Mini-Lecture 8.1

Introduction to Radicals

Learning Objectives:

1. Find square roots.
2. Find cube roots.
3. Find *n*th roots.
4. Approximate square roots.
5. Simplify radicals containing variables.

Examples:

1. Find each square root.

   a) $\sqrt{49}$ b) $\sqrt{\frac{1}{36}}$ c) $-\sqrt{9}$

   d) $\sqrt{-100}$ e) $\sqrt{\frac{25}{121}}$ f) $\sqrt{0.64}$

2. Find each cube root.

   a) $\sqrt[3]{8}$ b) $\sqrt[3]{-216}$ c) $\sqrt[3]{-\frac{8}{27}}$

3. Find each root.

   a) $\sqrt[4]{16}$ b) $\sqrt[3]{-27}$ c) $-\sqrt[4]{\frac{81}{625}}$

4. Approximate each square root to three decimal places.

   a) $\sqrt{12}$ b) $\sqrt{22}$ c) $-\sqrt{120}$

5. Find each root. Assume that all variables represent positive numbers.

   a) $\sqrt{x^2}$ b) $\sqrt{a^4}$ c) $\sqrt{m^8}$

   d) $\sqrt{81x^4}$ e) $\sqrt{x^{10}y^8z^2}$ f) $\sqrt[3]{27a^6b^9c^3}$

Teaching Notes:

- Many students confuse 1c), 1d), and 2b).
- Students have a hard time understanding $\sqrt{x^2} = |x|$ even though we assume that all variable represent positive numbers.
- It is very important to stress that using a calculator gives an *approximation* and leaving an answer in radical form is an *exact* value.

Answers: *1a) 7; 1b) 1/6; 1c) -3; 1d) not a real number; 1e) 5/11; 1f) 0.8; 2a) 2; 2b) -6; 2c) -2/3; 3a) 2; 3b) -3; 3c) -3/5; 4a) 3.464; 4b) 4.69; 4c) -10.954; 5a) x; 5b)* a^2*; 5c)* m^4*; 5d)* $9x^2$*; 5e)* x^5y^4z *; 5f)* $3a^2b^3c$

Mini-Lecture 8.2

Simplifying Radicals

Learning Objectives:

1. Use the product rule to simplify square roots.
2. Use the quotient rule to simplify square roots.
3. Simplify radicals containing variables.
4. Simplify higher roots.
5. Key Vocabulary: *perfect squares.*

Examples:

1. Use the product rule to simplify each radical.

   a) $\sqrt{18}$ b) $\sqrt{12}$ c) $\sqrt{33}$

   d) $\sqrt{160}$ e) $5\sqrt{16}$ f) $-3\sqrt{50}$

2. Use the quotient rule and the product rule to simplify each radical.

   a) $\sqrt{\frac{25}{16}}$ b) $\sqrt{\frac{99}{4}}$ c) $\sqrt{\frac{125}{144}}$

3. Simplify each radical. Assume that all variables represent positive numbers.

   a) $\sqrt{x^5}$ b) $\sqrt{y^9}$ c) $\sqrt{a^{13}}$

   d) $\sqrt{\frac{18}{x^2}}$ e) $\sqrt{36y^3}$ f) $\sqrt{80y^{12}}$

   g) $\sqrt{\frac{98}{p^6}}$ h) $\sqrt{\frac{300}{x^{20}}}$ i) $\sqrt{\frac{16x}{z^{10}}}$

4. Simplify each radical.

   a) $\sqrt[3]{40}$ b) $\sqrt[3]{300}$ c) $\sqrt[3]{\frac{625}{216}}$

Teaching Notes:

- Many students have trouble with radicals.
- When simplifying, students get confused where to write the numbers – outside the radical symbol or in the radicand.
- A common error is to evaluate "$\sqrt{16} = \sqrt{4} = 2$". Many students do not know when to stop!
- Each section in the text has 3 worksheets in the Extra Practice featuring differentiated learning.

<u>*Answers:*</u> *1a)* $3\sqrt{2}$; *1b)* $2\sqrt{3}$; *1c)* $\sqrt{33}$; *1d)* $4\sqrt{10}$; *1e) 20; 1f)* $-15\sqrt{2}$; *2a) 5/4; 2b)* $\frac{3\sqrt{11}}{2}$; *2c)* $\frac{5\sqrt{5}}{12}$; *3a)* $x^2\sqrt{x}$; *3b)* $y^4\sqrt{y}$; *3c)* $a^6\sqrt{a}$; *3d)* $\frac{3\sqrt{2}}{x}$; *3e)* $6y\sqrt{y}$; *3f)* $4y^6\sqrt{5}$; *3g)* $\frac{7\sqrt{2}}{p^3}$; *3h)* $\frac{10\sqrt{3}}{x^{10}}$; *3i)* $\frac{4\sqrt{x}}{z^5}$; *4a)* $2\sqrt[3]{5}$; *4b)* $\sqrt[3]{300}$; *4c)* $\frac{5\sqrt[3]{5}}{6}$

Mini-Lecture 8.3

Adding and Subtracting Radicals

Learning Objectives:

1. Add or subtract like radicals.
2. Simplify radical expressions, and then add or subtract any like radicals.

Examples:

1. Add or subtract as indicated.

   a) $20\sqrt{5}+3\sqrt{5}$ b) $11\sqrt{7}-3\sqrt{7}$ c) $-7\sqrt{11}-5\sqrt{11}$

   d) $11\sqrt{3}-12\sqrt{3}+35+3\sqrt{3}$ e) $3\sqrt{7}+5\sqrt{21}-8\sqrt{21}-10\sqrt{7}$

2. Add or subtract by first simplifying each radical and then combining any like radicals. Assume that all variables represent positive numbers.

   a) $8\sqrt{5}+9\sqrt{20}$ b) $-7\sqrt{2}+9\sqrt{50}$ c) $-8\sqrt{3}-3\sqrt{75}$

   d) $-10\sqrt{48}-3\sqrt{75}$ e) $-5\sqrt{8x}-6\sqrt{18x}$ f) $-5\sqrt{x^2}+3x+8\sqrt{x^2}$

3. Simplify each radical expression.

   a) $5\sqrt[3]{7}+8\sqrt[3]{7}$ b) $-3\sqrt[3]{12}+8\sqrt[3]{12}-10$ c) $2\sqrt[3]{25}-7\sqrt[3]{5}+6\sqrt[3]{25}$

   d) $\sqrt[3]{40}+6\sqrt[3]{135}$ e) $\sqrt[3]{128}-5\sqrt[3]{250}$ f) $7\sqrt[3]{x}+\sqrt[3]{64x}$

Teaching Notes:

- Many students need extra practice in identifying like radicals.
- Some students combine the coefficients and multiply the like radicals.
- Many students confuse $\sqrt{}$ *and* $\sqrt[3]{}$. In fact, a common error is to evaluate $\sqrt[3]{4}=2$ *or* $\sqrt[3]{36}=6$. Encourage students to be cautious determining the index.
- Each section in the text has 3 worksheets in the Extra Practice featuring differentiated learning.

Answers: *1a)* $23\sqrt{5}$; *1b)* $8\sqrt{7}$; *1c)* $-12\sqrt{11}$; *1d)* $2\sqrt{3}+35$; *1e)* $-3\sqrt{21}-7\sqrt{7}$; *2a)* $26\sqrt{5}$; *2b)* $38\sqrt{2}$; *2c)* $-23\sqrt{3}$; *2d)* $-55\sqrt{3}$; *2e)* $-28\sqrt{2x}$; *2f)* $6x$; *3a)* $13\sqrt[3]{7}$; *3b)* $5\sqrt[3]{12}-10$; *3c)* $8\sqrt[3]{25}-7\sqrt[3]{5}$; *3d)* $20\sqrt[3]{5}$; *3e)* $-21\sqrt[3]{2}$; *3f)* $11\sqrt[3]{x}$

Mini-Lecture 8.4
Multiplying and Dividing Radicals

Learning Objectives:

1. Multiply radicals.
2. Divide radicals.
3. Rationalize denominators.
4. Rationalize using conjugates.
5. Key Vocabulary: *product rule for radicals, quotient rule for radicals, rationalizing, conjugates.*

Examples:

1. Multiply and simplify. Assume that all variables represent positive real numbers.

   a) $\sqrt{3}\bullet\sqrt{5}$ b) $\sqrt{5x}\bullet\sqrt{5x}$ c) $\sqrt{2}\bullet\sqrt{6}$

   d) $\left(3\sqrt{x}\right)^2$ e) $\sqrt{5x^3}\bullet\sqrt{15x}$ f) $\sqrt{6}\left(\sqrt{3}+\sqrt{2}\right)$

   g) $\left(\sqrt{7}+3\right)\left(\sqrt{7}-3\right)$ h) $\left(8\sqrt{5}+9\right)\left(9\sqrt{5}+3\right)$ i) $\left(4\sqrt{3}-8\right)^2$

2. Divide and simplify. Assume that all variables represent positive real numbers.

   a) $\dfrac{\sqrt{12}}{\sqrt{3}}$ b) $\dfrac{\sqrt{50}}{\sqrt{2}}$ c) $\dfrac{\sqrt{50y^3}}{\sqrt{2y}}$

3. Rationalize each denominator and simplify. Assume that all variables represent positive real numbers.

   a) $\dfrac{\sqrt{7}}{\sqrt{5}}$ b) $\sqrt{\dfrac{5}{12}}$ c) $\dfrac{3x}{\sqrt{2}}$

4. Rationalize each denominator and simplify. Assume that all variables represent positive real numbers.

   a) $\dfrac{2}{6-\sqrt{3}}$ b) $\dfrac{7}{\sqrt{5}+2}$ c) $\dfrac{15}{3+\sqrt{x}}$

Teaching Notes:

- Many students have trouble with problem 1i. They tend to square each term in the binomial rather than squaring the binomial.
- Most students are able to rationalize a denominator with one term.
- Many students have difficulty rationalizing a denominator with 2 terms.
- Each section in the text has 3 worksheets in the Extra Practice featuring differentiated learning.

Answers: *1a)* $\sqrt{15}$; *1b)* $5x$; *1c)* $2\sqrt{3}$; *1d)* $9x$; *1e)* $5x^2\sqrt{3}$; *1f)* $3\sqrt{2}+2\sqrt{3}$; *1g) -2;* *1h)* $387+105\sqrt{5}$; *1i)* $112-64\sqrt{3}$; *2a) 2; 2b) 5; 2c) 5y; 3a)* $\dfrac{\sqrt{35}}{5}$; *3b)* $\dfrac{\sqrt{15}}{6}$; *3c)* $\dfrac{3x\sqrt{2}}{2}$; *4a)* $\dfrac{12+2\sqrt{3}}{33}$; *4b)* $7\sqrt{5}-14$; *4c)* $\dfrac{45-15\sqrt{x}}{9-x}$

Mini-Lecture 8.5

Solving Equations Containing Radicals

Learning Objectives:

1. Solve radical equations by using the squaring property of equality once.
2. Solve radical equations by using the squaring property of equality twice.

Examples:

1. Solve each equation.

   a) $\sqrt{x}=5$ b) $\sqrt{x}-3=13$ c) $3\sqrt{x}-15=60$

   d) $2\sqrt{x}+11=9$ e) $1+\sqrt{y+4}=11$ f) $\sqrt{y+9}=y+3$

   g) $\sqrt{5x-2}=\sqrt{2x+1}$ h) $\sqrt{x+8}-x=2$ i) $\sqrt{9x^2+5x-20}=3x$

2. Solve each equation.

   a) $\sqrt{x}+3=\sqrt{x+21}$ b) $\sqrt{x-27}=\sqrt{x}-3$ c) $\sqrt{x}-1=\sqrt{x-9}$

   Mixed Practice. Solve each equation.

   d) $\sqrt{5x-1}=3$ e) $x+3=\sqrt{2x}+7$ f) $\sqrt{x+11}=\sqrt{6x-9}$

Teaching Notes:

- Many students have to be reminded to isolate the radical before squaring both sides.
- Refer students to the textbook for *To Solve a Radical Equation Containing Square Roots.*
- Many students find the concept of extraneous solutions confusing.
- Show students a simple example of an extraneous solution, such as:
  $x=5\rightarrow$ square both sides $\rightarrow x^2=25\rightarrow x=\pm 5\rightarrow x=-5$ is extraneous.
- Each section in the text has 3 worksheets in the Extra Practice featuring differentiated learning.

Answers: *1a) 25; 1b) 256; 1c) 625; 1d) not real; 1e) 96; 1f) 0; 1g) 1; 1h) 1; 1i) 4; 2a) 4; 2b) 36; 2c) 25; 2d) 2; 2e) 8; 2f) 4*