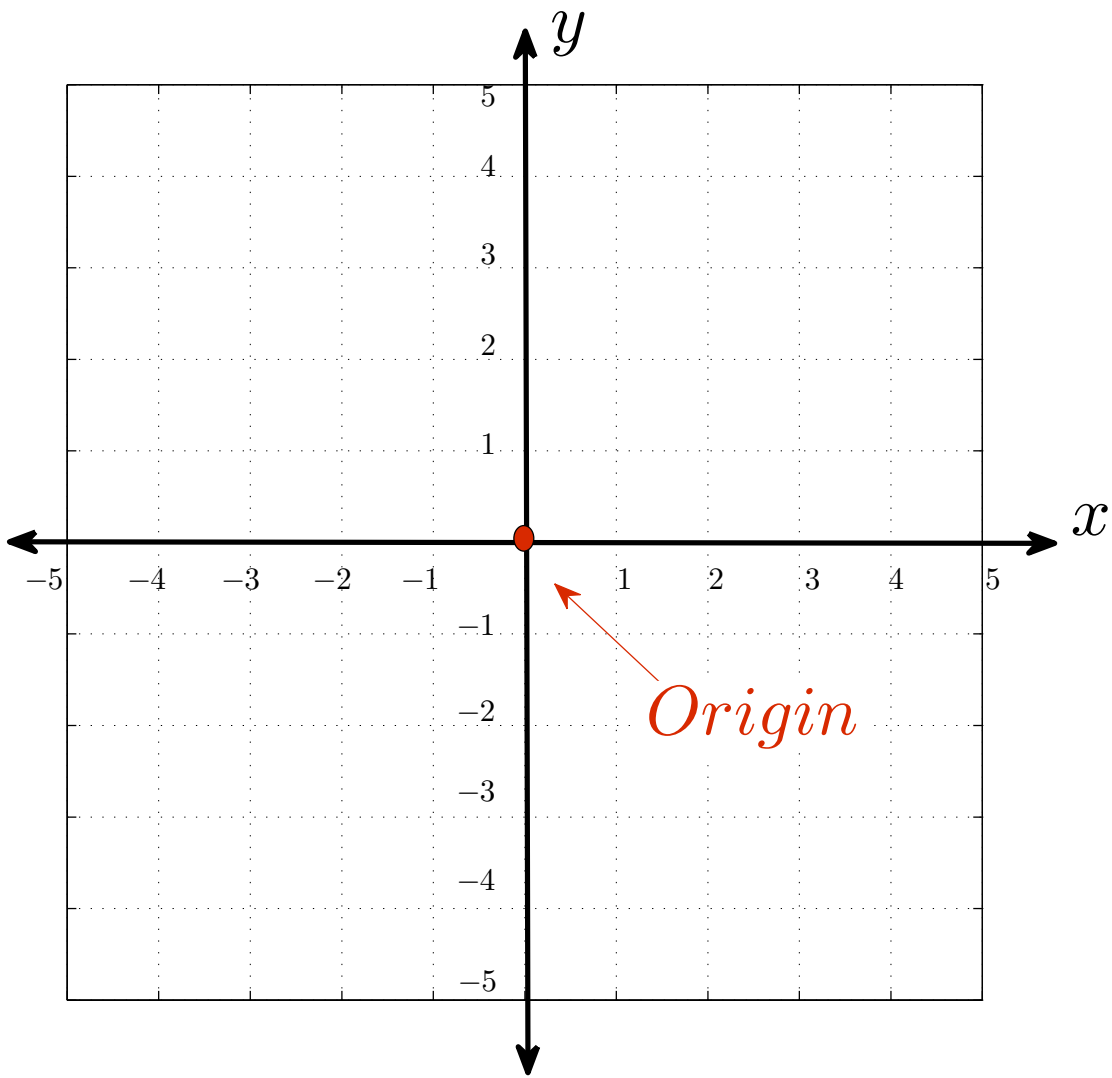
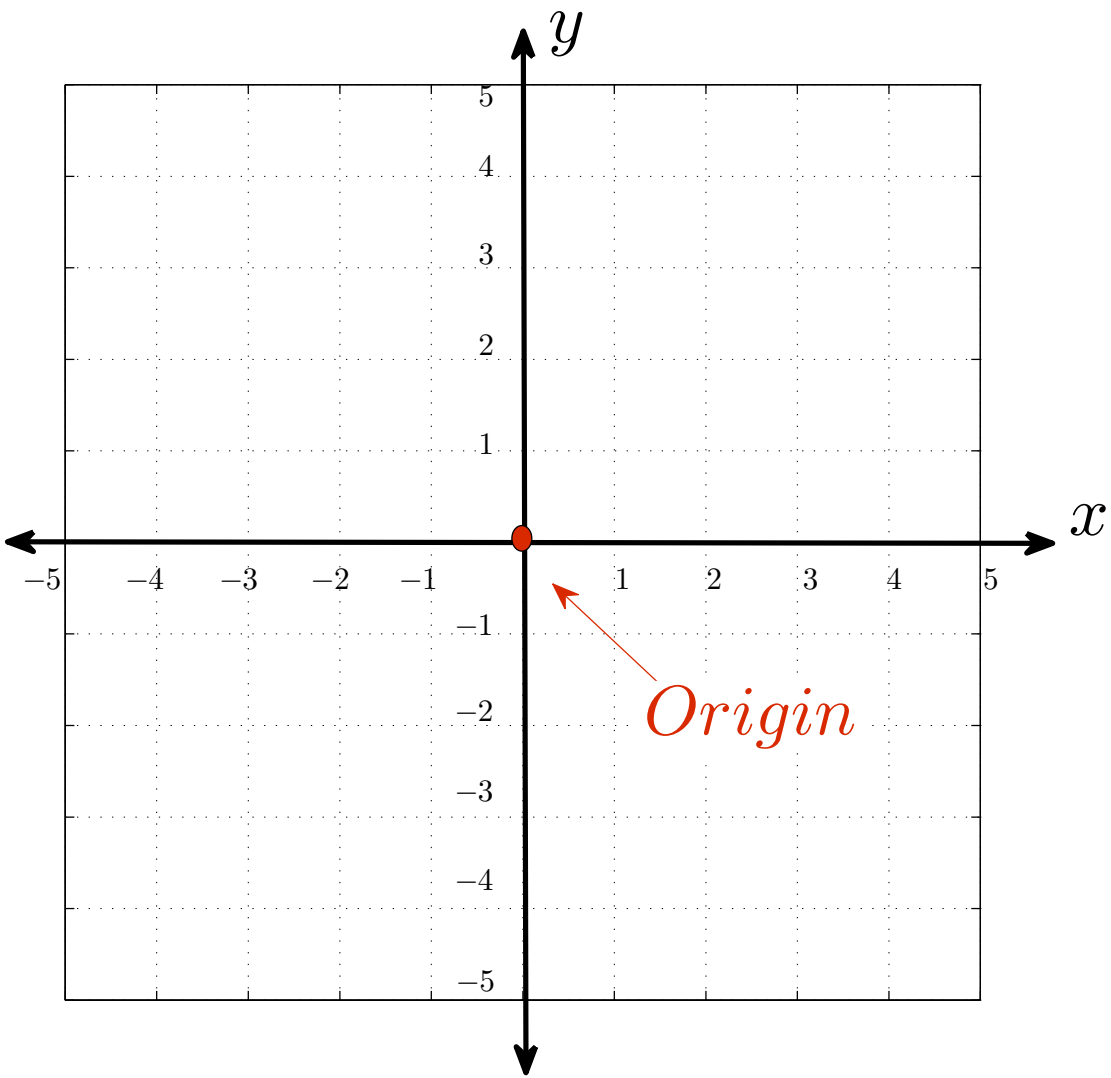


To plot $(0,-2)$, begin at the origin. Travel along the x-axis 0 units.



From that point (the origin), move up 2 spaces in the negative y direction (downwards).



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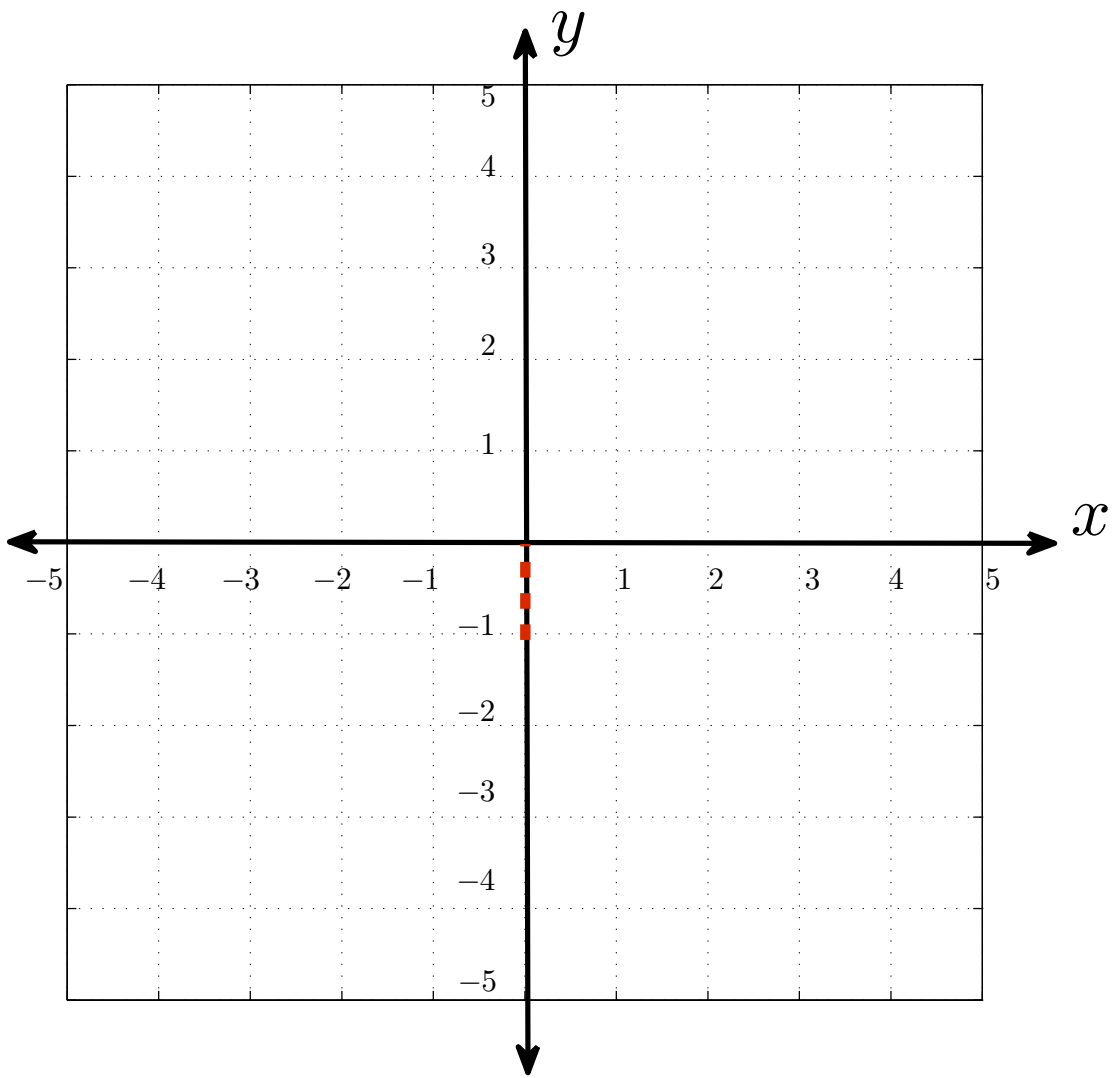

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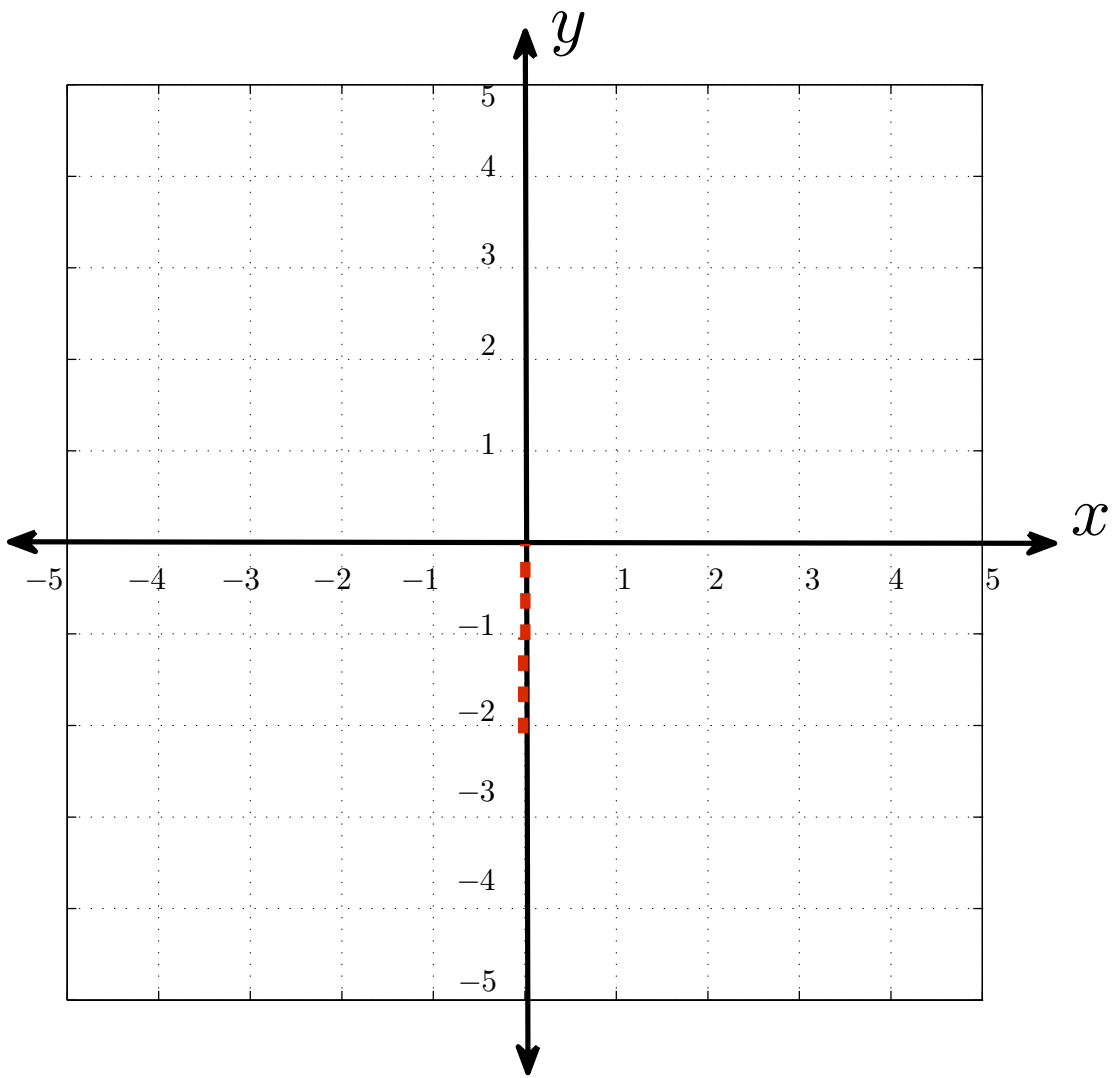

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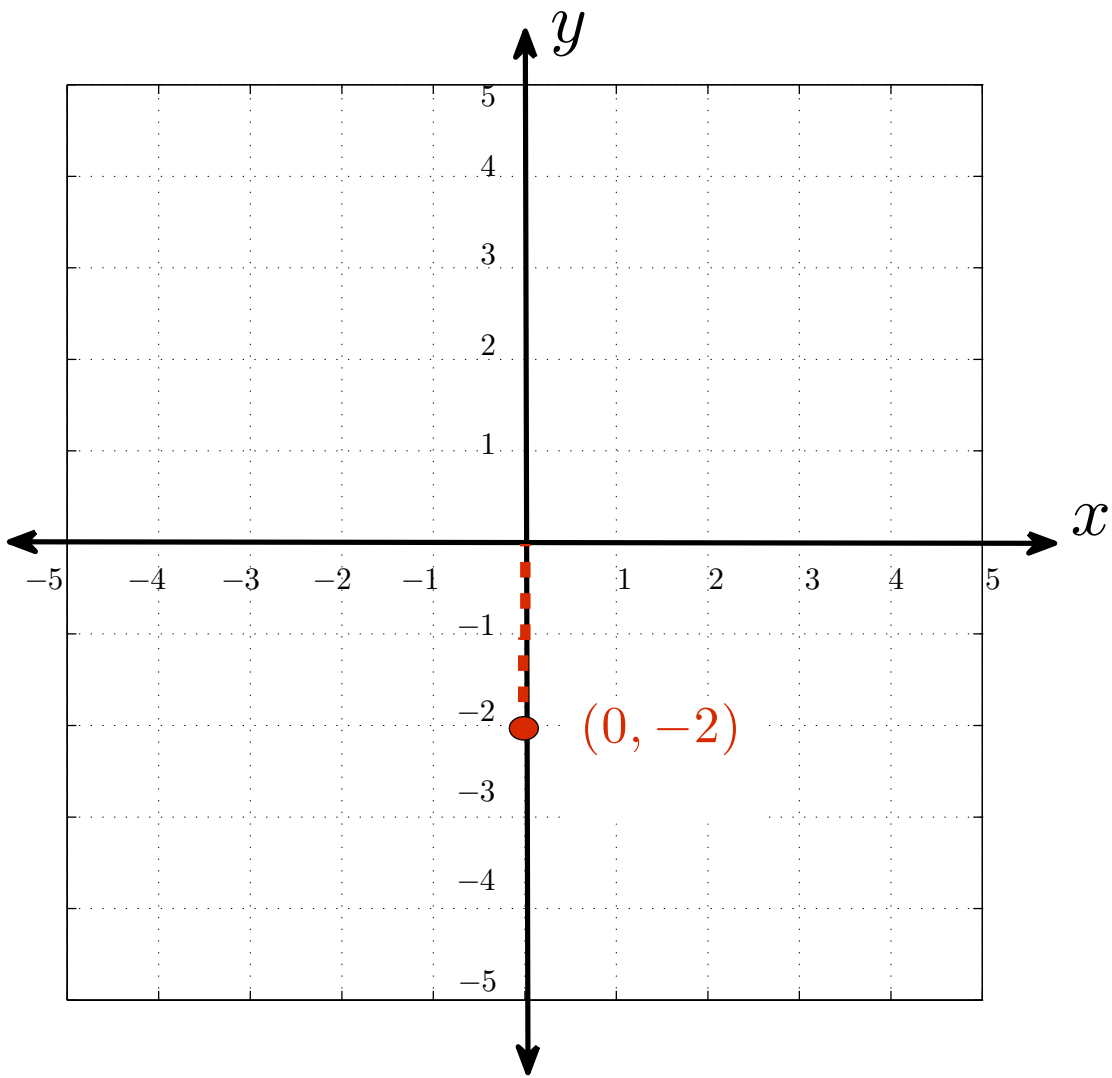
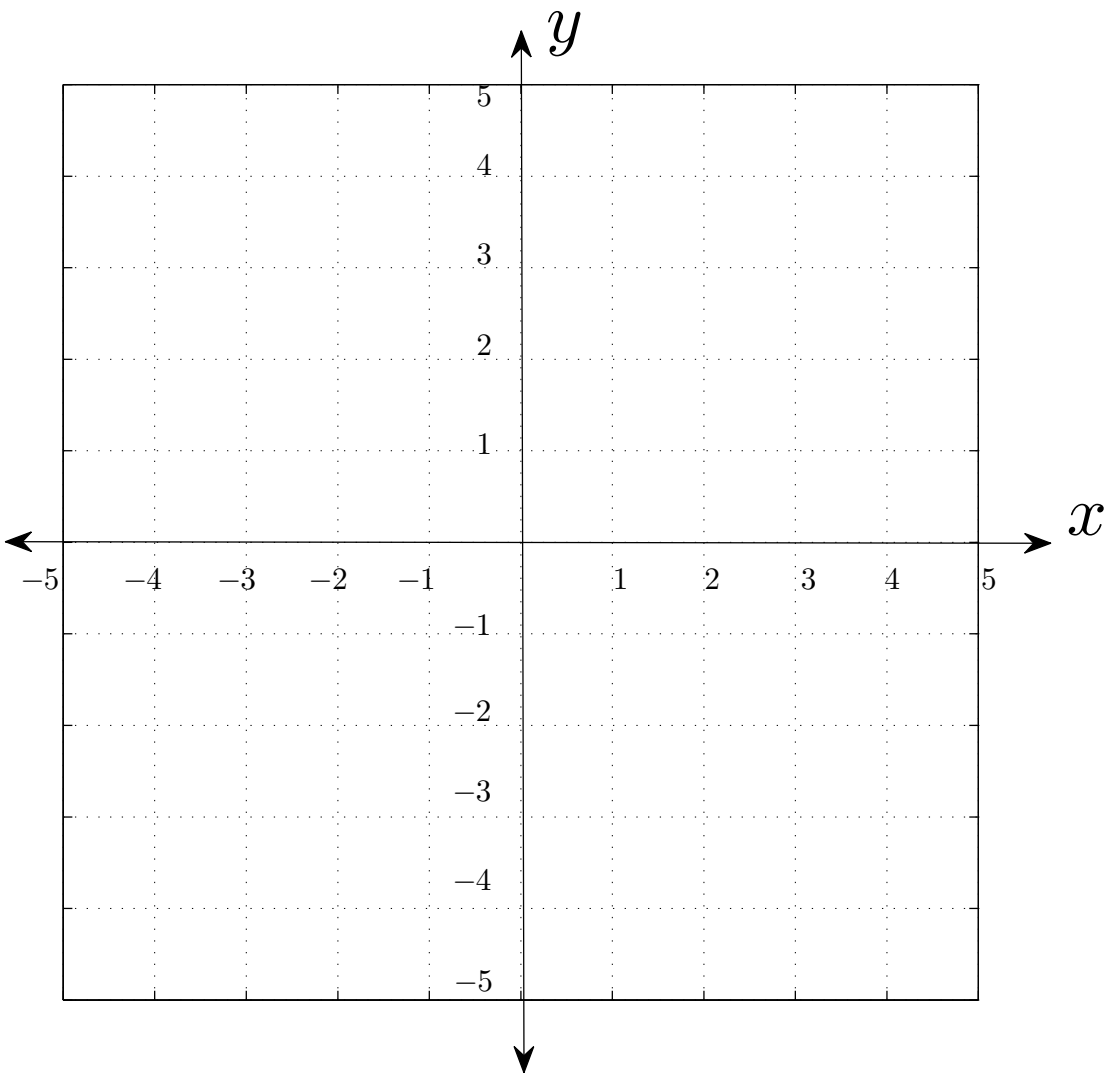


Plot (graph) $(3, 4)$, $(-4, 3)$, $(-1, -4)$ and $(5, -4)$



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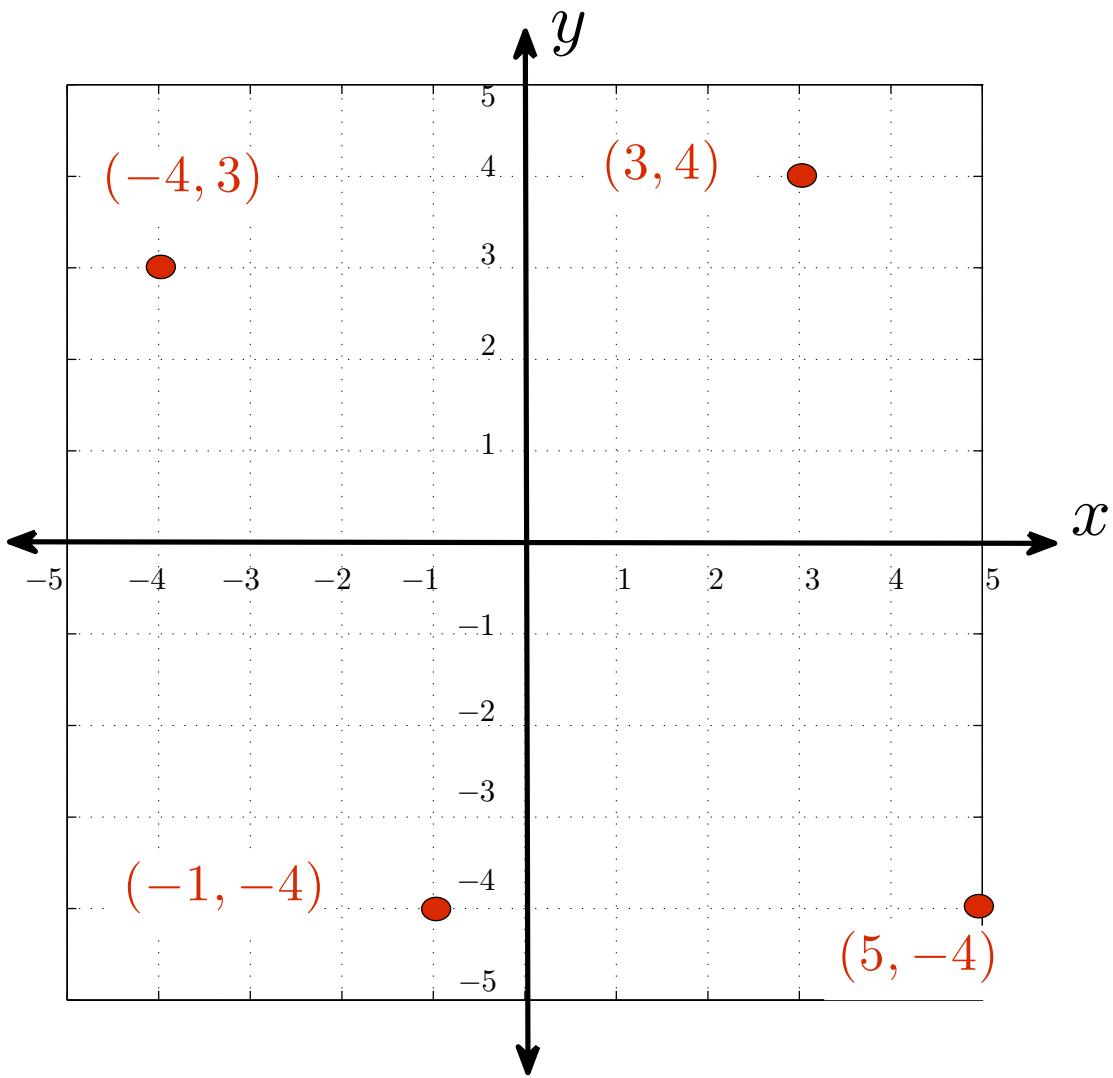
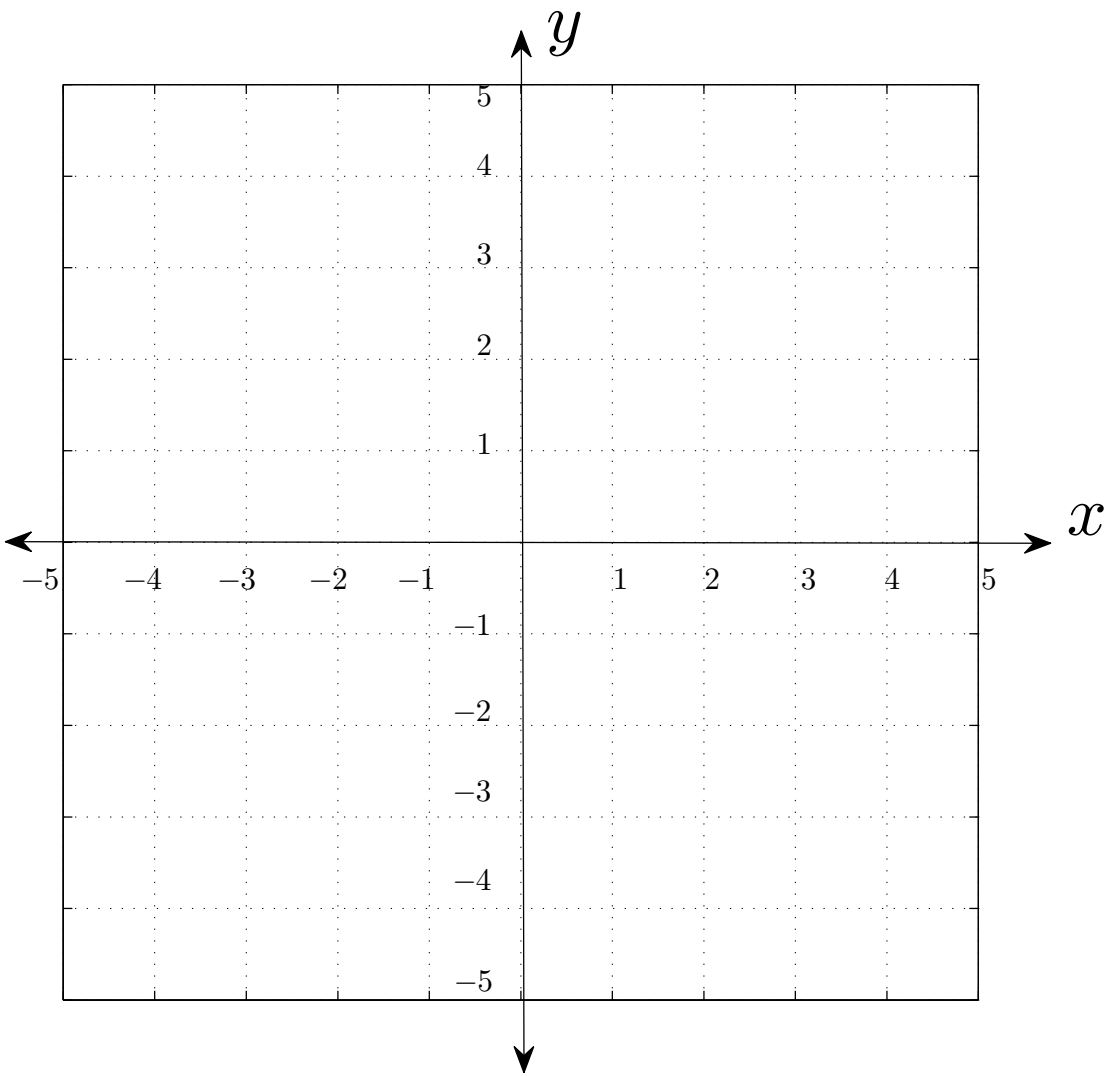


Plot (graph) $(4, 0)$, $(0, -3)$, $(-1, 0)$, and $(0, 5)$



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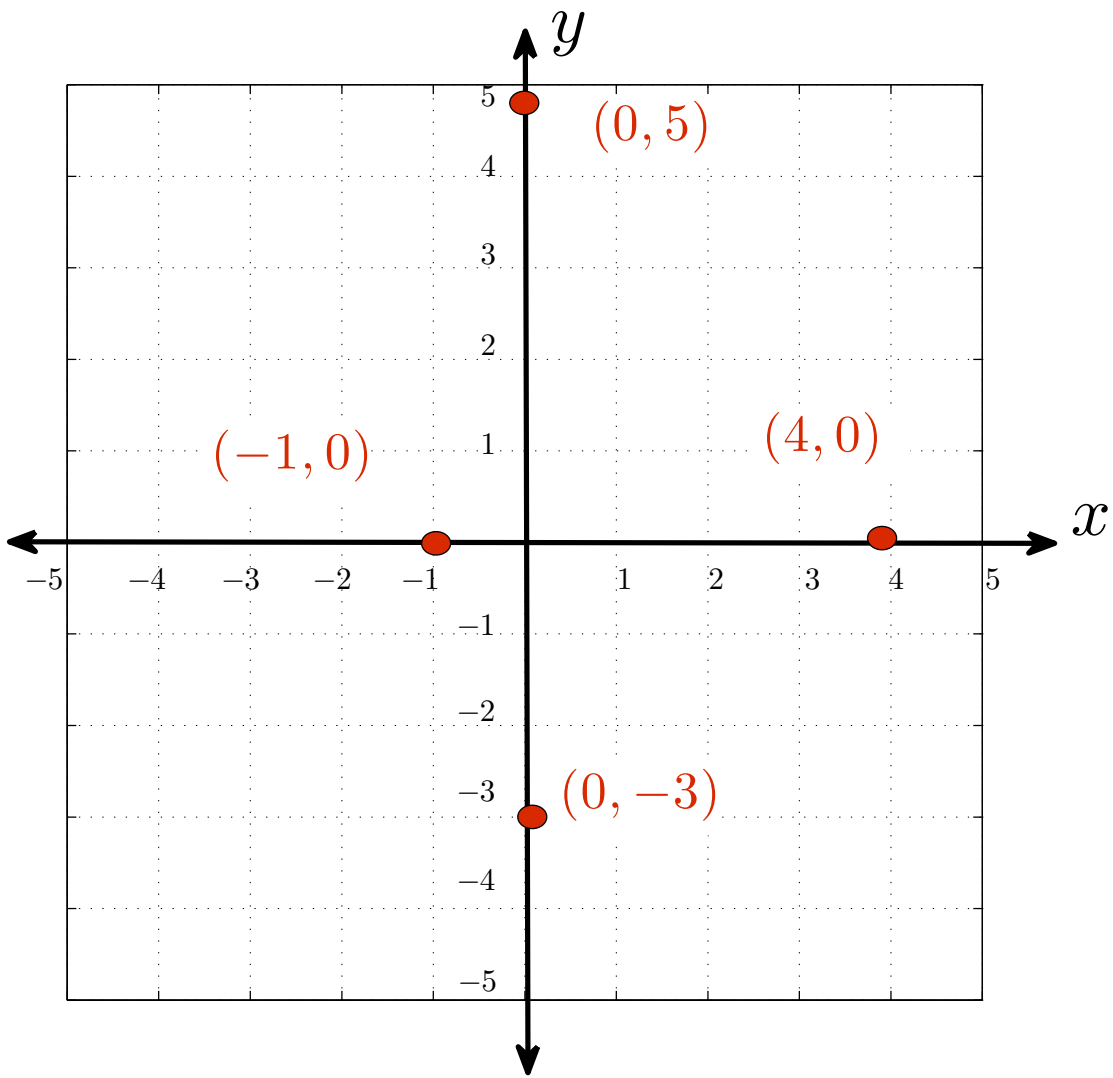

Definition

Suppose A, B and C represent any real numbers. A **linear equation in two variables** is an equation having the *form*

$$A\,x + B\,y = C,$$

For example, $2\,x + 3\,y = 1$ is a linear equation in the two variables x and y.

Solutions of a linear equation in two variables

Any linear equation in two variables always has in infinite number of solutions, and solutions come in the form of ordered pairs.

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Terminology	Definition	Illustration
Solution of an equation in x and y	An ordered pair (a, b) that yields a true statement if $x = a$ and $y = b$	(1,4) is a solution of $y = 5x - 1$, since substituting x=1 and $y = 4$ renders the LHS = 4 and the RHS $= 5(1) - 1 = 4$

LHS is an abbreviation for "left-hand side" (of the equation)

RHS is an abbreviation for "right-hand side" (of the equation)

Equations and Graphs

Definition

For each ordered-pair solution, (a, b), of an equation in x and y there is a point (a, b) in a rectangular coordinate plane. The set of all such points is called a **graph of the equation**.

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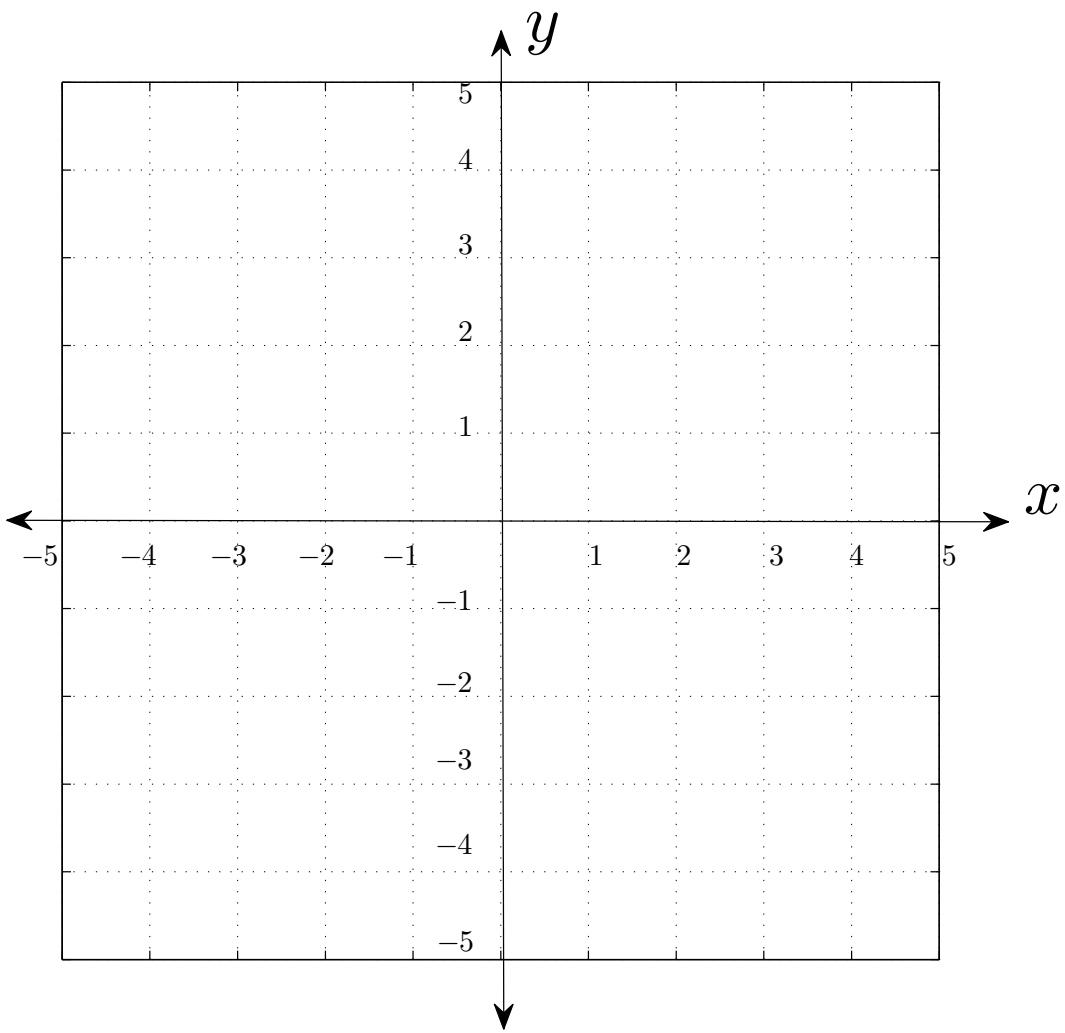
We can graph a linear equation by finding 3 ordered-pair solutions of the equation, plot the corresponding points on the rectangular grid, then draw a line between the three points.

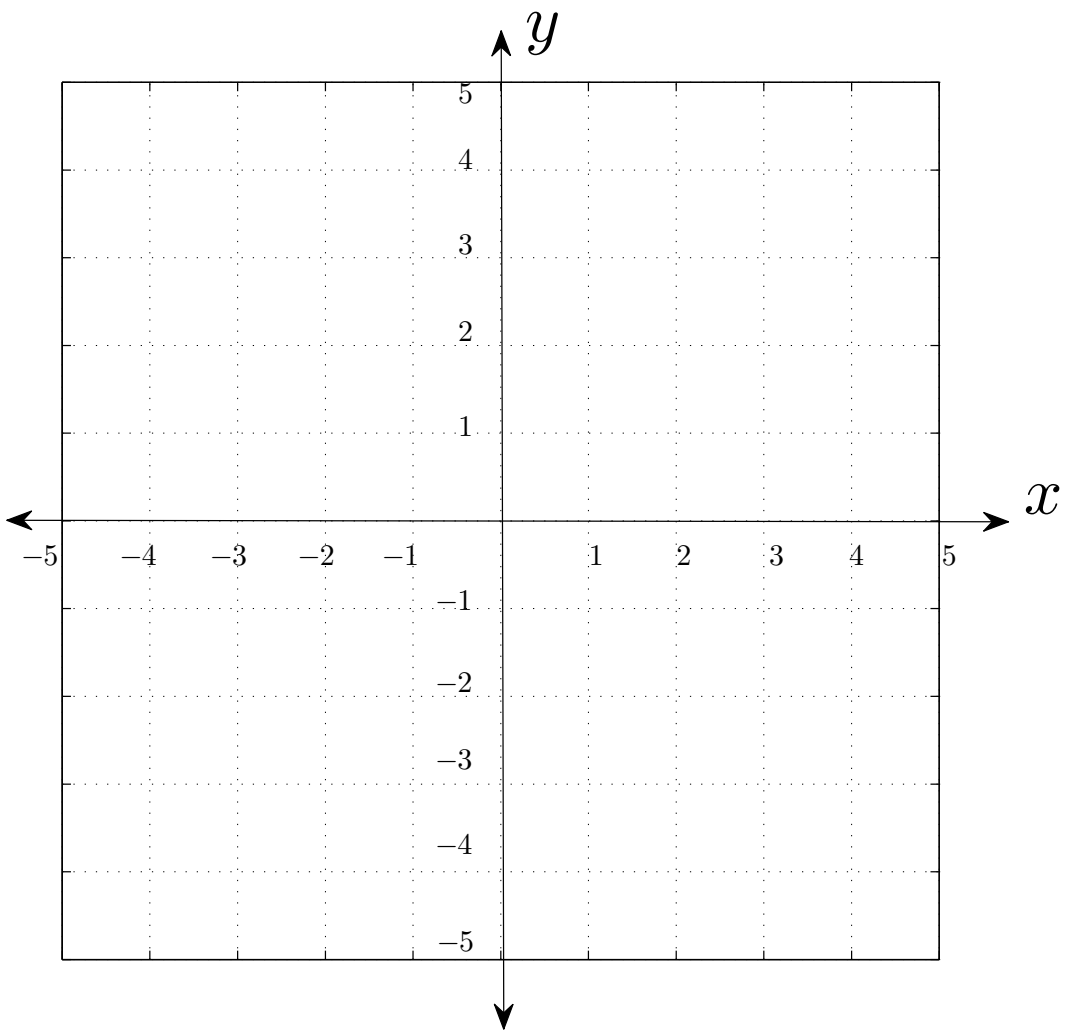
We use the third point for "insurance." If all three points line up in a straight we have not made a mistake!

Example 3: Graph the linear equation $y = -\frac{1}{2}x - 3$



Example 3: Graph the linear equation $y = -\frac{1}{2}x - 3$

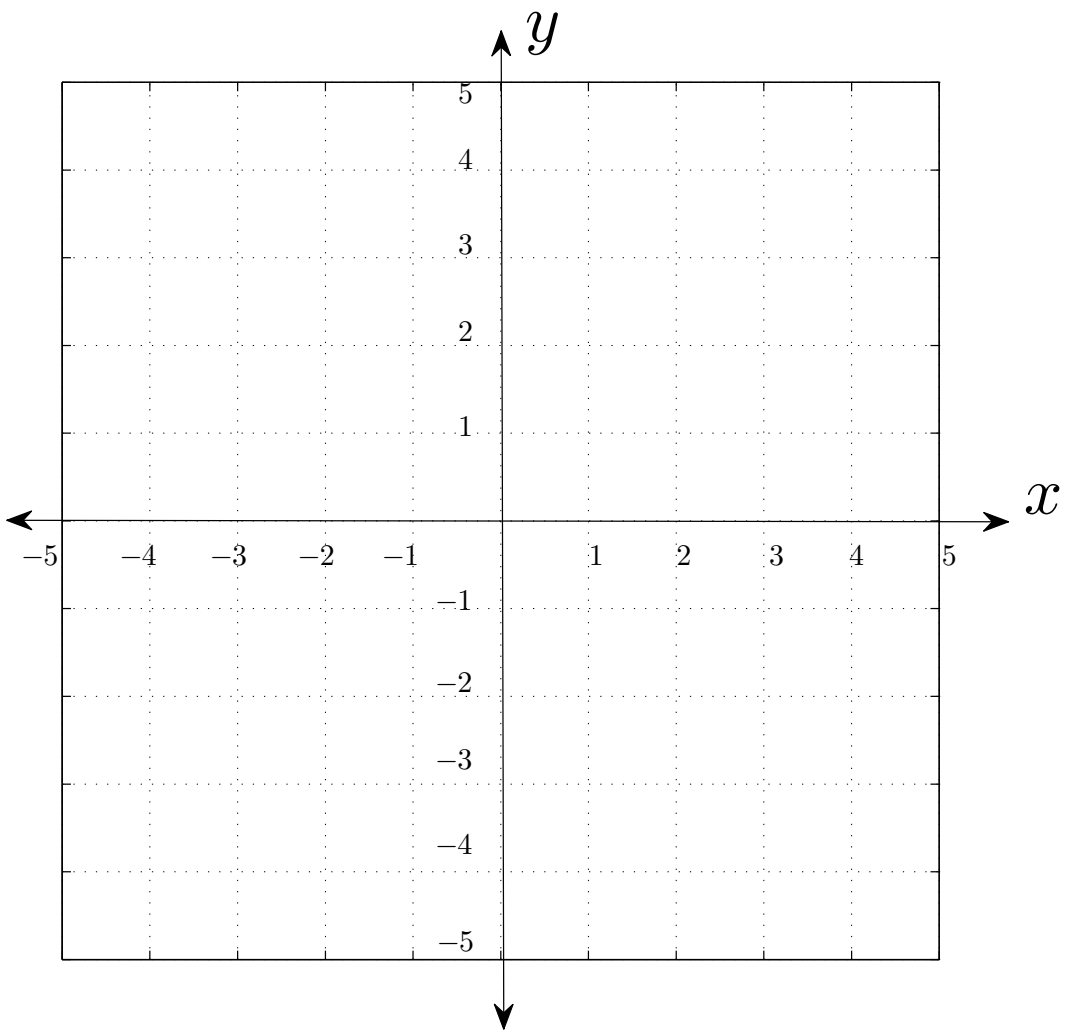
We begin by a making a table that summarizes x and y values. Since every value of x we substitute into the equation will be multiplied by $-\frac{1}{2}$, we use numbers for x that are divisible by 2.



x	y	(x, y)
-2		
0		
2		

Example 3: Graph the linear equation $y = -\frac{1}{2}x - 3$

That way, when we multiply by $-\frac{1}{2}$, the result will be an integer.



x	y	(x, y)
-2		
0		
2		

Example 3: Graph the linear equation $y = -\frac{1}{2}x - 3$

We let $x = -2$ in the equation to find the y-value of the ordered pair which is associated with x-coordinate -2.

x	y	(x, y)
-2		
0		
2		

Example 3: Graph the linear equation $y = -\frac{1}{2}x - 3$

We let $x = -2$ in the equation to find the y-value of the ordered pair which is associated with x-coordinate -2.

$$y = -\frac{1}{2} \cdot (x) - 3$$

$$= -\frac{1}{2} \cdot (-2) - 3$$

$$= 1 - 3$$

$$= -2$$

x	y	(x, y)
-2		
0		
2		

Example 3: Graph the linear equation $y = -\frac{1}{2}x - 3$

Upon simplification, we get the ordered pair solution (-2,-2)

$$y = -\frac{1}{2} \cdot (x) - 3$$

$$= -\frac{1}{2} \cdot (-2) - 3$$

$$= 1 - 3$$

$$= -2$$

x	y	(x, y)
-2	-2	(-2,-2)
0		
2		

Example 3: Graph the linear equation $y = -\frac{1}{2}x - 3$

Next, we let $x = 0$ in the equation to find the y-value of the ordered pair which is associated with x-coordinate 0.

$$y = -\frac{1}{2} \cdot (x) - 3$$

$$= -\frac{1}{2} \cdot (0) - 3$$

$$= 0 - 3$$

$$= -3$$

x	y	(x, y)
-2	-2	(-2,-2)
0		
2		

Example 3: Graph the linear equation $y = -\frac{1}{2}x - 3$

This gives us the ordered pair solution (0,-3)

$$y = -\frac{1}{2} \cdot (x) - 3$$

$$= -\frac{1}{2} \cdot (0) - 3$$

$$= 0 - 3$$

$$= -3$$

x	y	(x, y)
-2	-2	(-2,-2)
0	-3	(0,-3)
2		

Example 3: Graph the linear equation $y = -\frac{1}{2}x - 3$

Afterwards, we let $x = 2$ in the equation.

$$y = -\frac{1}{2} \cdot (x) - 3$$

$$= -\frac{1}{2} \cdot (2) - 3$$

$$= -1 - 3$$

$$= -4$$

x	y	(x, y)
-2	-2	(-2,-2)
0	-3	(0,-3)
2		

Example 3: Graph the linear equation $y = -\frac{1}{2}x - 3$

Upon simplification, we get the ordered pair solution (2,-4)

$$y = -\frac{1}{2} \cdot (x) - 3$$

$$= -\frac{1}{2} \cdot (2) - 3$$

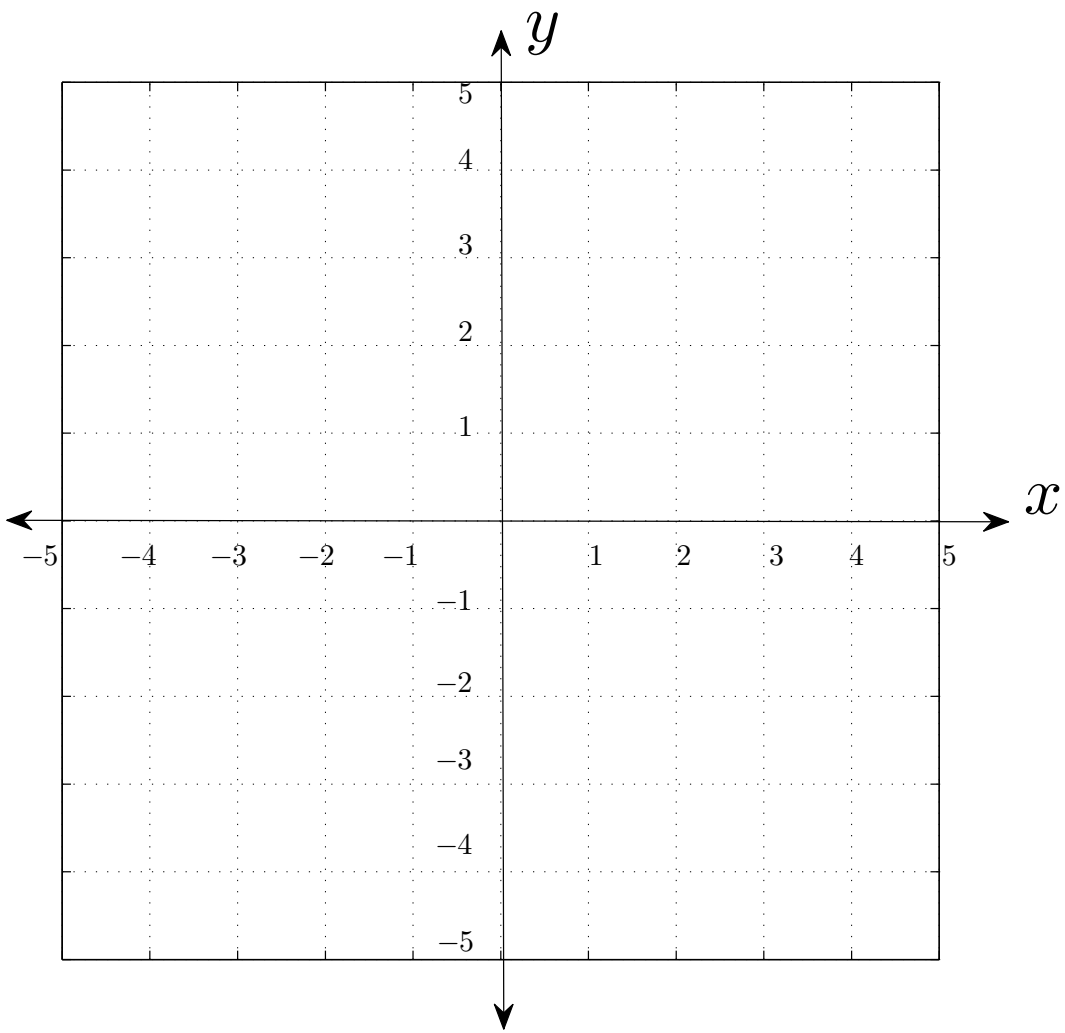
$$= -1 - 3$$

$$= -4$$

x	y	(x, y)
-2	-2	(-2,-2)
0	-3	(0,-3)
2	-4	(2,-4)

Example 3: Graph the linear equation $y = -\frac{1}{2}x - 3$

We now locate the three ordered pair solutions (points) on the rectangular coordinate grid, then draw a line through the solutions.



x	y	(x, y)
-2	-2	(-2, -2)
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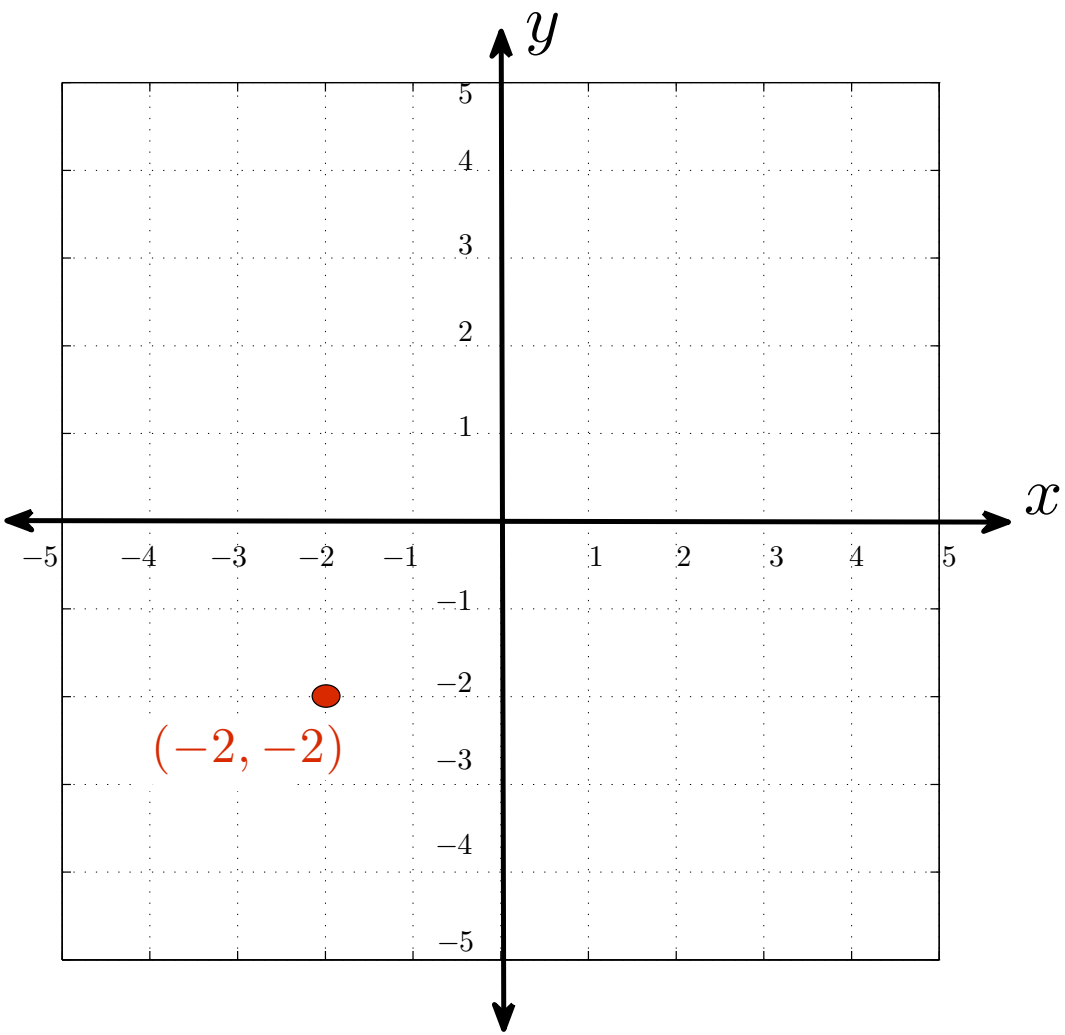

x	y	(x, y)
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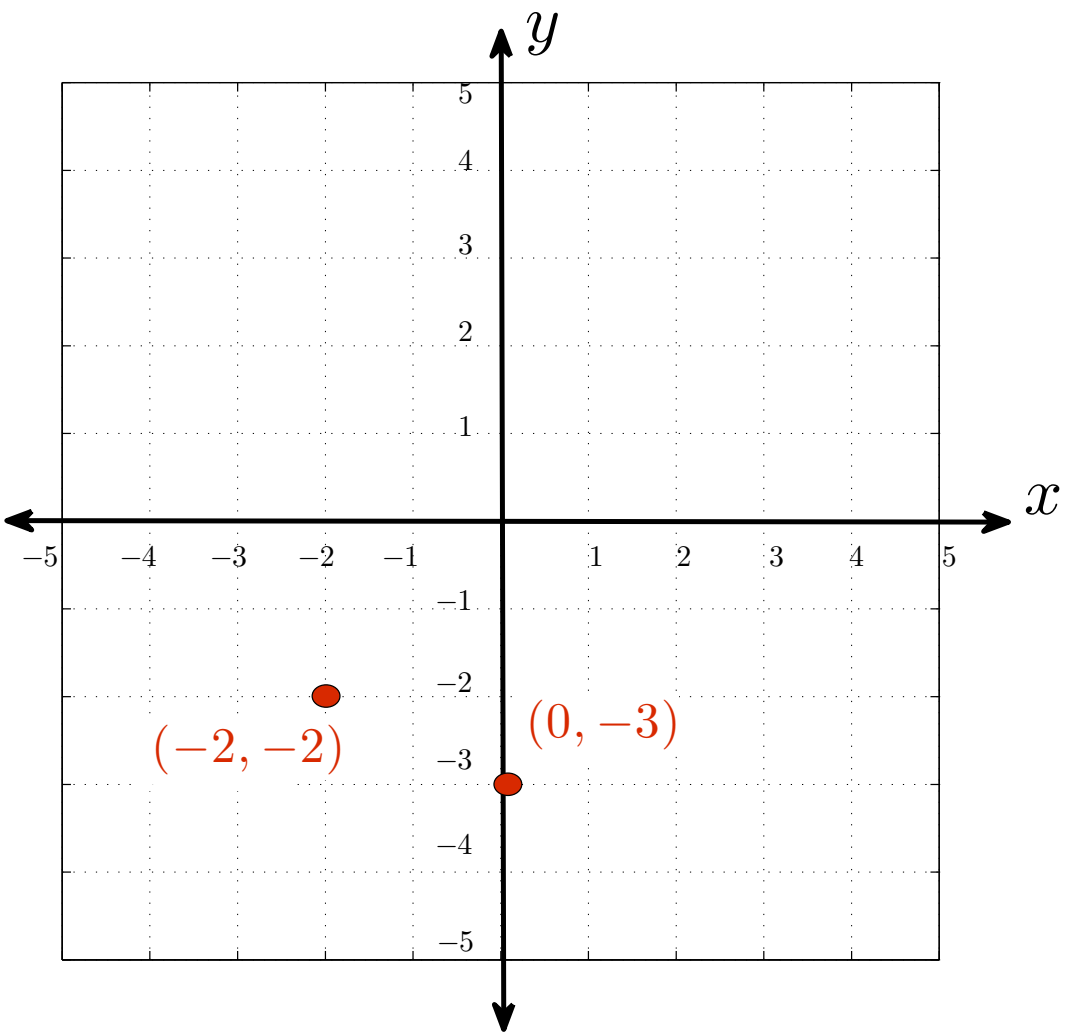
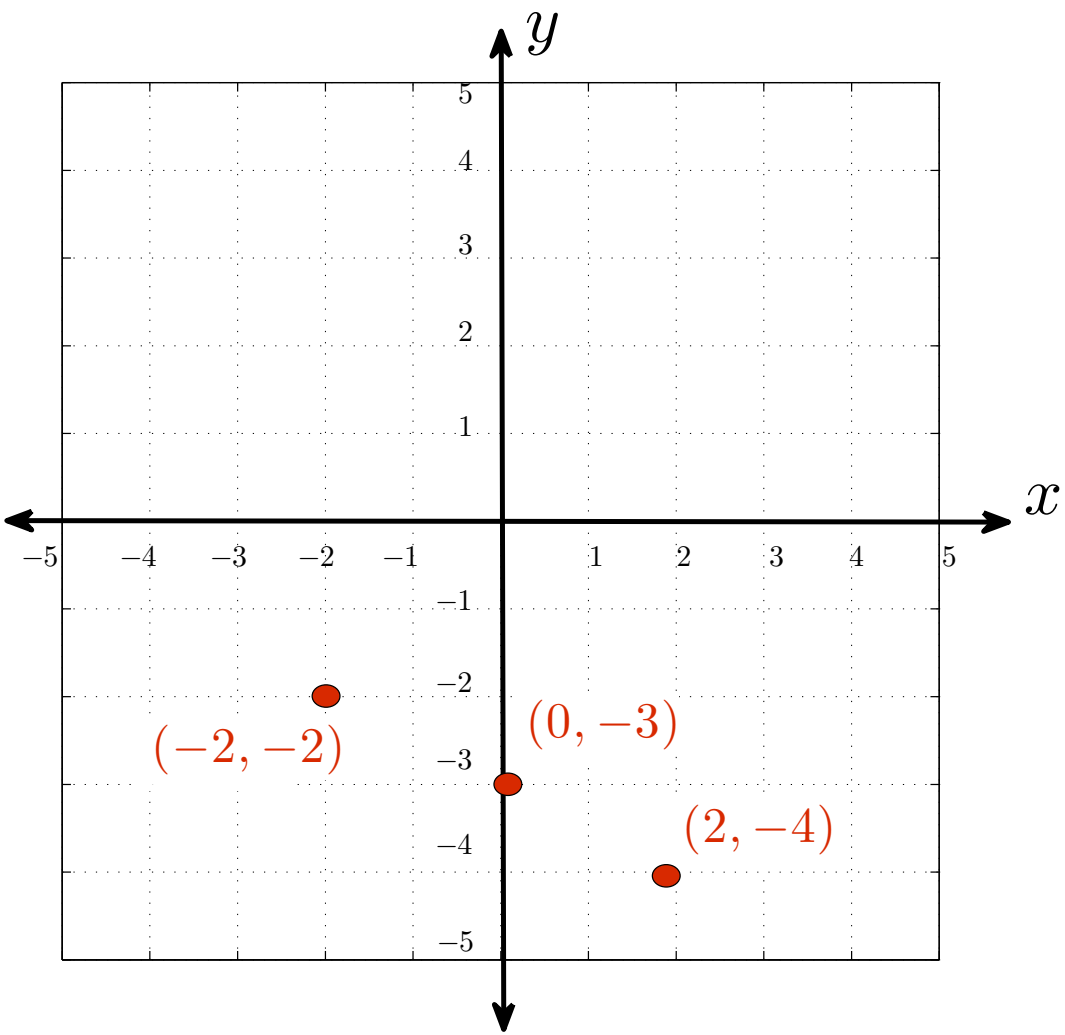


x	y	(x, y)
-2	-2	(-2, -2)
0	-3	(0,-3)
2	-5	(2, -4)

Example 3: Graph the linear equation $y = -\frac{1}{2}x - 3$

We now locate the three ordered pair solutions (points) on the rectangular coordinate grid, then draw a line through the solutions.



x	y	(x, y)
-2	-2	(-2, -2)
0	-3	(0,-3)
2	-5	(2, -4)

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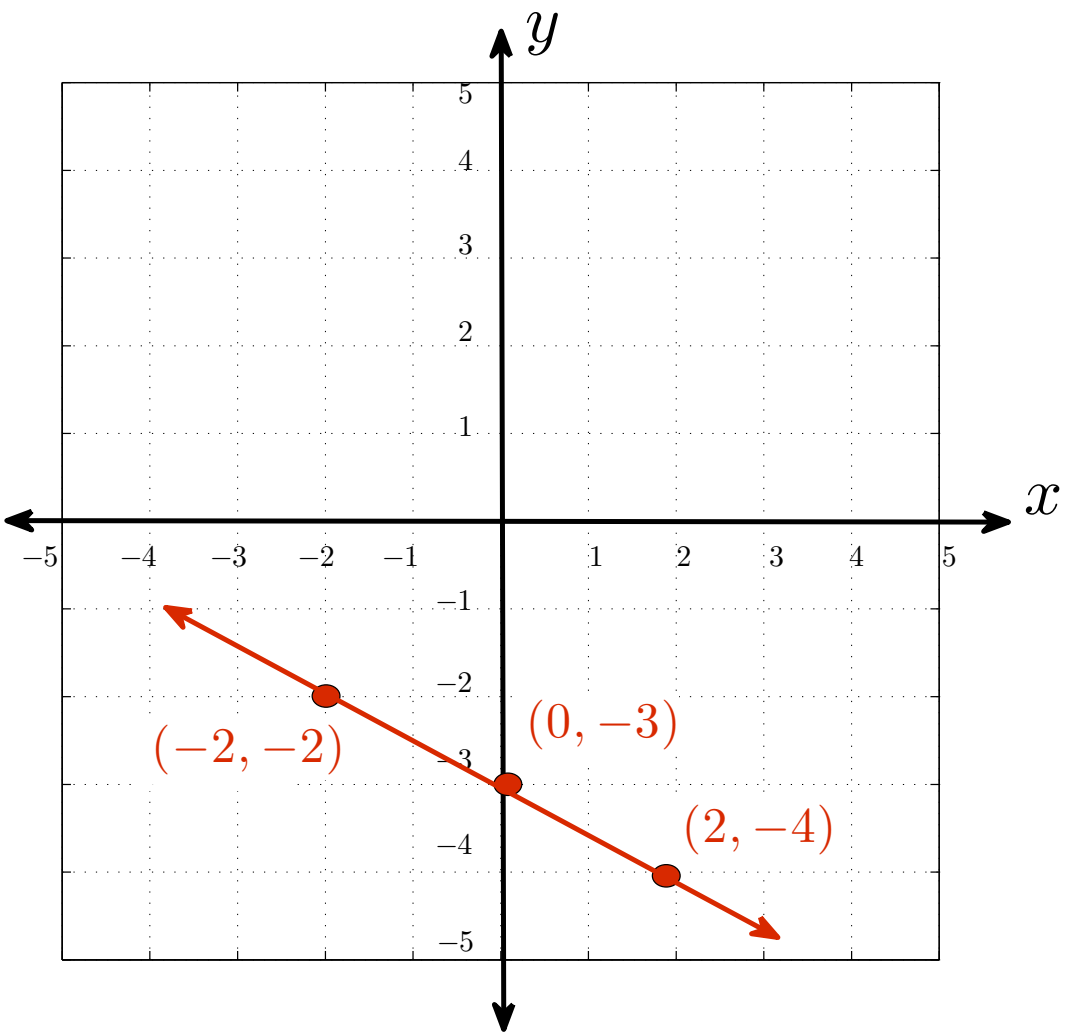

x	y	(x, y)
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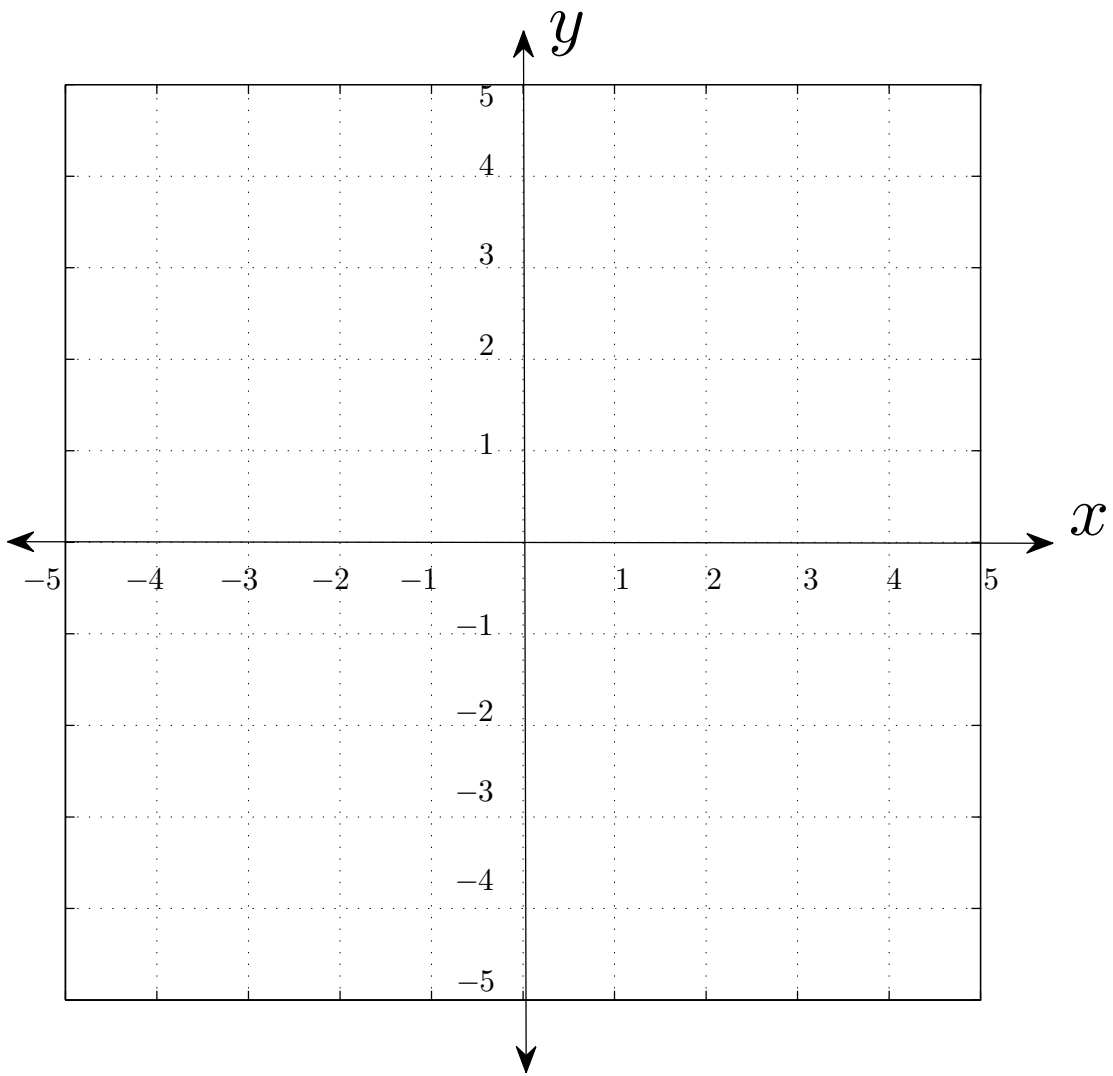
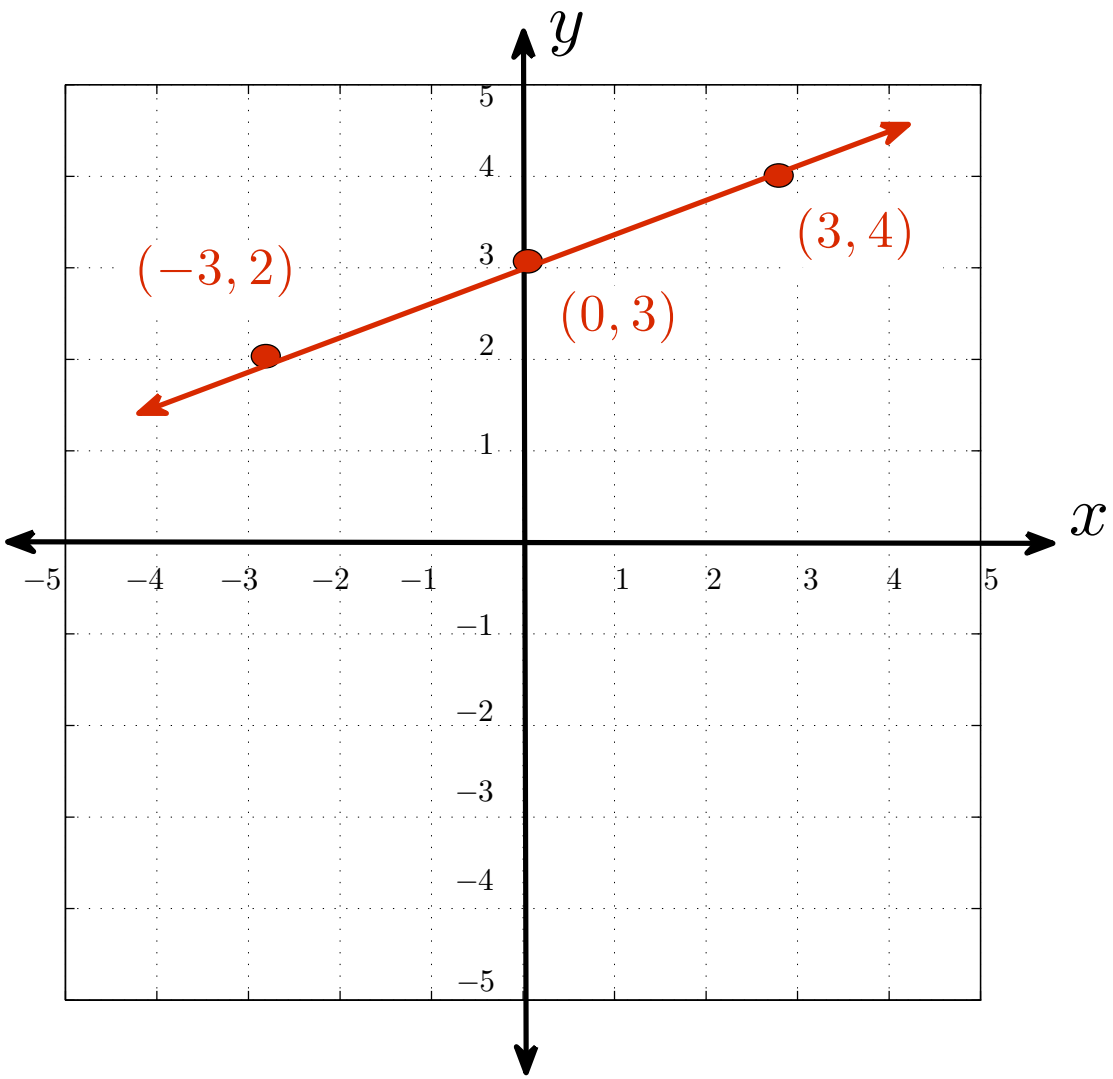
0	-3	(0,-3)
2	-5	(2, -4)

Concept Check: Graph $y = \frac{1}{3}x + 3$



x	y	(x, y)

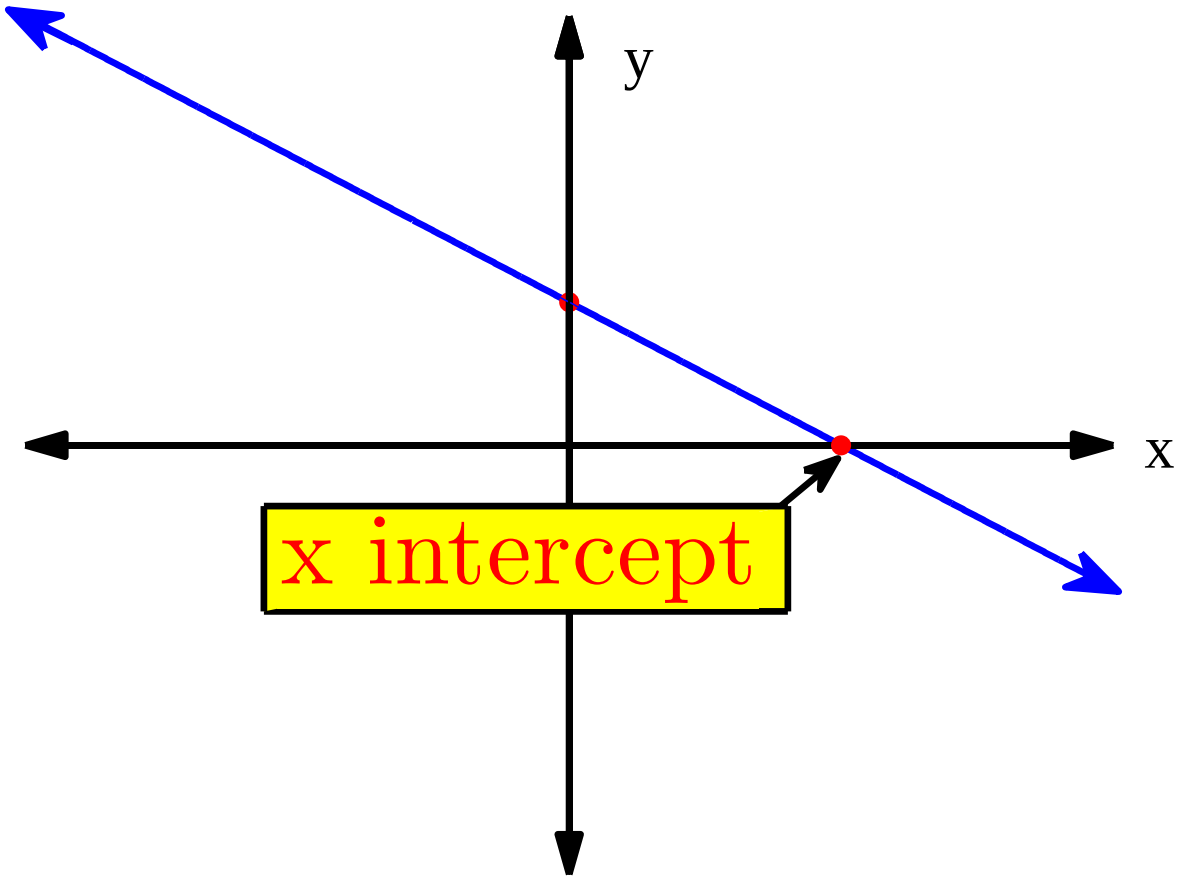
Concept Check: Graph $y = \frac{1}{3}x + 3$



x	y	(x, y)
-3	2	(-3,2)
0	3	(0,3)
3	4	(3,4)

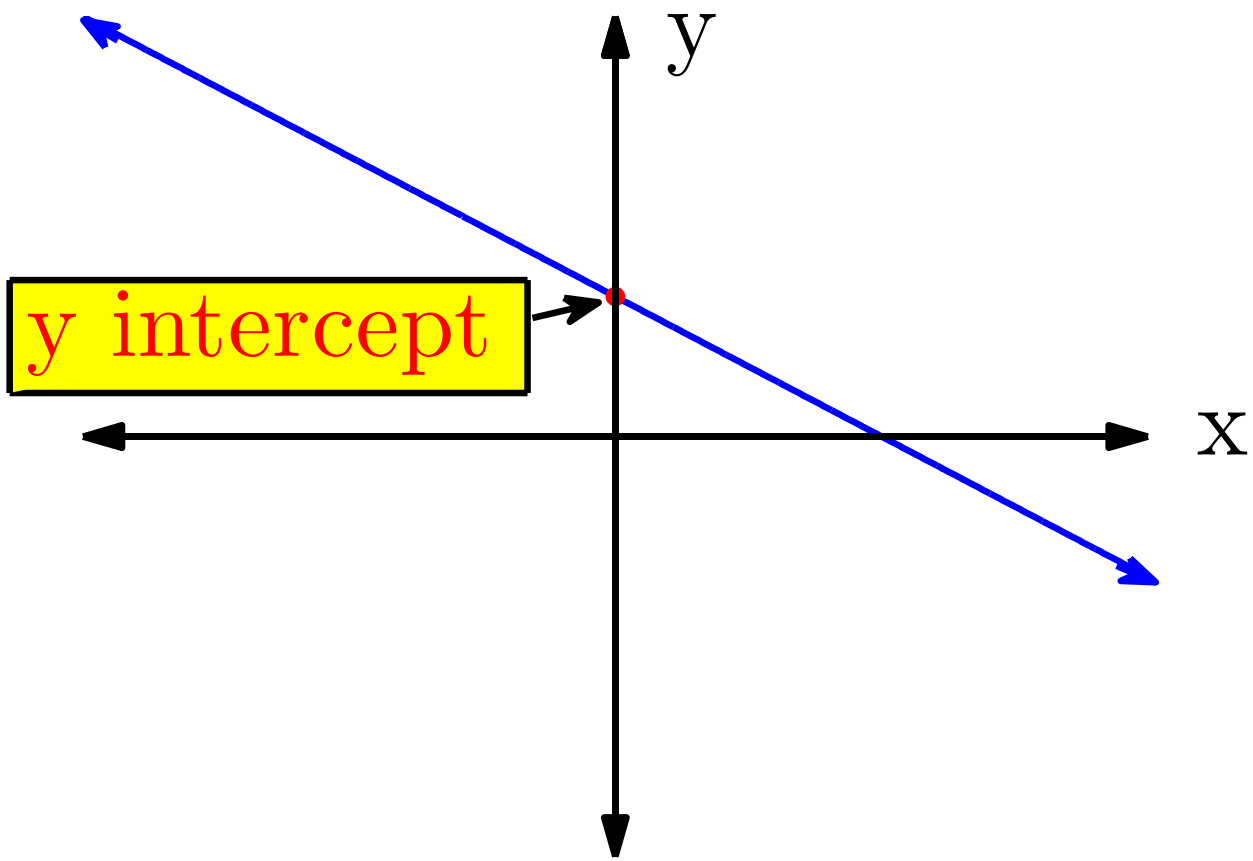
Definition

The graph of an equation has an **x-intercept** whenever the graph of the equation crosses the x axis. The x intercept always occurs when the value of y is equal to zero.



Definition

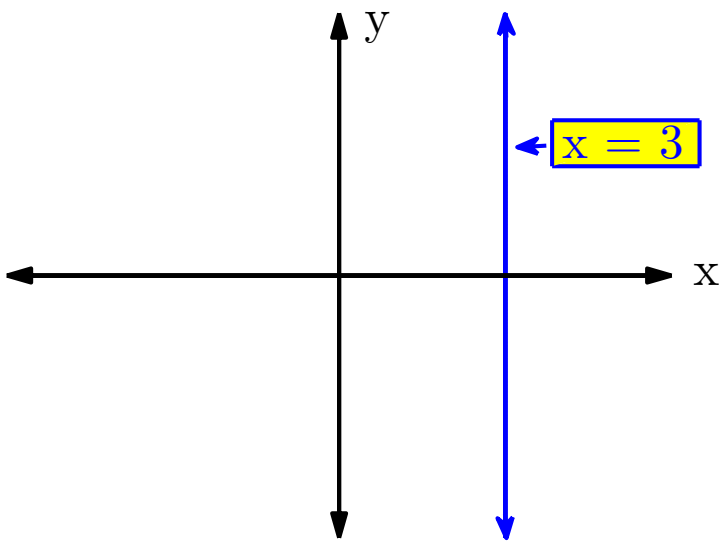
The graph of an equation has an **y-intercept** whenever the graph of the equation crosses the y axis. The y intercept always occurs when the value of x is equal to zero.



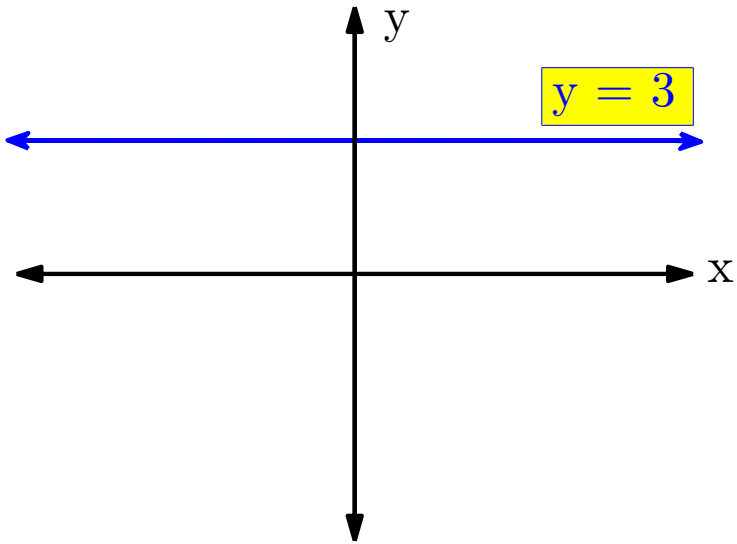
Example 4 Find the x- and y-intercepts for $5x - 7y = -35$, then graph the solution set.

Theorem

Suppose a and b are real numbers. Graphs of linear equations of the form $x = a$ are vertical lines and graphs of linear equations of the form $y = b$ are horizontal lines.



graph of $x = 3$



graph of $y = 3$

More Classroom Examples

Work Together! Stop at 8:35 a.m. for Quiz 1 Review
Graph each of the following lines:

- $y = \frac{1}{2}x$
- $x = -2$
- $y = -4$