Chapter 5 Polynomials

Professor Tim Busken

Department of Mathematics
Grossmont College

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Multiplying Polynomials

Multiplying Binomials using the Distributive Property

We can multiply two binomials using the Distributive Property,

$$a \cdot (b + c) = a \cdot b + a \cdot c.$$

Suppose we want to multiply $(2x - 3)$ times $(x + 4)$. Then we treat $(2x - 3)$ like it is playing the role of "a" in the distributive property:

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$$\begin{aligned}
(2x-3)\cdot(x+4) &= (2x-3)\cdot x + (2x-3)\cdot 4 & \\
&= x\cdot(2x-3) + 4\cdot(2x-3) & \text{since } a\cdot b = b\cdot a \\
&= x\cdot(2x) + x\cdot(-3) + 4\cdot(2x) + 4\cdot(-3) & \text{Distributive Prop.} \\
&= (2x)\cdot x + (-3)\cdot x + 4\cdot(2x) + (-3)\cdot 4 & \text{Commutative. Prop.} \\
&= 2x\cdot x + (-3)\cdot x + 4\cdot 2x + (-3)\cdot 4 & \text{Associative Prop.} \\
&= 2x^2 + (-3x) + 8x + (-12) & \text{Closure Prop.} \\
&= 2x^2 + 5x - 12 & \text{Combine Like Terms}
\end{aligned}$$

Multiplying Binomials using the FOIL Technique

The product of two binomials results in four terms before the like terms are combined. The acronym “Foil” stands for *FIRST, OUTSIDE, INSIDE, LAST*, and should remind you how to compute the product of two binomials. Consider the following product:

$$(2x - 3) \cdot (x + 4) = \underbrace{2x^2}_{\text{F}} + \underbrace{8x}_{\text{O}} + \underbrace{(-3x)}_{\text{I}} + \underbrace{(-12)}_{\text{L}}$$

Multiplying Binomials using the FOIL Technique

$2x^2$ comes from multiplying the *first* terms of each binomial.

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Multiplying Binomials using the FOIL Technique

$8x$ comes from multiplying the *outside* terms of each binomial.

$$(2x - 3) \cdot (x + 4) = \underbrace{2x^2}_{F} + \underbrace{8x}_{\substack{O\\U\\T\\S\\I\\D\\E}} + \underbrace{(-3x)}_{I} + \underbrace{(-12)}_{L}$$

Multiplying Binomials using the FOIL Technique

$-3x$ comes from multiplying the *inside* terms of each binomial.

$$(2x-3)\cdot(x+4)=\underbrace{2x^2}_{\text{F}}+\underbrace{8x}_{\text{O}}+\underbrace{(-3x)}_{\text{INSIDE}}+\underbrace{(-12)}_{\text{L}}$$

Multiplying Binomials using the FOIL Technique

-12 comes from multiplying the *last* terms of each binomial.

$$(2x-3)\cdot(x+4)=\underbrace{2x^2}_{\text{F}}+\underbrace{8x}_{\text{O}}+\underbrace{(-3x)}_{\text{I}}+\underbrace{(-12)}_{\substack{\text{L}\\\text{A}\\\text{S}\\\text{T}}}$$

Trinomial times Binomial

Example: Multiply $(x^2 - 3x + 4) \cdot (2x - 3)$

Solution

$(x^2 - 3x + 4) \cdot (2x - 3) =$

$= (2x - 3) \cdot (x^2 - 3x + 4)$ comm prop ×

$= 2x \cdot (x^2 - 3x + 4) + (-3) \cdot (x^2 - 3x + 4)$ distr. prop ×

$= 2x^3 - 6x^2 + 8x - 3x^2 + 9x - 12$ distr. prop ×

$= 2x^3 + (-6x^2 - 3x^2) + (8x + 9x) - 12$ comm., assoc. +

$= \boxed{2x^3 - 9x^2 + 17x - 12}$ addn closure prop

p5 Factoring Polynomials

#1 RULE: FACTOR OUT THE GCF

Factoring reverses multiplication. Consider the polynomial expression $6x^2 - 3x$, whose two terms have a greatest common factor, $3x$.

$$6x^2 - 3x = (3x) \cdot (2x) - (3x) \cdot (1)$$

$$= \quad 3x \cdot (2x - 1) \qquad \text{since} \quad a \cdot b - a \cdot c = a \cdot (b - c)$$

We can rewrite $6x^2 - 3x$ as a difference of two products. Afterwards, we can rewrite an equivalent expression using the distributive property. We call this process **factoring out the gcf**.

Definition (The #1 Rule of Factoring)

The first step to factoring any algebraic expression is to factor out the gcf (if there is one).

Definition

The **greatest common factor (GCF)** for a polynomial is the largest monomial that divides (is a factor of) each term of the polynomial.

Example: The greatest common factor for $25x^5 + 20x^4 - 30x^3$ is $5x^3$ since it is the largest monomial that is a factor of each term.

$$\begin{aligned} 25x^5 + 20x^4 - 30x^3 &= 5x^3 \cdot (5x^2) + 5x^3 \cdot (4x) - 5x^3 \cdot (6) \\ &= 5x^3 \cdot (5x^2 + 4x - 6) \end{aligned}$$

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3. Compute the product of the common factors found in Steps 1 and 2. This expression is the GCF of the polynomial.

Factor the greatest common factor from each of the following.

- $8x^3 - 8x^2 - 48x$
- $15a^7 - 25a^5 + 30a^3$
- $12x^4y^5 - 9x^3y^4 - 15x^5y^3$
- $4(a+b)^4 + 6(a+b)^3 + 16(a+b)^2$
- $x(x+7) + 2(x+7)$

Factoring Trinomials with a Leading Coefficient of 1

Earlier in the chapter, we multiplied binomials.

$$(x + 2) \cdot (x + 8) = x^2 + 10x + 16$$

$$(x + 6)(x + 3) = x^2 + 9x + 18$$

In each case, the product of the two binomials is a trinomial.

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Factoring Trinomials with a Leading Coefficient of 1

In general,

$$(x + a) \cdot (x + b) = x^2 + ax + bx + a \cdot b$$

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We can view this generalization as a factoring problem

$$x^2+(a+b)x+a\cdot b=(x+a)\cdot(x+b)$$

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To factor a trinomial with a leading coefficient of 1, we simply find the two numbers *a* and *b* whose sum is the coefficient of the middle term, and whose product is the constant term.

Factoring Trinomials with a Leading Coefficient of 1

Factor.

- $x^2 + 5x + 4$
- $x^2 + 7x + 6$
- $x^2 + 9x + 14$
- $x^2 + 11x + 24$
- $x^2 + 19x + 34$
- $x^2 + 12x + 27$
- $x^2 + 20x + 64$
- $x^2 + 18x + 65$
- $x^2 - x + 5$
- $x^2 + 5xy + 4y^2$
- $x^2 + 5xy + 6y^2$
- $x^2 + 12xy + 27y^2$
- $m^2 + 19mn + 60n^2$
- $x^2 + 2x - 15$
- $x^2 - 7x - 18$
- $x^2 + x - 20$

Factor By Grouping Method

Polynomials with four terms can sometimes be **factored by grouping**.

Example Factor $x^4 - 2x^3 - 8x + 16$

Solution:

$$
\begin{aligned}
x^4 - 2x^3 - 8x + 16 &= (x^4 - 2x^3) + (-8x + 16) \quad \text{(assoc. prop +)} \\
&= \Big[x \cdot (x^3) + (-2) \cdot (x^3)\Big] + \Big[(-8) \cdot x + (-8) \cdot (-2)\Big] \\
&= x^3(x - 2) - 8(x - 2) \quad \text{(distr. prop.)} \\
&= x^3(x - 2) - 8(x - 2) \quad \text{(identify common factor)} \\
&= \boxed{(x - 2)(x^3 - 8)} \quad \text{(distr. prop)}
\end{aligned}
$$

Factor By Grouping Method

Factor each using the grouping technique.

- $5x + 5y + x^2 + xy$
- $ab^3 + b^3 + 6a + 6$
- $15 - 5y^4 + 3x^3 + x^3y^4$
- $x^3 + 5x^2 + 3x + 15$

ac-grouping method for quadratics

Problem: Factor $ax^2 + bx + c$	Problem: Factor $10x^2 - 11x - 6$
(1) Multiply a times c.	(1) $a = 10$, $b = -11$, $c = -6$, so clearly $a \cdot c = -60$
(2) List all possible pairs of numbers whose product is ac	-60 $\quad$ -60 $\swarrow\searrow$ $\quad$ $\swarrow\searrow$ $-6 \cdot 10$ $\quad$ $-1 \cdot 60$ $6 \cdot (-10)$ $\quad$ $1 \cdot (-60)$ $3 \cdot (-20)$ $\quad$ $-5 \cdot 12$ $-3 \cdot 20$ $\quad$ $5 \cdot (-12)$ $\boxed{-15 \cdot 4}$ $\quad$ $15 \cdot (-4)$
(3) Box the pair whose sum is $b \nearrow$	(3) $b = -11$, and $-15 + 4 = -11$

ac-grouping method for quadratics

Problem: Factor $ax^2 + bx + c$

(4) Replace b with the sum of the circled pair. Distribute x into this quantity

(5) Now factor by grouping: Use parenthesis to group the first two terms, and another () to group the second two terms.

Problem: Factor $10x^2 – 11x – 6$

(4) $10x^2 – 11x – 6$

$= 10x^2 + (–15x + 4x) – 6$

$= 10x^2 – 15x + 4x – 6$

(5) $(10x^2 – 15x) + (4x – 6)$

$= 5x(2x – 3) + 2(2x – 3)$

$= 5x(2x – 3) + 2(2x – 3)$

$= (2x – 3)\cdot(5x + 2)$