Mini-Lecture 2.5

An Introduction to Problem Solving

Learning Objectives:

Apply the steps for problem solving as we

1. Solve problems involving direct translation.
2. Solve problems involving relationships among unknown quantities.
3. Solve problems involving consecutive integers.

Examples:

1. Solve.
   a) Eight is added to a number and the sum is doubled, the result is –11 less than the number. Find the number.

   b) Three times the difference of a number and 2 is equal to 8 subtracted from twice a number. Find the number.

2. Solve.

   a) A college graduating class is made up of 450 students. There are 206 more girls than boys. How many boys are in the class?

   b) A 22-ft pipe is cut into two pieces. The shorter piece is 7 feet shorter than the longer piece. What is the length of the longer piece?

   c) A triangle has three angles, A, B, and C. Angle C is 18° greater than angle B. Angle A is 4 times angle B. What is the measure of each angle?
   (Hint: The sum of the angles of a triangle is 180°).

3. Solve.
   a) The room numbers of two adjacent hotel rooms are two consecutive odd numbers. If their sum is 1380, find the hotel room numbers.

   b) When you open a book, the left and right page numbers are two consecutive natural numbers. The sum of their page numbers is 349. What is the number of the page that comes first?

Teaching Notes:

- Many students find application problems challenging.
- Encourage students, whenever possible, to draw diagrams, charts, etc.
- Encourage students to use algebra to solve a problem even though they may be able to solve without it.
- Refer students to ***General Strategy for Problem Solving*** section 2.4.
- Each section in the text has 3 worksheets in the Extra Practice featuring differentiated learning.

Answers: 1a) -27; 1b) -2; 2a) 122 boys; 2b) 14.5 feet; 2c) A=108°, B=27°, C=45°; 3a) 689, 691; 3b) 174

Mini-Lecture 2.6

Formulas and Problem Solving

Learning Objectives:

1. Use formulas to solve problems.
2. Solve a formula or equation for one of its variables.

Examples:

1. Substitute the given values into each given formula and solve for the unknown variable, If necessary, round to one decimal place.

   a) Distance Formula
   $d = rt$; $t = 9$, $d = 63$

   b) Perimeter of a rectangle
   $P = 2l + 2w$; $P = 32$, $w = 7$

   c) Volume of a pyramid
   $V = \frac{1}{3}Bh$; $V = 40$, $h = 8$

   d) Simple interest
   $I = prt$; $I = 23$, $p = 230$, $r = 0.02$

   e) Convert the record high temperature of 102°F to Celsius. ($F = \frac{9}{5}C + 32$)

   f) You have decided to fence an area of your backyard for your dog. The length of the area is 1 meter less than twice the width. If the perimeter of the area is 70 meters, find the length and width of the rectangular area.

   g) For the holidays, Chris and Alicia drove 476 miles. They left their house at 7 a.m. and arrived at their destination at 4 p.m. They stopped for 1 hour to rest and re-fuel. What was their average rate of speed?

2. Solve each formula for the specified variable.

   a) Area of a triangle
   $A = \frac{1}{2}bh$ for b

   b) Perimeter of a triangle
   $P = s_1 + s_2 + s_3$ for s_3

   c) Surface area of a special rectangular box
   $S = 4lw + 2wh$ for l

   d) Circumference of a circle
   $C = 2\pi r$ for r

Teaching Notes:

- Most students will only need algebra reminders when working with a formula given values.
- Refer students to ***Solving Equations for a Specified Variable*** chart in the textbook.
- Most students have problems with applications. Refer them back to section 2.4 and the ***General Strategy for Problem Solving*** in the textbook.
- Each section in the text has 3 worksheets in the Extra Practice featuring differentiated learning.

Answers: 1a) 7; 1b) 9; 1c) 15; 1d) 5; 1e) 38.9°C; 1f) l=23, w=12; 1g) 59.5 mph; 2a) $b = \frac{2A}{h}$; *2b)* $s_3 = P - s_1 - s_2$; *2c)* $\frac{S - 2wh}{4w}$; *2d)* $r = \frac{C}{2\pi}$

Mini-Lecture 2.7

Percent and Mixture Problem Solving

Learning Objectives:

1. Solve percent equations.
2. Solve discount and mark-up problems.
3. Solve percent increase and percent decrease problems.
4. Solve mixture problems.

Examples:

1. Find each number described.

   a) 5% of 300 is what number?
   b) 207 is 90% of what number?
   c) 15 is 1% of what number?
   d) What percent of 350 is 420?

2. Solve the following discount and mark-up problems. If needed, round answers to the nearest cent.

   a) A "Going-Out-Of-Business" sale advertised a 75% discount on all merchandise. Find the discount and the sale price of an item originally priced at $130.

   b) Recently, an anniversary dinner cost $145.23 excluding tax. Find the total cost if a 15% tip is added to the cost.

3. Solve the following percent increase and decrease problems.

   a) The number of minutes on a cell phone bill went from 1200 minutes in March to1600 minutes in April. Find the percent increase. Round to the nearest whole percent.

   b) In 2004, a college campus had 8,900 students enrolled. In 2005, the same college campus had 7,600 students enrolled. Find the percent decrease. Round to the nearest whole percent.

   c) Find the original price of a pair of boots if the sale price is $120 after a 20% discount.

4. A lab has a 20% acid solution and a 50% acid solution. How many liters of each are required to obtain 600 liters of a 30% solution?

Teaching Notes:

- Most students find problem solving challenging. Encourage students to make a list of all appropriate formulas.
- Each section in the text has 3 worksheets in the Extra Practice featuring differentiated learning.

Answers: 1a) 15; 1b) 230; 1c) 1500; 1d) 120%; 2a) discount - $97.50, sale price - $32.50; 2b) $167.01; 3a) 33%; 3b) 15%; 3c) $150; 4) 400 liters of 20%, 200 liters of 50%

Mini-Lecture 2.8

Further Problem Solving

Learning Objectives:

1. Solve problems involving distance.
2. Solve problems involving money.
3. Solve problems involving interest.

Examples:

1. How long will it take a car traveling 60 miles per hour to overtake an activity bus traveling 45-miles per hour if the activity bus left 2 hours before the car?

	D	***r***	***t***
Car			
Activity Bus			

2. A collection of dimes and quarters and nickels are emptied from a drink machine. There were four times as many dimes as quarters, and there were ten less nickels than there were quarters. If the value of the coins was$19.50, find the number of quarters, the number of dimes, and the number of nickels.

	Number	**Value of each**	**Total value**	
Quarters				
Dimes				
Nickels				
Entire Collection				

3. Jeff received a year end bonus of $80,000. He invested some of this money at 8% and the rest at 10%. If his yearly earned income was $7,300, how much did Jeff invest at 10%? Use the following table to model the situation.

	Principal	· Rate	· Time	= Interest
8% Fund				
10% Fund				
Total				

Teaching Notes:

- Most students find problem solving challenging. Encourage students to make a list of all appropriate formulas.
- Each section in the text has 3 worksheets in the Extra Practice featuring differentiated learning.

Answers: 1) 6 hours; 2) Number of Quarters = 40, Number of dimes = 80, number of nickels = 303; 3) $45,000

2.5

(1a) $(x+8)\cdot 2 = x-11$

$2x+16 = x-11$

$-x \quad -16 \qquad -x \quad -16$

$x+0 = 0-27$

$x = -27$

(1b) $3(x-2) = 2x-8$

$3x-6 = 2x-8$

$-2x+6 \qquad -2x+6$

$x+0 \qquad 0-2$

$x = -2$

(2a) boys + girls = 450

girls = boys + 206

boys

(2a) (numb. boys) + (numb. girls) = 450

(numb. girls) = 206 + (numb. boys) } given

Find the numb. of boys. Let x = numb. boys. Then numb. girls = $x+6$

$450 = (\text{numb}_{\text{boys}}) + (\text{numb. girls})$

and $450 = x + (206 + x)$, or $450 = 2x + 206$.

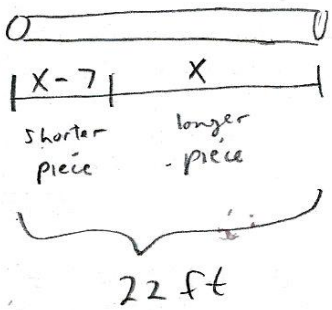
$-206 \qquad -206$ Subtract

$244 = 2x + 0$

So $2x = 244$, or

$\frac{2x}{2} = \frac{244}{2}$, or $\boxed{x = 122 \text{ boys}}$

(2b)



unknown: the length of the longer piece

Let x = the length of the longer piece.

Then, $x-7$ = the length of the shorter piece.

(Shorter piece) + (longer piece) = 22 ft

$(x-7) + x = 22$

$2x-7 = 22$

$+7 \qquad +7$

$2x+0 = 29$

$\frac{2x}{2} = \frac{29}{2}$

$\boxed{x = 14.5 \text{ ft}}$

(2C) (angle C) = 18 + (angle B)

(angle A) = 4·(angle B)

Let x = the measure of angle B.

The sum of the angle measures is 180°. We can write this sum in terms of x.

$$x + (18 + x) + (4x) = 180$$

$$6x + 18 = 180$$

$$-18 \quad -18$$

$$6x = 162$$

$$\frac{6x}{6} = \frac{162}{6}$$

$$x = 27°$$

$A = 108°$

$B = 27°$

$C = 45°$

(3a) $x + (x+2) = 1380$

$2x + 2 = 1380$

$-2 \quad -2$

$2x = 1378$

$\frac{2x}{2} = \frac{1378}{2}$

$x = 2\overline{)1378}$ = 689

-12

17

-16

18

689, 691

(3b) $x + (x+1) = 349$

$2x + 1 = 349$

$-1 \quad -1$

$2x = 348$

$\frac{2x}{2} = \frac{348}{2}$

$x = 174$

2.6

(1a) $d = rt$

$63 = (r)(9)$

$9r = 63$

$\frac{9r}{9} = \frac{63}{9}$

$\boxed{r = 7}$

(1b) $P = 2l + 2w$

$32 = 2l + 2(7)$

$32 = 2l + 14$

$32 - 14 = 2l$

$18 = 2l$

$\frac{18}{2} = \frac{2l}{2}$

$\boxed{l = 9}$

(1c) $V = \frac{1}{3}BH$

$40 = \frac{1}{3} \cdot B \cdot 8$

$\frac{8}{3}B = 40$

$\frac{3}{8} \cdot \frac{8}{3}B = \frac{3}{8} \frac{40}{1}^{5}$

$\boxed{B = 15}$

(1d) $I = prt$

$23 = (230)(0.02)t$

$23 = 4.6t$

$\frac{23}{4.6} = \frac{4.6}{4.6}t$

$\boxed{t = 5}$

(1e) $F = \frac{9}{5}C + 32$

$102 = \frac{9}{5}C + 32$

$102 - 32 = \frac{9}{5}C$

$70 = \frac{9}{5}C$

$\frac{5}{9} \cdot \frac{70}{1} = \frac{5}{9} \cdot \frac{9}{5}C$

$\boxed{C \doteq 38.9°}$

(1f) $l = 2w - 1$

$P = 70$

$P = 2w + 2l$

$P = 2w + 2(2w - 1)$

$70 = 2w + 4w - 2$

$70 = 6w - 2$

$72 = 6w$

$\frac{72}{6} = w$

$\boxed{w = 12}$

$\boxed{l = 23}$

(2a) $A = \frac{1}{2}bh$

$2A = 2 \cdot \frac{1}{2}bh$

$2A = bh$

$\frac{2A}{h} = b$

(2b) $P = S_1 + S_2 + S_3$

$-S_1 - S_2 \quad -S_1 - S_2$

$P - S_1 - S_2 = S_3$

$\boxed{S_3 = P - S_1 - S_2}$

(2c) $S = 4lw + 2wh$

$-2wh \quad -2wh$

$S - 2wh = 4lw + 0$

$\frac{4lw}{4w} = \frac{S - 2wh}{4w}$

$\boxed{l = \frac{S - 2wh}{4w}}$

(2d) $C = 2\pi r$

$\frac{C}{2\pi} = \frac{2\pi r}{2\pi}$

(1a) 5% of 300 is what number

↓ ↓ ↓ ↓ ↓

0.05 · 300 = x

300
× .05
15.00

15

(1b) 207 is 90% of what number?

↓ ↓ ↓ ↓ ↓

207 = 0.9 · x

$\frac{207}{0.9} = \frac{0.9x}{0.9}$

$x = .9\overline{)207.0} = 9\overline{)2070}$

230
−18
27
−27
00

x = 230

(1c) 15 is 1% of what number

↓ ↓ ↓ ↓ ↓

15 = 0.01 · x

$\frac{15}{0.01} = \frac{0.01x}{0.01}$, $x = 0.01\overline{)15.00}$

$= 1\overline{)1500} = 1500$

(1d) What percent of 350 is 420?

p · 350 = 420

350p = 420

$\frac{350}{350}p = \frac{420}{350}$; p = 1.2 = 120%

(2a) Take 75% off of $130.

75% of $130

.75 × $130 = $97.5 discount

$130.0
−97.5
$32.5

$32.50 sale price

(2b) 15% of $145.23

tip: 145.23 × 0.15 = $21.7845

total: 145.23 + 21.78 = $167.01

(3a) percent of original value = amount of increase

p · 1200 = 1600 − 1200

1200p = 400

$\frac{1200}{1200}p = \frac{400}{1200}$

p = 0.333...

= 33%

(3b) percent of original = amount of decrease

8900p = 8900 − 7600

8900p = 1300

$\frac{8900p}{8900} = \frac{1300}{8900}$

p ≈ 0.146 ≈ 15%

(3c) (Original price) − 20% of the original price = $120

Let x = the original price.

$$x - 0.2x = 120$$

$$0.8x = 120$$

$$\frac{0.8x}{0.8} = \frac{120}{0.8}$$

$$\boxed{x = 150}$$

---

(4) (20% acid soln)·(numb of liters of 20% acid soln) + (50% acid soln)(numb. liters of 50% soln) = (30% acid soln)(600 liters)

Let x = numb. liters of 20% soln
then 600 − x = numb. liters 50% soln.

$$0.2x + 0.5(600 - x) = (0.3)(600)$$

$$0.2x + 300 - 0.5 = 180$$

$$-0.3x + 300 = 180$$

$$-300 \quad -300$$

$$-0.3x = -120$$

$$\frac{-0.3x}{-0.3} = \frac{-120}{-0.3}$$

$$\boxed{x = 400 \text{ liters}}$$

$$600 - x = 600 - 400$$

$$= \boxed{200 \text{ liters}}$$

2.8

①

	D	r	t
car		60 mph	x
bus		45 mph	x+2

Distance is the same for both vehicles when the car overtakes the bus.

Let x = the number of hours it takes the car to overtake the bus.

The moment the car overtakes the bus, the bus has been on the road (x+2) hours.

Distance is the same for both vehicles too.

We can write distance for both vehicles in terms of x, then equate the distances.

$$60x = 45(x+2)$$

$$60x = 45x + 90$$

$$-45x \quad -45x$$

$$15x = 90$$

$$\frac{15x}{15} = \frac{90}{15}$$

$$x = 6 \text{ hours}$$