

Mini-Lecture 7.1

Simplifying Rational Expressions

Learning Objectives:

1. Find the value of a rational expression given a replacement number.
2. Identify values for which a rational expression is undefined.
3. Simplify or write rational expressions in lowest terms.
4. Write equivalent rational expressions of the form $-\frac{a}{b} = \frac{-a}{b} = \frac{a}{-b}$.

Examples:

1. Find the value of the expression for the given value (s).

a) $\frac{x+4}{x-2}; x=4$

b) $\frac{y^2}{y-3}; y=5$

c) $\frac{x^2+5x-2}{x^2-x-2}; x=-3$

2. Find any numbers for which each rational expression is undefined.

a) $\frac{7}{4a}$

b) $\frac{3y^3}{y^2-5y}$

c) $\frac{x}{2x^2-7x-4}$

3. Simplify each expression.

a) $\frac{(y+3)(y-1)}{(y-1)(y+5)}$

b) $\frac{6-a}{a-6}$

c) $\frac{-2x-2y}{x+y}$

d) $\frac{6x-12}{3x^2-12}$

e) $\frac{y^2+8y+15}{y^2+9y+18}$

f) $\frac{x^2-xy+5x-5y}{x^2+5x}$

4. List four equivalent forms for each rational expression.

a) $-\frac{x-2}{x+5}$

b) $-\frac{y+3}{y-11}$

c) $-\frac{5y+2}{3y-1}$

Teaching Notes:

- Some students need a review of simplifying numerical fractions before applying to rational expressions.
- Many students need to be reminded to factor completely before simplifying.
- Each section in the text has 3 worksheets in the Extra Practice featuring differentiated learning.

Answers: 1a) 4; 1b) 25/2; 1c) -4/5; 2a) 0; 2b) 0, 5; 2c) -1/2, 4; 3a) $\frac{y+3}{y+5}$; 3b) -1; 3c) -2; 3d) $\frac{2}{x+2}$;
 3e) $\frac{y+5}{y+6}$; 3f) $\frac{x-y}{x}$; 4a) $\frac{-(x-2)}{x+5}, \frac{2-x}{x+5}, \frac{x-2}{-(x+5)}, \frac{x-2}{-x-5}$; 4b) $\frac{-(y+3)}{y-11}, \frac{-y-3}{y-11}, \frac{y+3}{-(y-11)}, \frac{y+3}{11-y}$;
 4c) $\frac{-(5y+2)}{3y-1}, \frac{-5y-2}{3y-1}, \frac{5y+2}{-(3y-1)}, \frac{5y+2}{1-3y}$

Mini-Lecture 7.2

Multiplying and Dividing Rational Expressions

Learning Objectives:

1. Multiply rational expressions.
2. Divide rational expressions.
3. Multiply or divided rational expressions.

Examples:

1. Find each product and simplify if possible.

$$\text{a) } \frac{8p-8}{p} \cdot \frac{7p^2}{9p-9} \qquad \text{b) } \frac{x^2+5x+6}{x^2+11x+24} \cdot \frac{x^2+8x}{x^2-4x+3} \qquad \text{c) } \frac{x^3-x^2+x}{x^3+1} \cdot \frac{-45x-45}{5x}$$

2. Find each quotient and simplify.

$$\text{a) } \frac{-4k^2}{4k^5} \div \frac{16k^4}{12k^6} \qquad \text{b) } \frac{2-2x}{x} \div \frac{5x-5}{3x^2} \qquad \text{c) } \frac{y^2-11y+30}{y^2-36} \div \frac{y^2-9}{y^2-3y-18}$$

3. Multiply or divide. Simplify if possible.

$$\text{a) } \frac{x^2-11x+10}{x^2-11x+28} \cdot \frac{x^2-9x+20}{x^2-16x+60} \qquad \text{b) } \frac{2k^2+8k+6}{k^2-9} \div \frac{4k^2+18k+14}{2k-6}$$

Teaching Notes:

- Many students need to review multiplying and dividing numerical fractions.
- When dividing, remind students to change division to multiplication by the reciprocal first, then begin factoring. Many students will begin factoring and forget that they are dividing.
- Some students will need additional practice using unit fractions.
- Each section in the text has 3 worksheets in the Extra Practice featuring differentiated learning.

Answers: 1a) $\frac{56p}{9}$; 1b) $\frac{x(x+2)}{(x-3)(x-1)}$; 1c) -9; 2a) $\frac{-3}{4k}$; 2b) $\frac{-6x}{5}$; 2c) $\frac{(y-5)(y-6)}{(y+6)(y-3)}$;
3a) $\frac{(x-5)(x-1)}{(x-6)(x-7)}$; 3b) $\frac{2}{2k+7}$;

Mini-Lecture 7.3

Adding and Subtracting Rational Expressions with Common Denominators and Least Common Denominator

Learning Objectives:

1. Add and subtract rational expressions with the same denominator.
2. Find the least common denominator of a list of rational expressions.
3. Write a rational expression as an equivalent expression whose denominator is given.
4. Key Vocabulary: *least common denominator (LCD)*, *equivalent expressions*.

Examples:

1. Add or subtract as indicated. Simplify the result if possible.

a) $\frac{3}{12x} + \frac{4}{12x}$

b) $\frac{8a+2b}{2} - \frac{8a-2b}{2}$

c) $\frac{7y^2}{y-1} + \frac{-7y}{y-1}$

d) $\frac{m^2-7m}{m-3} + \frac{12}{m-3}$

e) $\frac{3x+2}{x^2+4x-21} - \frac{2x-5}{x^2+4x-21}$

2. Find the LCD for each list of rational expressions.

a) $\frac{1}{20x^5}, \frac{1}{36x^4}$

b) $\frac{2}{x+3}, \frac{3}{x-5}$

c) $\frac{4}{3a+27}, \frac{6}{a^2+9a}$

3. Rewrite each rational expression as an equivalent rational expression with the given denominator.

a) $\frac{3}{8m} = \frac{?}{72m}$

b) $\frac{x}{x+4} = \frac{?}{5x+20}$

c) $\frac{a}{a+3b} = \frac{?}{a^2-9b^2}$

d) $\frac{-25}{x^3+2x^2-3x} = \frac{?}{x(x-1)(x+3)(x+5)}$

Teaching Notes:

- Most students need a review of finding the LCD for numerical fractions.
- Many students have a difficult time finding the LCD. Refer them back to the process they use when finding the LCD for numerical fractions.
- Each section in the text has 3 worksheets in the Extra Practice featuring differentiated learning.

Answers: 1a) $\frac{7}{12x}$; 1b) $2b$; 1c) $7y$; 1d) $m-4$; 1e) $\frac{1}{x-3}$; 2a) $180x^5$; 2b) $x^2 - 2x - 15$; 2c) $3a^2 + 27a$;
3a) 27 ; 3b) $5x$; 3c) $a^2 - 3ab$; 3d) $-25x - 125$

Mini-Lecture 7.4

Adding and Subtracting Rational Expressions With Unlike Denominators

Learning Objectives:

1. Add and subtract rational expressions with unlike denominators.

Examples:

1. Perform each indicated operation. Simplify if possible.

a) $\frac{x}{9} + \frac{8}{7}$

b) $\frac{x}{5} - \frac{3}{11}$

c) $\frac{9}{40} - \frac{2}{5x}$

d) $\frac{6}{z^2} - \frac{4}{z}$

e) $\frac{5}{r} + \frac{9}{r-3}$

f) $\frac{9-5y}{63} - \frac{8-7y}{18}$

g) $\frac{-8x+3}{x} + \frac{-8x+2}{5x}$

h) $\frac{6x}{x+6} + \frac{3}{x-6}$

i) $\frac{7}{6-m} + \frac{2}{m-6}$

j) $\frac{-5}{y-4} - \frac{7}{4-y}$

k) $\frac{m-5}{m^2+9m+20} + \frac{4m-1}{m^2+7m+10}$

l) $\frac{x}{x^2-16} - \frac{6}{x^2+5x+4}$

m) $\frac{3}{y^2-3y+2} + \frac{5}{y^2-1}$

Teaching Notes:

- Most students have difficulty with adding and subtracting rational expressions, mainly with finding the LCD.
- Remind students that with subtraction, they are subtracting the entire numerator (i.e. Distributive Property).
- Some students may find working in a vertical format easier to build equivalent expressions.
- Each section in the text has 3 worksheets in the Extra Practice featuring differentiated learning.

Answers: 1a) $\frac{7x+72}{63}$; 1b) $\frac{11x-15}{55}$; 1c) $\frac{-9x-16}{40x}$; 1d) $\frac{6-4z}{z^2}$; 1e) $\frac{14r-15}{r^2-3r}$; 1f) $\frac{39y-38}{126}$;

1g) $\frac{-48x+17}{5x}$; 1h) $\frac{6x^2-33x+18}{x^2-36}$; 1i) $\frac{6}{6-m}$; 1j) $\frac{2}{y-4}$; 1k) $\frac{5m^2+12m-14}{m^3+11m^2+38m+40}$;

1l) $\frac{x^2-5x+24}{x^3+x^2-16x-16}$; 1m) $\frac{8y-7}{y^3-2y^2-y+2}$

Mini-Lecture 7.5

Solving Equations Containing Rational Expressions

Learning Objectives:

1. Solve equations containing rational expressions.
2. Solve equations containing rational expressions for a specified variable.
3. Key Vocabulary: *rational expressions*.

Examples:

1. Solve each equation and check each solution.

a) $\frac{x}{3} - \frac{x}{9} = 8$

b) $\frac{4x}{5} - 8 = x$

c) $\frac{x-3}{7} = \frac{x+4}{3}$

d) $\frac{x+10}{x-5} = \frac{9}{5-x}$

e) $\frac{4}{x-4} + \frac{4}{2x-8} = 6$

f) $\frac{x}{x-7} + 6 = \frac{7}{x-7}$

g) $\frac{-5x}{3x+3} = \frac{2x}{6x+6} + \frac{6x-4}{x+1}$

h) $\frac{1}{x} + \frac{1}{x-8} = \frac{x-7}{x-8}$

i) $\frac{-2}{m+5} - \frac{3}{m-5} = \frac{15}{m^2-25}$

j) $\frac{x+4}{x^2+2x-15} - \frac{4}{x^2+10x+25} = \frac{x-4}{x^2+2x-15}$

2. Solve each equation for the indicated variable.

a) $\frac{1}{a} + \frac{1}{b} = c$ for b

b) $A = \frac{1}{2}h(B+b)$ for B

c) $F = \frac{-GMm}{r^2}$ for m

Teaching Notes:

- Remind students to always determine the values for x that will make the denominators of the equation equal to zero.
- Some students prefer to create equivalent expressions using the LCD then set numerators equal.
- Most students have difficulty solving a formula for a specific variable.
- To help students focus on solving a formula for a specific variable, encourage them to write the variable with a different color pencil.

Answers: 1a) 36; 1b) -40; 1c) -37/4; 1d) -19; 1e) 5; 1f) no solution; 1g) 1/2; 1h) 1; 1i) -4; 1j) -13;

2a) $b = \frac{a}{ac-1}$; 2b) $B = \frac{2A}{h} - b$ or $\frac{2A-bh}{h}$; 2c) $m = -\frac{Fr^2}{GM}$

Mini-Lecture 7.6

Proportion and Problem Solving with Rational Equations

Learning Objectives:

1. Solve proportions.
2. Use proportions to solve problems.
3. Solve problems about numbers.
4. Solve problems about work.
5. Solve problems about distance.

Examples:

1. Solve each proportion.

a) $\frac{x}{39} = \frac{6}{13}$

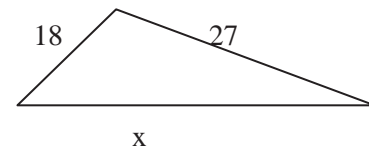
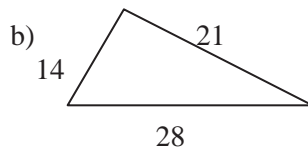
b) $\frac{2}{7} = \frac{5}{x}$

c) $\frac{8+x}{5} = \frac{5+x}{3}$

d) $\frac{7}{2} = \frac{z-5}{z-3}$

2. Solve. Find the unknown length (x) in the pair of similar triangles.

- a) There are 170 calories in 3 peanut butter cookies. How many calories are in 7 cookies?



3. Solve the following: problems about numbers.

a) Five divided by the difference of a number and one equals the quotient of ten and the sum of the number and twelve. Find the number.

b) If three times a number added to five is divided by the number plus nine, the result is four thirds. Find the number.

4. Solve the following: problems about work.

a) A painter can paint a sign in 4 hours. His assistant can paint the same sign in 6 hours. How long will it take them to paint the sign if they paint it together.

b) A conveyer belt can move 1000 cans of soup to the loading area in 7 minutes. A smaller conveyer belt can move the same number of cans in 11 minutes. How long will it take to move 1000 cans of soup if both conveyer belts are used.

5. Solve the following: problems about distance.

a) A cyclist bikes at a constant speed for 18 miles. His return trip is a different route of 23 miles long and will take 1 hour longer. Find the speed.

Teaching Notes:

- Most students will struggle with applications.
- Refer students to Chapter 2, Section 2.4, **General Strategy for Problem Solving**.
- Each section in the text has 3 worksheets in the Extra Practice featuring differentiated learning.

Answers: 1a) 18; 1b) $35/2$; 1c) $-1/2$; 1d) $11/5$; 2a) $396\frac{2}{3}$; 2b) 36; 3a) 14; 3b) $21/5$; 4a) 2.4 hours; 4b) $4\frac{5}{18}$ min.; 5a) 5 mph.