Mini-Lecture 5.1

Exponents

Learning Objectives:

1. Evaluate exponential expressions.
2. Use the product rule for exponents.
3. Use the power rule for exponents.
4. Use the power rule for products and quotients.
5. Use the quotient rule for exponents, and define a number raised to the 0 power.
6. Decide which rule(s) to use to simplify an expression.

Examples:

1. Evaluate each expression.

   a) 3^3 b) $(-7)^2$ c) -6^2 d) $-4y^2$ when $y = -5$

2. Use the product rule to simplify each expression. Write the results using exponents.

   a) $x^5 \cdot x^3$ b) $(4z^3)(9z^5)$ c) $(-3x^3y^2)(-5x^4y^6)$ d) $(9ab^2c^4)(-11a^3b)(-2b^2c^5)$

3. Use the power rules to simplify each expression.

   a) $(x^7)^3$ b) $(y^3)^{11}$ c) $(xy)^5$ d) $(5x^3y^2z)^3$

4. Use the power rule for products and quotients.

   a) $(-7a^3b^3)^2$ b) $\left(\frac{ab}{c}\right)^7$ c) $\left(\frac{-3xy}{z^3}\right)^4$ d) $\left(\frac{3x^2y^4}{-2z^3}\right)^2$

5. Use the quotient rule and simplify each expression.

   a) $\frac{x^5}{x^2}$ b) $\frac{(-6)^{11}}{(-6)^9}$ c) $\frac{x^{12}y^5}{x^8y^4}$ d) $\frac{8a^3b^8c^3}{18ab^5c^2}$

   Simplify each expression.

   e) 8^0 f) $\left(\frac{3}{7}\right)^0$ g) $(5x^2y)^0$ h) $x^0 + 9^0$

6. Mixed practice. Decide which rules to use and simplify each expression.

   a) $(8a^3b^2c^0)^2$ b) $\left(\frac{-3x^2y^5}{2xz^2}\right)^3$ c) $(-4a^2c^3)(-6a^3b^2c^7)$ d) $\frac{(12ab)^4}{(6a^2b^2)^2}$

Teaching Notes:

- Most students need a lot of practice to master these rules.
- Each section in the text has 3 worksheets in the Extra Practice featuring differentiated learning.

Answers: *1a) 27; 1b) 49; 1c) -36; 1d) -100; 2a) x^8; 2b) $36z^8$; 2c) $15x^7y^8$; 2d) $198a^4b^5c^9$; 3a) x^{21}; 3b) y^{33}; 3c) x^5y^5; 3d) $125x^9y^6z^3$; 4a) $49a^6b^6$; 4b) $\frac{a^7b^7}{c^7}$; 4c) $\frac{81x^4y^4}{z^{12}}$; 4d) $\frac{9x^4y^8}{4z^6}$; 5a) x^3; 5b) 36; 5c) x^4y; 5d) $\frac{4a^2b^3c}{9}$; 5e) 1; 5f) 1; 5g) 1; 5h) 2; 6a) $64a^6b^4$; 6b) $\frac{-27x^4y^{15}}{4z^6}$; 6c) $24a^5b^2c^{10}$; 6d) 576*

Mini-Lecture 5.2

Adding and Subtracting Polynomials

Learning Objectives:

1. Define polynomial, monomial, binomial, trinomial, and degree.
2. Find the value of a polynomial given replacement values for the variables.
3. Simplify a polynomial by combining like terms.
4. Add and subtract polynomials.

Examples:

1. Find the degree of each polynomial and determine whether it is a monomial, binomial, trinomial, or none of these.

   a) $x^2 + x - 6$ b) $3x + 10$ c) $10x^3y^2z$ d) $8z^5 + 9$

   Identify the degrees of the terms and degree of the polynomial.

   e) $2xy - 5x + 6xy$ f) $4a^3 - 3a + 6$ g) $x^3y - x^2y^2 + xy^3$ h) $s^5t^2 - 3s^4t + 5st$

2. Evaluate each polynomial when (a) y = 0; and (b) y = -2

   a) $4y - 7$ b) $y^3 - 4$ c) $3y^2 + 8y - 9$ d) $-13 - 4y - y^2$

3. Simplify each expression buy combining like terms.

   a) $3x - 10x$ b) $3y^3 - 6x^2 + 2x^2 - 5y^3$ c) $3.7x^3 - 6.3x + 11.6 + 1.8x - x^3 - 8.2$

4. Perform the indicated operation.

   a) $(4x - 3) + (2x - 7)$ b) $(5x^2 + 3x - 7) - (-5x - 3)$ c) Subtract $(3x + 2y)$ from $(5x - 7y)$

Teaching Notes:

- Most students find these objectives easy.
- Some students, when identifying the degree of a polynomial, get confused when one term is made of different variables.
- Each section in the text has 3 worksheets in the Extra Practice featuring differentiated learning.

Answers: 1a) 2, trinomial; 1b) 1, binomial; 1c) 3, monomial; 1d) 5, binomial; 1e) 2,1,2,2; 1f) 3, 1, 3; 1g) 4, 4, 4, 4; 1h) 7, 5, 2, 7; 2a) -7, -15; 2b) -4, -12; 2c) -9, -13; 2d) –13, -9; 3a) -7x; 3b) $-2y^2-4x^2$; 3c) $2.7x^3-4.5x+3.4$; 4a) 6x-10; 4b) $5x^2+8x-4$; 4c) 2x-9y

Mini-Lecture 5.3

Multiplying Polynomials

Learning Objectives

1. Use the distributive property to multiply polynomials.
2. Multiply polynomials vertically.

Examples:

1. Multiply the following monomials.

   a) $4x^3 \cdot 2x^6$ b) $(-3t^4)(5t^3)$ c) $(-4.2x^3)(5.1x^5)$ d) $\left(-\frac{2}{7}a^4\right)\left(\frac{7}{8}a^7\right)$

   Multiply the monomial by the polynomial.

   e) $5a(-12a - 6)$ f) $4x^3(-7x + 1)$

   g) $-6y^5(8y^4 - 12y^2)$ h) $3ab^7\left(3ab^3 - 12b^2 - 4a\right)$

   Multiply the following binomials.

   i) $(x + 3)(x - 5)$ j) $(3x^2 - 4)(2x^3 + 5)$ k) $\left(x + \frac{3}{4}\right)^2$ l) $(1 - 5x)(2 - 3x)$

   m) $(y - 12)(y^2 + 6y - 3)$ n) $(x - 8)(4 - 5x - x^2)$ o) $(8ab - b)^2$

2. Multiply vertically.

   a) $(x - 3y)(4x - 5y)$ b) $(y - 2)(3y^2 + 4y - 1)$ c) $(x^2 + x + 7)(x^2 + x + 1)$

Teaching Notes:

- Most students find this section relatively easy.
- Remind students to be cautious with signs when distributing.
- In 3c) and 3g), many students will "distribute" the exponent to each term in the base instead of squaring the binomial.
- Some students are very hesitant to work vertically.
- Each section in the text has 3 worksheets in the Extra Practice featuring differentiated learning.

Answers: *1a) $8x^9$; 1b) $-15t^7$; 1c) $-21.42x^8$; 1d) $-1/4a^{11}$; 1e) $-60a^2-30a$; 1f) $-28x^4+4x^3$; 1g) $-48y^9+72y^7$; 1h) $9a^2b^{10}-36ab^9-12a^2b^7$; 1i) $x^2-2x-15$; 1j) $6x^5-8x^3-15x^2-20$; 1k) $x^2+\frac{6}{4}x+\frac{9}{16}$; 1l) $15x^2-13x+2$; 1m) $y^3-6y^2-75y+36$; 1n) $-x^3-13x^2-36x-32$; 1o) $64a^2b^2-16ab^2+b^2$; 2a) $4x^2-17xy+15y^2$; 2b) $3y^3-2y^2-9y+2$; 2c) $x^4 + 2x^3 + 9x^2 + 8x + 7$*

Mini-Lecture 5.4

Special Products

Learning Objectives:

1. Multiply two binomials using the FOIL Method.
2. Square a binomial.
3. Multiply the sum and difference of two terms.

Examples:

1. Multiply using FOIL.

   a) $(x + 7)(x - 12)$ b) $(3x - 1)(2x + 5)$ c) $(a - 2b)(a + 12b)$ d) $\left(x + \frac{3}{7}\right)\left(x - \frac{1}{6}\right)$

2. Multiply. (Square a binomial).

   a) $(x + 4)^2$ b) $(3x - 5)^2$ c) $(5x - 3y)^2$ d) $(7a^3 - 4)^2$

3. Multiply the sum and difference of two terms.

   a) $(y - 3)(y + 3)$ b) $(5x - 1)(5x + 1)$ c) $\left(2x - \frac{3}{5}\right)\left(2x + \frac{3}{5}\right)$ d) $(10x - 7y)(10x + 7y)$

4. Mixed practice. Multiply using special products.

   a) $(n + 13)^2$ b) $(a - 4y)(a + 11y)$ c) $(t - 2)(t + 13)$ d) $(3x + 5)(3x - 5)$

   e) $\left(-5a^2 + 10b\right)\left(-5a^2 - 7b\right)$ f) $\left(2x - \frac{4}{7}\right)\left(2x + \frac{4}{7}\right)$ g) $(4x + 13y)^2$

Teaching Notes:

- Many students find FOIL easy.
- In examples 2, some students will incorrectly "distribute" the exponent rather than squaring the binomial. For example: $(x+4)^2 = x^2 + 4^2$.
- Encourage students to recognize the special products rather than just "FOIL"-ing them.
- Each section in the text has 3 worksheets in the Extra Practice featuring differentiated learning.

Answers: *1a)* $x^2 - 5x - 84$; *1b)* $6x^2+13x-5$; *1c)* $a^2+10ab-24b^2$; *1d)* $x^2 + \frac{11}{42}x - \frac{1}{14}$; *2a)* $x^2+8x+16$; *2b)* $9x^2-30x+25$; *2c)* $25x^2-30xy+9y^2$; *2d)* $49a^6-56a^3+16$; *3a)* y^2-9; *3b)* $25x^2-1$; *3c)* $4x^2 - \frac{9}{25}$; *3d)* $100x^2 - 49y^2$; *4a)* $n^2+26n+169$; *4b)* $a^2+7ay-44y^2$; *4c)* $t^2+11t-26$; *4d)* $9x^2-25$; *4e)* $25a^4-15a^2b-70b^2$; *4f)* $4x^2 - \frac{16}{49}$; *4g)* $16x^2+104xy+169y^2$

Mini-Lecture 5.5

Negative Exponents and Scientific Notation

Learning Objectives:

1. Simplify expressions containing negative exponents.
2. Use all the rules and definitions for exponents to simplify exponential expressions.
3. Write numbers in scientific notation.
4. Convert numbers from scientific notation to standard form.

Examples:

1. Simplify each expression. Write each result using positive exponents only.

   a) 3^{-2} b) $8a^{-3}$ c) $\dfrac{x^{-4}}{y^{-3}}$ d) $4^{-2}+4^{0}$

2. Simplify each expression. Write each result using positive exponents only.

   a) $a^{-6}\cdot a^{-3}\cdot a\cdot a^{-2}$ b) $\dfrac{\left(2x^{4}\right)^{3}}{x^{15}}$ c) $\left(x^{-2}y^{8}\right)^{-2}$ d) $\dfrac{-6x^{2}y^{-3}}{-12x^{5}y^{-6}}$

   e) $\dfrac{m^{2}\left(m^{-4}\right)^{-2}}{\left(m^{-2}\right)^{5}}$ f) $\left(\dfrac{3xy^{5}}{2x^{4}y^{7}}\right)^{-3}$ g) $\left(\dfrac{3a^{-3}b^{-2}}{6a^{-2}b^{-5}}\right)^{0}$ h) $\left(-2r^{-3}s^{-2}t\right)\left(-5t^{-4}\right)$

3. Convert the following numbers in standard form to scientific form.

   a) 83,000 b) 1,250,000 c) 0.000154 d) 0.00000689

4. Convert the following numbers in scientific form to standard form.

   a) 1.03×10^{6} b) 8.7×10^{-5} c) 6.003×10^{10} d) 2.02×10^{-3}

Teaching Notes:

- Many students move the numerical coefficient along with the variable. For example, in 1b) a common incorrect answer is $8a^{-3} = \dfrac{1}{8a^{3}}$
- Overall, students need a lot of practice with these rules to master these objectives.
- Each section in the text has 3 worksheets in the Extra Practice featuring differentiated learning.

Answers: 1a) 1/9; 1b) $8/a^{3}$; 1c) y^{3}/x^{4}; 1d) 17/16; 2a) $1/a^{10}$; 2b) $8/x^{3}$; 2c) x^{4}/y^{16}; 2d) $y^{3}/2x^{3}$; 2e) m^{20}; 2f) $\dfrac{8x^{9}y^{6}}{27}$; 2g) 1; 2h) $\dfrac{10}{r^{3}s^{2}t^{3}}$; 3a) $8.3\text{x}10^{4}$; 3b) $1.25\text{x}10^{6}$; 3c) $1.54\text{x}10^{-4}$; 3d) $6.89\text{x}10^{-6}$; 4a) 1,030,000; 4b) 0.000087; 4c) 60,030,000,000; 4d) 0.00202

Mini-Lecture 5.6

Dividing Polynomials

Learning Objectives:

1. Divide a polynomial by a monomial.
2. Use long division to divide a polynomial by another polynomial.

Examples:

1. Perform each division.

a) $\dfrac{10x^6 - 40x^3}{5x^2}$ b) $\dfrac{6a^7 - 10a^5}{-2a^7}$ c) $\dfrac{-14x^7 + 6x^6 - 6x^5}{-2x^5}$

2. Find each quotient using long division.

a) $\dfrac{x^2 + 9x + 20}{x + 5}$ b) $\dfrac{6m^3 + 26m^2 - 17m + 15}{m + 5}$

c) $\dfrac{-20x^3 + 17x^2 + 15x + 13}{-5x - 2}$ d) $\left(4m^3 + 14m^2 - 5m + 12\right) \div \left(m + 4\right)$

Find each quotient using long division. Don't forget to write the polynomials in descending order and fill in any missing terms.

e) $\left(x^4 + 81\right) \div \left(x - 3\right)$ f) $\dfrac{9 - 5x - 25x^3 - 15x^2}{-5x + 2}$

Teaching Notes:

- Encourage students to write out each step before simplifying in 1a), 1b), 1c). Many students will "cancel" the monomial and one of the terms instead of dividing.
- Most students will need slow, methodical modeling to understand the concept of dividing by a monomial.
- Many students need to see a numerical long division done in parallel with long division of polynomials.
- Each section in the text has 3 worksheets in the Extra Practice featuring differentiated learning.

Answers: *1a)* $2x^4-8x$; *1b)* $-3+\dfrac{5}{a^2}$; *1c)* $7x^2-3x+3$; *2a)* $x+4$; *2b)* $6m^2-4m+3$; *2c)* $4x^2-5x-1+\dfrac{11}{-5x-2}$; *2d)* $4m^2-2m+3$; *2e)* $x^3+3x^2+9x+27+\dfrac{162}{x-3}$; *2f)* $5x^2+5x+3+\dfrac{3}{-5x+2}$