Mini-Lecture 6.1

The Greatest Common Factor and Factoring by Grouping

Learning Objectives:

1. Find the greatest common factor of a list of integers.
2. Find the greatest common factor of a list of terms.
3. Factor out the greatest common factor from a polynomial.
4. Factor a polynomial by grouping.

Examples:

1. Find the greatest common factor for each list.

   a) 16, 6 b) 18, 24 c) 15, 21 d) 12, 28, 40

2. Find the GCF for each list.

   a) $15m^2, 25m^5$ b) $40x^2$, $20x^7$ c) $-28x^4$, $56x^5$ d) $21m^2n^5$, $35mn^4$

3. Factor out the GCF from each polynomial.

   a) $5a + 15$ b) $56z + 8$ c) $y^3 + 2y$

   d) $5x^3 + 10x^4$ e) $16z^5 + 8z^3 – 12z$ f) $x(y^2 – 2) + 3(y^2 – 2)$

   g) $6a^8b^9 – 8a^3b^4 + 2a^2b^3 + 4a^5b^3$

4. Factor each four-term polynomial by grouping.

   a) $8y^2 – 12y + 10y - 15$ b) $15a^6 – 25a^3 – 6a^3 + 10$ c) $15x^3 – 25x^2y –6xy^2 + 10y^3$

Teaching Notes:

- Many students remove common factors, not the *greatest* common factor.
- Encourage students to factor in a step-by-step manner: first factor out the GCF for the coefficients, then the GCF for each variable.
- Most students have trouble factoring by grouping when it entails factoring a negative from the second group. Encourage students to always write a sign and check by distributing. If the check has the correct terms but wrong sign; switch the sign.
- Remind students that they can check their work by multiplying.
- Each section in the text has 3 worksheets in the Extra Practice featuring differentiated learning.

Answers: *1a) 2; 1b) 6; 1c) 3; 1d) 4; 2a) $5m^2$; 2b) $20x^2$; 2c) $-28x^4$; 2d) $7mn^4$; 3a) $5(a+3)$; b) $8(7z+1)$; 3c) $y(y^2+2)$; 3d) $5x^3(1+2x)$; 3e) $4z(4z^4+2z^2-3)$; 3f) $(y^2-2)(x+3)$; 3g) $2a^2b^3(3a^6b^6-4ab+1+2a^3)$; 4a) $(2y-3)(4y+5)$; 4b) $(3a^3-5)(5a^3-2)$; 4c) $(3x-5y)(5x^2-2y^2)$*

Mini-Lecture 6.2

Factoring Trinomials of the Form $x^2 + bx + c$

Learning Objectives:

1. Factor trinomials of the form $x^2 + bx + c$.
2. Factor out the greatest common factor and then factor a trinomial of the form $x^2 + bx + c$.

Examples:

1. Factor each trinomial completely. If a polynomial can't be factored, write "prime".

   a) $x^2 + 11x + 30$ b) $y^2 + 7y + 10$ c) $x^2 + 3x - 4$

   d) $x^2 - 4x - 21$ e) $x^2 - 13x + 30$ f) $x^2 - x + 32$

   g) $m^2 + 17m + 16$ h) $5x - 14 + x^2$ i) $a^2 + 13ab + 40b^2$

2. Factor each trinomial completely. Some of these trinomials contain a greatest common factor (other than 1). Don't forget to factor out the GCF first.

   a) $2x^2 - 18x + 28$ b) $3x^2 + 6x - 9$ c) $x^2 + 10x + 24$

   d) $2x^2 + 20x - 22$ e) $5x^2 + 20x + 15$ f) $-x^3 + 3x^2 + 10x$

   g) $4x^4 - 36x^3 + 56x^2$ h) $x^3y + 10x^2y^2 + 24xy^3$ i) $\frac{1}{3}y^2 - \frac{8}{3}y - 11$

Teaching Notes:

- When factoring trinomials of this form, many students find it helpful to make a table listing all possible factor pairs for c in the first column and their sums in the second column.
- Some students have trouble factoring a trinomial when the last term is negative.
- Remind students that when the last term (the constant) of a trinomial is positive, the factors have the same sign. When the last term (the constant) of a trinomial is negative, the factors have different signs.
- Refer students to: ***To Factor a Trinomial of the Form $x^2 + bx + c$*** in the textbook.
- Remind students that they can always check their work by multiplication.
- Each section in the text has 3 worksheets in the Extra Practice featuring differentiated learning.

<u>Answers:</u> 1a) (x+6)(x+5); 1b) (y+5)(y+2); 1c) (x+4)(x-1); 1d) (x-7)(x+3); 1e) (x-10)(x-3); 1f) prime; 1g) (m+16)(m+1); 1h) (x+7)(x-2); 1i) (a+8b)(a+5b); 2a) 2(x-7)(x-2); 2b) 3(x+3)(x-1); 2c) (x+6)(x+4); 2d) 2(x+11)(x-1); 2e) 5(x+3)(x+1); 2f) -x(x-5)(x+2); 2g) $4x^2(x-7)(x-2)$; 2h) xy(x+6y)(x+4y);

2i) $\frac{1}{3}(y-11)(y+3)$

Mini-Lecture 6.3

Factoring Trinomials of the Form $ax^2 + bx + c$ and Perfect Square Trinomials

Learning Objectives

1. Factor trinomials of the form $ax^2 + bx + c$, where $a \neq 1$.
2. Factor out the GCF before factoring a trinomial of the form $ax^2 + bx + c$.
3. Factor perfect square trinomials.

Examples;

1. Complete each factored form.

   a) $3x^2 + 8x + 4 = (3x + 2)(\qquad)$ b) $2y^2 + 7y - 15 = (2y - 3)(\qquad)$

   Factor each trinomial completely.

   c) $2x^2 + 7x + 3$ d) $5x^2 + 17x + 6$ e) $8x^2 + x - 7$

   f) $20r^2 + 31r - 7$ g) $6x^2 + 19x - 11$ h) $3x^2 - 7x - 20$

2. Factor each trinomial completely. If necessary, factor out the GCF first.

   a) $14x^2 + 4x - 10$ b) $9x^2 - 6x - 15$ c) $14x^3 + 66x^2 - 20x$

   d) $25x^3 - 15x^2 - 10x$ e) $4x^2y^2 - xy^2 - 105y^2$ f) $12x^2 - 25xt + 12t^2$

   g) $-7x^2 - 33x + 10$ h) $18x^4 - 3x^3 - 21x^2$ i) $2x^5 - x^3y^2 - 15xy^4$

3. Factor each Perfect Square Trinomial completely.

   a) $x^2 + 2x + 1$ b) $4x^2 - 12x + 9$ c) $25x^2 + 60xy + 36y^2$

   d) $16x^3 - 8x^2y + xy^2$ e) $5x^3 - 10x^2 + 5x$ f) $2a - 24ay + 72ay^2$

Teaching Notes:

- Some students remember factoring from high school and are able to use the trial-and-error method to factor.
- Some students may need to see Section 4.4, *Factoring Trinomials by Grouping* before being able to factor successfully.
- Encourage students to use strategies when factoring. For example, identify any prime numbers to reduce the number of combinations.
- Many students will forget to put the GCF in their final answer.
- Remind students that they can check their work by multiplying.
- Each section in the text has 3 worksheets in the Extra Practice featuring differentiated learning.

Answers 1a) $(x+2)$; 1b) $(y+5)$; 1c) $(2x+1)(x+3)$; 1d) $(5x+2)(x+3)$; 1e) $(8x-7)(x+1)$; 1f) $(5r-1)(4r+7)$; 1g) $(3x+11)(2x-1)$; 1h) $(3x+5)(x-4)$; 2a) $2(7x-5)(x+1)$; 2b) $3(3x-5)(x+1)$; 2c) $2x(7x-2)(x+5)$; 2d) $5x(5x+2)(x-1)$; 2e) $y^2(x+5)(4x-21)$; 2f) $(4x-3t)(3x-4t)$; 2g) $(-7x+2)(x+5)$; 2h) $3x^2(6x-7)(x+1)$; 2i) $x(2x^2+5y^2)(x^2-3y^2)$, 3a) $(x+1)^2$, 3b) $(2x-3)^2$, 3c) $(5x+6y)^2$ 3d) $x(4x-y)^2$, 3e) $5x(x-1)^2$, 3f) $2a(1-6y)^2$

Mini-Lecture 6.4

Factoring Trinomials of the Form $ax^2 + bx + c$ by Grouping

Learning Objectives

1. Use the grouping method to factor trinomials of the form $ax^2 + bx + c$.

Examples:

1. Factor the following trinomial by grouping. Complete the outlined steps.

   a) $12y^2 + 17y + 6$

   Find two numbers whose product is $72\ (12 \bullet 6)$ and whose sum is 17: ______
   Write 17y using the factors from previous step: ______
   Factor by grouping: ________

   b) $10x^2 + 9x - 9$

   Find two numbers whose product is $-90\left[10 \bullet (-9)\right]$ and whose sum is (-9): ______
   Write (-9x) using the factors from part (a): ______
   Factor by grouping: ________

   Factor by grouping.

   c) $8x^2 + 18x + 9$ d) $6x^2 + 7x - 3$ e) $7x^2 - 19x - 6$

   f) $4x^2 - 12x + 9$ g) $6x^2 - 17x + 5$ h) $20x^2 - 15x - 50$

   i) $45x^3 + 45x^2 - 50x$ j) $x - 15 + 6x^2$ k) $10z^2 - 12z - 1$

Teaching Notes:

- Most students appreciate seeing the grouping method. This method gives the student a step-by-step guide to factoring.
- Encourage students to use whatever method works for them (trial-and-error or grouping).
- Remind students to put the trinomial into standard form before attempting to factor.
- Encourage students to check their factoring answers by multiplication.
- Each section in the text has 3 worksheets in the Extra Practice featuring differentiated learning.

Answers: *1a) 9,8; 9y+8y; (4y+3)(3y+2); 1b) -6, 15; -6x+15x; (5x-3)(2x+3); 1c) (4x+3)(2x+3); 1d) (2x+3)(3x-1); 1e) (7x+2)(x-3); 1f0 $(2x-3)^2$; 1g) (2x-5)(3x-1); 1h) 5(x-2)(4x+5); 1i) 5x(3x-2)(3x+5); 1j) (3x+5)(2x-3); 1k) prime*

Mini-Lecture 6.5

Factoring Binomials

Learning Objectives:

1. Factor the difference of two squares.
2. Factor the sum or difference of two cubes.

Examples:

1. Factor each binomial completely.

a) $x^2 - 9$ b) $x^2 - 25$ c) $y^2 - 64$

d) $4a^2 - 9$ e) $49x^2 - 1$ f) $9a^2 + 16b^2$

g) $36m^2 - 100n^2$ h) $\frac{1}{4}x^2 - 1$ i) $64 - \frac{9}{25}a^2$

2. Factor each sum or difference of two cubes completely.

a) $8x^3 + 1$ b) $a^3 - 1$ c. $64x^3 + 27y^3$

d) $54y^4 - 2y$ e) $125b^5 + b^2$ f) $a^6 - 1$

Teaching Notes:

- Some students will have a better understanding of a difference of two squares if they are first shown 3a) and 3b) with a middle term of 0*x*.
- Encourage students to become proficient with special case factoring as it will be important for future algebra topics such as completing the square.
- Each section in the text has 3 worksheets in the Extra Practice featuring differentiated learning.

Answers: 1a) (x+3)(x-3); 1b) (x+5)(x-5); 1c) (y+8)(y-8); 1d) (2a+3)(2a-3); 1e) (7x+1)(7x-1); 1f) cannot be factored; 1g) (6m+10n)(6m-10n); 1h) (1/2x+1)(1/2x-1); 1i) (8+3/5a)(8-3/5a); 2a) $(2x+1)(4x^2-2x+1)$; *2b)* $(a-1)(a^2+a+1)$; *2c)* $(4x+3y)(16x^2-12xy+9y^2)$; *2d)* $2y(3y-1)(9y^2+3y+1)$; *2e)* $b^2(5b+1)(25b^2-5b+1)$; *2f)* $(a^2-1)(a^4+a^2+1)$

Mini-Lecture 6.6

Solving Quadratic Equations by Factoring

Learning Objectives:

1. Solve quadratic equations by factoring.
2. Solve equations with degree greater than 2 by factoring.
3. Find the x-intercepts of the graph of a quadratic equation in two variables.

Examples:

1. Solve each equation.

   a) $(x-1)(x+4)=0$ b) $(x+5)(x+9)=0$ c) $(x-10)(x+8)=0$

   d) $5x(x-15)=0$ e) $(2x-5)(x+3)=0$ f) $\left(x-\frac{2}{7}\right)\left(x-\frac{1}{3}\right)=0$

   g) $x^2-x-30=0$ h) $x^2-9x+20=0$ i) $y^2-y-42=0$

   j) $x^2-7x=0$ k) $x^2=25$ l) $x^2+2x=15$

   m) $5x^2-30x+40=0$ n) $x(x-4)=21$ o) $x(x-6)=16$

2. Solve the following equations with degree greater than 2 by factoring.

   a) $y^3+14y^2+49y=0$ b) $24x^3-4x^2-20x=0$ c) $(x-4)(x^2-3x+2)=0$

   d) $16a^3-9a=0$ e) $49t^3-4t=0$ f) $(9x+5)(10x^2-3x-4)=0$

3. Find the x-intercepts of the graph.

   a) $y=(x-4)(x+5)$ b) $y=(x-1)(x+1)$ c) $y=x^3-x^2-4x-4$

Teaching Notes:

- Remind students to always put the equation into standard form.
- Some students try to use the zero factor property before the equation is in standard form. For example 1n) $x(x-4)=21 \rightarrow x=21, x-4=21, etc.$
- Students will find this section challenging.
- Remind students to always check their answers.
- Each section in the text has 3 worksheets in the Extra Practice featuring differentiated learning.

Answers: *1a) 1, -4; 1b) -5, -9; 1c) 10, -8; 1d) 0, 15; 1e) 5/2, -3; 1f) 2/7, 1/3; 1g) 6, -5; 1h) 4, 5; 1i) 7, -6; 1j) 0, 7; 1k) 5, -5; 1l) -5, 3; 1m) 2, 4; 1n) 7, -3; 1o) –2, 8; 2a) 0, -7; 2b) 0, -5/6, 1; 2c) 4, 2, 1; 2d) 0, ¾, -3/4; 2e) 0, 2/7, -2/7; 2f) -5/9, -1/2, 4/5, 3a) (4,0), (-5,0), 3b) (1,0), (-1,0), 3c) (2,0), (-2,0), (1,0)*