Mini-Lecture 2.1
Simplifying Algebraic Expressions

Learning Objectives:

1. Identify terms, like terms, and unlike terms.
2. Combine like terms.
3. Use the distributive property to remove parentheses.
4. Write word phrases as algebraic expressions.

Examples

1. Identify the numerical coefficient of each term.

    a) $9x$ b) $-3y$ c) $-x$ d) $2.7x^2y$

    Indicate whether the terms in each list are like or unlike.

    e) $6x$, $-3x$ f) $-xy^2$, $-x^2y$ g) $5ab$, $-\dfrac{1}{2}ba$ h) $2x^3yz^2$, $-x^3yz^3$

2. Simplify each expression by combining any like terms.

    a) $7x - 2x + 4$ b) $-9y + 2 - 1 + 6 + y - 7$ c) $1.6x^5 + 0.9x^2 - 0.3x^5$

3. Simplify each expression. Use the distributive property to remove any parentheses.

    a) $3(x + 6)$ b) $-(-5m + 6n - 2p)$ c) $\dfrac{1}{3}(6x - 9)$

    Remove parentheses and simplify each expression.

    d) $14(2x + 6) - 4$ e) $10a - 5 - 2(a - 3)$ f) $3(2x - 5) - (x + 7)$

4. Write each phrase as an algebraic expression. Simplify if possible.

    a) Add $-4y + 3$ to $6y - 9$ b) Subtract $2x -1$ from $3x + 7$

    c) Triple a number, decreased by six   d) Six times the sum of a number and two

Teaching Notes:

- Students will need repeated practice with identifying terms and like terms.
- Some students do not know that a variable without a numerical coefficient actually has a coefficient of 1.
- Some students will forget to distribute the minus sign in 3b), 3e), and 3f). Some students might need to write a 1 in front of the parentheses in 3b) and 3f).
- Each section in the text has 3 worksheets in the Extra Practice featuring differentiated learning.

Answers: *1a) 9; 1b) -3; 1c) -1; 1d) 2.7; 1e) like; 1f) unlike; 1g) like; 1h) unlike; 2a) 5x+4; 2b) -8y;*
2c) 1.3x⁵+0.9x²; 3a) 3x+18; 3b) 5m-6n+2p; 3c) 2x-3; 3d) 28x+80; 3e) 8a+1; 3f) 5x-22;
4a) (-4y+3) + (6y-9) = 2y-6; 4b) (3x+7) – (2x-1) = x + 8; 4c) 3x-6; 4d) 6(x + 2)

Mini-Lecture 2.2
The Addition Property of Equality

Learning Objectives:

1. Define linear equations and use the addition property of equality to solve linear equations
2. Write word phrases as algebraic expressions.

Examples:

1. Solve each equation. Check each solution.

   a) $y - 6 = 18$ b) $-18 = t - 5$ c) $8.1 + y = 13.9$ d) $a + \dfrac{2}{3} = -\dfrac{3}{4}$

   Solve each equation. If possible, be sure to first simplify each side of the equation. Check each solution.

   e) $5(y + 2) = 6(y - 3)$ f) $10x = 4x + 9 + 5x$

   g) $-8z + 5 + 6z = -3z + 10$ h) $-5x + 4 + 6x = 15 - 28$

   i) $-\dfrac{1}{6}x - \dfrac{1}{3} = \dfrac{5}{6}x + \dfrac{1}{2}$ j) $-14.9 + 4a - 2.7 + 2a = 5.1 + 7a + 1.5$

2. Write each algebraic expression described.

   a) Two numbers have a sum of 72. If one number is z, express the other number in terms of *z*.

   b) During a recent marathon, Tom ran 8 more miles than Judy ran. If Judy ran *x* miles, how many miles did Tom run?

   c) On a recent car trip, Raymond drove *x* miles on day one. On day two, he drove 170 miles more than he did on day one. How many miles, in terms of *x*, did Raymond drive for both days combined?

Teaching Notes:

- Some students need a quick review of "like terms".
- Advise students to write out each step until they have mastered this concept. Avoid shortcuts!
- Some students need to be taught how to work a problem in sequential order showing each step.
- Encourage students to take their time and organize their work. This will help when the problems become more complex.
- Each section in the text has 3 worksheets in the Extra Practice featuring differentiated learning.

Answers: 1a) 24; 1b) -13; 1c) 5.8; 1d) -17/12; 1e) 28; 1f) 9; 1g) 5; 1h) -17; 1i) -5/6; 1j) 11;
2a) 72 – z; 2b) x+8; 2c) 2x+ 170

Mini-Lecture 2.3
The Multiplication Property of Equality

Learning Objectives:

1. Use the multiplication property of equality to solve linear equations.
2. Use both the addition and multiplication properties of equality to solve linear equations.
3. Write word phrases as algebraic expressions.

Examples:

1. Use the multiplication property of equality to solve the following linear equations. Check each solution.

   a) $-8x = -24$

   b) $7x = 0$

   c) $-z = 19$

   d) $3x = -22$

   e) $\dfrac{2}{5}a = 12$

   f) $\dfrac{y}{-11} = 2.5$

   g) $\dfrac{-3}{8}b = 0$

   h) $-10.2 = -3.4c$

2. Use the addition property of equality and the multiplication property of equality to solve the following linear equations. Check each solution.

   a) $5x + 6 = 46$

   b) $\dfrac{a}{9} - 7 = 11$

   c) $-24 = -3x - 9$

   d) $\dfrac{1}{3}y - \dfrac{1}{3} = -6$

   e) $-5.8z + 1.9 = -32.5 - 1.5z$

   f) $8y + 7 = 6 - 2y - 10y$

   g) $4(4x - 1) = (-8) - (-24)$

3. Write each algebraic expression described. Simplify if possible.

   a) If z represents the first of two consecutive even integers, express the sum of the two integers in terms of z.

   b) If x represents the first of three consecutive even integers, express the sum of the first and third integer in terms of x.

   c) Houses on one side of a street are all numbered using consecutive odd integers. If the first house on the street is numbered $x,$ write an expression in x for the sum of five house numbers in a row.

Teaching Notes:

- Review "like terms" with students.
- Many students do not combine like terms before using one of the properties.
- Encourage students to always take the time to check their solution.
- Each section in the text has 3 worksheets in the Extra Practice featuring differentiated learning.

Answers: *1a) 3; 1b) 0; 1c) -19; 1d) -22/3; 1e) 30; 1f) -27.5; 1g) 0; 1h) 3; 2a) 8; 2b) 162; 2c) 5; 2d) -17; 2e) 8; 2f) -1/20; 2g) 5/4; 3a) 2z+2; 3b) 2x+4; 3c) 5x+20*

Mini-Lecture 2.4
Solving Linear Equations

Learning Objectives:

1. Apply a general strategy for solving a linear equation.
2. Solve equations containing fractions.
3. Solve equations containing decimals.
4. Recognize identities and equations with no solution.

Examples:

1. Solve the following linear equations.

   a) $6a - (5a - 1) = 4$

   b) $4(3b - 1) = 16$

   c) $4z = 8(2z + 9)$

   d) $2(x + 8) = 3(x - 5)$

   e) $3(2a - 3) = 5(a + 4)$

   f) $12(4c - 2) = 3c - 4$

2. Solve each equation containing fractions.

   a) $\dfrac{y}{6} - 4 = 1$

   b) $\dfrac{1}{4}x - \dfrac{3}{8}x = 5$

   c) $\dfrac{-6x + 5}{4} + 1 = -\dfrac{5x}{4}$

   Solve each equation containing decimals.

   d) $0.05x + 0.06(x - 1,500) = 570$

   e) $0.4(x + 7) - 0.1(3x + 6) = -0.8$

3. Solve each equation. Indicate if it is an identity or an equation with no solution.

   a) $6(z + 7) = 6z + 42$

   b) $3 + 12x - 1 = 8x + 4x - 1$

   c) $\dfrac{x}{3} - 3 = \dfrac{2x}{6} + 1$

Teaching Notes:

- Refer students to the beginning of this section in the textbook for steps: To Solve Linear Equations in One Variable.
- Most students find solving equations with fractions or decimals difficult.
- Common error: When multiplying equations with fractions by the LCD, some students multiply only the terms with fractions instead of all terms.
- Common error: When solving equations with decimals and parentheses (examples 2d and 2e), some students multiply terms both inside parentheses and outside parentheses by a power of 10.
- Each section in the text has 3 worksheets in the Extra Practice featuring differentiated learning.

Answers: 1a) 3; 1b) 5/3; 1c) -6; 1d) 31; 1e) 29; 1f) 4/9; 2a) 30; 2b) -40; 2c) 9; 2d) 6,000 2e) -30; 3a) identify; 3b) no solution; 3c) no solution

Mini-Lecture 2.9
Solving Linear Inequalities

Learning Objectives:

1. Define linear inequality in one variable, graph solution sets on a number line, and use interval notation.
2. Solve linear inequalities.
3. Solve compound inequalities.
4. Solve inequality applications.
5. Key Vocabulary: *inequality, $<, \leq, >, \geq$, addition property of inequality, multiplication property of inequality, at least, no less than, at most, no more than, is less than, is greater than.*

Examples:

1. Graph each inequality on a number line and write it in interval notation.

   a) $x \geq -5$ b) $y < 7$ c) $-\dfrac{3}{2} \geq m$ d) $x > -\dfrac{2}{5}$

2. Using the addition property of inequality, solve each inequality. Graph the solution set and write it in interval notation.

   a) $x + 7 \leq 12$ b) $x - 10 > -3$ c) $-4z - 2 > -5z + 1$ d) $18 - 2x \leq -3x + 24$

   Using the multiplication property of inequality, solve each inequality. Graph the solution set and write it in interval notation.

   e) $-8 \geq \dfrac{x}{3}$ f) $3x < 73$ g) $0 < \dfrac{y}{8}$ h) $-\dfrac{3}{5}z \leq 9$

   Using both properties, solve each inequality.

   i) $3(3x - 16) < 12(x - 2)$ j) $-18(z - 2) \geq -21z + 24$ k) $\dfrac{8}{21}(x + 2) > \dfrac{1}{7}(x + 3)$

3. Solve each inequality. Graph the solution set and write it in interval notation.
   a) $-5 < t \leq 0$ b) $-12 \leq 2x < -8$ c) $3 \leq 4x - 9 \leq 7$

4. Solve the following.

   a) Eight more than twice a number is less than negative twelve. Find all numbers that make this statement true.

   b) One side of a triangle is six times as long as another side and the third side is 8 inches long. If the perimeter can be no more than 106 inches, find the maximum lengths of the other two sides.

Teaching Notes:
- Remind students to reverse the direction of the inequality symbol when multiplying or dividing by a negative number.
- Suggest students keep the coefficient of the variable positive whenever possible.
- Each section in the text has 3 worksheets in the Extra Practice featuring differentiated learning.

Answers: *1a) – 3c) graph answers at end of mini-lectures; 1a) [-5,∞); 1b) (∞,7); 1c) [- 3/2, ∞); 1d) (-2/5, ∞);
2a) (−∞,5] ; 2b) (7,∞) ; 2c) (3,∞) ; 2d) (−∞,6] ; 2e) (−∞,−24] ; 2f) (−∞,24 1/3) ; 2g) (0,∞) ;
2h) [−15,∞) ; 2i) (−8,∞) ; 2j) [−4,∞) ; 2k) (7/5, ∞); 3a) (−5,0] ; 3b) [-6,4); 3c) [3,4]; 4a) x<-10;
4b) 14, 84*