Mini-Lecture 4.1

Solving Systems of Linear Equations by Graphing

Learning Objectives:

1. Determine if an ordered pair is a solution of a system of equations in two variables.
2. Solve a system of linear equations by graphing.
3. Without graphing, determine the number of solutions of a system.

Examples:

1. Determine whether the ordered pair is a solution of the system of linear equations.

a) $(-4,-5)\begin{cases} x+y=-9 \\ x-y=1 \end{cases}$

b) $(-5,-3)\begin{cases} 2x+y=-7 \\ 3x+2y=-9 \end{cases}$

c) $(2,-4)\begin{cases} 4x=4-y \\ 2x=-12-4y \end{cases}$

d) $(-3,1)\begin{cases} 3x=10-y \\ 4x=15-3y \end{cases}$

2. Solve each system of linear equations by graphing. Note: All systems have a solution.

a) $\begin{cases} 4x+y=-4 \\ 5x+2y=-2 \end{cases}$

b) $\begin{cases} 3x+2y=22 \\ 2x+4y=28 \end{cases}$

c) $\begin{cases} x=6 \\ \frac{1}{6}x-y=1 \end{cases}$

d) $\begin{cases} 2x+5y=32 \\ 3y=20-2x \end{cases}$

3. Without graphing, determine the number of solutions of a system.
Note: the systems have no solution or an infinite number of solutions.

a) $\begin{cases} 4x-16y=12 \\ y=\frac{1}{4}x-\frac{3}{4} \end{cases}$

b) $\begin{cases} -x=y \\ x=6-y \end{cases}$

Teaching Notes:

- Many students need to be reminded to use graph paper and be very neat with their graphing skills.
- Remind students to substitute their solution into the original equations to check their results.
- Many students get confused between consistent and inconsistent systems and its meaning for the solution.
- Each section in the text has 3 worksheets in the Extra Practice featuring differentiated learning.

Answers: 1a) yes; 1b) no; 1c) yes; 1d) no; 2a) – 2d) see mini-lecture graphing answers; 3a) infinite; 3b) no solution

Mini-Lecture 4.2

Solving Systems of Linear Equations by Substitution

Learning Objectives:

1. Use the substitution method to solve a system of linear equations.

Examples:

1. Solve each system of equations by the substitution method. Note: the following systems have one equation already solved for one variable.

a) $\begin{cases} x + y = 9 \\ y = 2x \end{cases}$

b) $\begin{cases} x = y - 2 \\ x + y = 6 \end{cases}$

Solve each system of equations by the substitution method.

c) $\begin{cases} x + 6y = 2 \\ 4x + 5y = -11 \end{cases}$

d) $\begin{cases} x - 3y = 3 \\ -5x - 2y = 2 \end{cases}$

e) $\begin{cases} x - 4y = -1 \\ 6x - 3y = -6 \end{cases}$

f) $\begin{cases} 6x + 7y = 33 \\ 3x - 3y = -42 \end{cases}$

g) $\begin{cases} 4x - 3y = 30 + x \\ 4x = -(y + 2) + 3x \end{cases}$

h) $\begin{cases} x - y = -4 \\ \frac{1}{2}x + \frac{1}{2}y = -3 \end{cases}$

i) $\begin{cases} 4x + y = 11 \\ 12x + 3y = 33 \end{cases}$

j) $\begin{cases} -6x - 24y = -10 \\ 5x + 20y = 0 \end{cases}$

Teaching Notes:

- Remind students to check their solution in the *original* equations.
- Many students write their final answer as x = a number and y = a number rather than an ordered pair (x, y).
- Many students find working with fractional coefficients challenging.
- Refer students to the textbook's summary "To Solve a System of Two Linear Equations by the Substitution Method".
- Each section in the text has 3 worksheets in the Extra Practice featuring differentiated learning.

Answers: 1a) (3, 6); 1b) (2, 4); 1c) (-4, 1); 1d) (0, -1); 1e) (-1, 0); 1f) (-5, 9) ; 1g) (4, -6); 1h) (-5, -1); 1i) infinite; 1j) no solution.

Mini-Lecture 4.3

Solving Systems of Linear Equations by Addition

Learning Objectives:

1. Use the addition method to solve a system of linear equations.

Examples:

1. Solve each system of equations by the addition method.

a) $\begin{cases} x+y=5 \\ x-y=11 \end{cases}$

b) $\begin{cases} -x+4y=28 \\ -6x-4y=-56 \end{cases}$

Solve each system of equation by the addition method. If a system contains fractions or decimals, you may want to first clear each equation of fractions or decimals.

c) $\begin{cases} x+5y=49 \\ -7x+4y=-31 \end{cases}$

d) $\begin{cases} x+3y=2 \\ 4x+2y=18 \end{cases}$

e) $\begin{cases} -2x-7y=-6 \\ 5x-3y=-26 \end{cases}$

f) $\begin{cases} 5x+8y=1 \\ 2x+3y=2 \end{cases}$

g) $\begin{cases} -x-2y=-4 \\ 5x+10y=8 \end{cases}$

h) $\begin{cases} 4x-6y=1 \\ 20x-30y=3 \end{cases}$

i) $\begin{cases} 3x+\frac{1}{3}y=10 \\ 2x+\frac{2}{3}y=4 \end{cases}$

j) $\begin{cases} 3.5x+0.3y=-18.7 \\ 0.7x+0.9y=-7.1 \end{cases}$

Teaching Notes:

- Encourage students to discuss which variable is the easiest to eliminate and what number an equation should be multiplied by to make the elimination possible.
- Remind students that there can be more than one way to solve a system.
- Remind students to check their solution.
- Each section in the text has 3 worksheets in the Extra Practice featuring differentiated learning.

Answers: *1a) (8, -3); 1b) (4, 8); 1c) (9, 8); 1d) (5, -1); 1e) (-4, 2); 1f) (13, -8); 1g) no solution; 1h) no solution; 1i) (4, -6); 1j) (-5, -4)*

Mini-Lecture 4.4

Systems of Linear Equations and Problem Solving

Learning Objectives:

1. Use a system of equations to solve problems.

Examples:

1. Solve .

a) ***Finding Unknown Numbers:*** The sum of two numbers is 7. Three times the first number equals 4 times the second number. Find the two numbers.

b) ***Finding Unknown Numbers:*** One number is four more than a second number. Two times the first number is 2 more than four times the second number.

c) ***Solving a Problem about Prices:*** Alicia purchased tickets to a local comedy club for 5 adults and 2 children. The total cost was $161. The cost of a child's ticket was $7 less than the cost of an adult's ticket. Find the price of an adult's ticket and a child's ticket.

d) ***Solving a Problem about Prices:*** Allison throws loose change found in the laundry into container. After one month, she finds it contains all nickels and dimes. In fact, there are 4 times as many dimes as nickels, and the value of the dimes is $3.50 more than the value of the nickels. How many nickels and dimes does Allison have?

e) ***Finding Rates:*** Kyle and Jason live 28 miles apart in Central Massachusetts. They decide to bicycle towards each other and meet somewhere in between. Kyle' rate of speed is 40% of Jason's. They start out at the same time and meet 2 hours later. Find Kyle's rate of speed.

f) ***Finding Amounts of Solutions:*** Amy has 3 liters of a 35% solution of sodium hydroxide in a container. What is the amount and concentration of sodium hydroxide solution she must add to this in order to end up with 7 liters of 27% solution?

Teaching Notes:

- Most students struggle with word problems.
- Refer students to the textbook's *Problem-Solving Steps* for guidance.
- Encourage students to draw and label diagrams or construct charts whenever possible.
- Entertain a discussion around which algebraic method, substitution or addition, is appropriate for the word problem.
- Remind students to always check their answer.
- Each section in the text has 3 worksheets in the Extra Practice featuring differentiated learning.

Answers: 1a) 4,3; 1b) 7,3; 1c) $25, $18; 1d) 10 nickels, 40 dimes; 1e) 4 mph; 1f) 4 liters of 21% solution

Mini-Lecture 4.5

Graphing Linear Inequalities

Learning Objectives:

1. Graph a linear inequality in two variables.

Examples:

1. Determine whether the ordered pairs given are solutions of the linear inequality in two variables.

   a) $x - y > -2;\ (0, -1),\ (1, 4)$ b) $2x + 4y \geq 6;\ (4, -1), (-3, 3)$

   c) $x > -y;\ (0, 0),\ (3, -2)$ d) $y > \frac{1}{3}x - 1;\ (0, 0),\ (-3, -1)$

   Graph each inequality.

   e) $x + y \geq 2$ f) $y < -\frac{1}{5}x$ g) $x - y > -3$

   h) $2x + y \leq -5$ i) $-2x - 3y < 6$ j) $x > y$

   k) $y \geq 2$ l) $x < 5$ m) $y \geq 0$

Teaching Notes:

- Most students who are good at graphing equalities will find this section easy.
- Although many students do not understand the region they are testing in problems 1a) – 1d), most need practice in testing before they begin graphing inequalities.
- Remind students to always use a test point from their proposed solution region to check their work.
- Remind students that the boundary line is dashed for < or > and solid for $\leq$ or $\geq$.
- Refer students to the gray instruction block: *To Graph a Linear Inequality in Two Variables.*
- Each section in the text has 3 worksheets in the Extra Practice featuring differentiated learning.

Answers: 1a) yes, no; 1b) no, yes; 1c) no, yes; 1d) yes, yes; 1e) – 1m) see mini-lecture graphing answers.

Mini-Lecture 4.6

Systems of Linear Inequalities

Learning Objectives:

1. Graph a system of linear inequalities.

Examples:

1. Graph the solution to the following system.

a) $\begin{cases} 2x \le y \\ x + y \ge 2 \end{cases}$ b) $\begin{cases} x - y > 3 \\ y < 2 \end{cases}$

c) $\begin{cases} x \ge -3 \\ y < 2 \end{cases}$ d) $\begin{cases} 3x > -6 \\ x + y \le -2 \end{cases}$

Teaching Notes:

- Students may have difficulty finding the solution region even when both inequalities are graphed correctly. Have students shade each inequality with a different color pencil or shading each at a different angle.
- Each section in the text has three worksheets in the Extra Practice featuring differentiated learning.

Answers

1a-d) graph answers at end of mini-lectures