Simplifying Rational Expressions and Functions

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Recall: The Number Types

Definition

The set of whole numbers,

$$\mathbb{W} = \{0, 1, 2, 3, 4, \dots\}$$

is the set of natural numbers unioned with zero, written $\mathbb{W} = \mathbb{N} \cup \{0\}$.

Recall: The Number Types

Definition

The set of integers,

$$\mathbb{Z} = \{\ldots, -4, -3, -2, -1, 0, 1, 2, 3, 4, \ldots\}$$

is also known as all the positive and negative whole numbers.

Recall: The Number Types

Definition

A rational number is any number that can be expressed as the ratio of two integers. The set of rational numbers is written symbolically as

$$\mathbb{Q} = \left\{ \frac{a}{b} \,\middle|\, \text{a and b are any integers, and } b \neq 0 \right\}$$

Note that any integer "a" is a rational number since $a = \frac{a}{1}$.

Rational expressions

A rational expression is defined similarly as any expression that can be written as the ratio of two polynomials.

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Some examples of rational expressions are

$$\frac{1}{x}, \qquad \frac{2m-3}{6n-7}, \qquad \frac{x^2-3x-1}{x^2-3x-5}, \qquad \frac{x-y}{y-x}$$

Basic Properties

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We use this property to reduce fractions to lowest terms. For example,

$$\frac{6}{8} = \frac{3 \cdot \cancel{2}}{4 \cdot \cancel{2}} = \frac{3}{4}$$

Using Basic Properties

In a similar fashion, we reduce rational expressions to lowest terms by

1. first factoring the numerator and denominator,
2. and then dividing both numerator and denominator by any factors they have in common.

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Example: Reduce $\frac{x^2 - 25}{x - 5}$ to lowest terms.

Solution:

$$\frac{x^2 - 25}{x - 5} = \frac{(x-5)\cdot(x+5)}{x-5} = \frac{\cancel{(x-5)}\cdot(x+5)}{\cancel{(x-5)}} = x + 5$$

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Solution:

$$\frac{x-5}{x^2-10+25} = \frac{x-5}{(x-5)^2} = \frac{1\cdot(x-5)}{(x-5)\cdot(x-5)} = \frac{1\cdot\cancel{(x-5)}}{(x-5)\cancel{(x-5)}} = \frac{1}{x-5}$$

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Solution:

$$\frac{-3+5x}{25x^2-9} = \frac{5x-3}{(5x)^2-3^2} = \frac{1\cdot(5x-3)}{(5x+3)\cdot(5x-3)} = \frac{1\cdot\cancel{(5x-3)}}{(5x+3)\cancel{(5x-3)}} = \frac{1}{5x+3}$$

Try This One! Reduce $\frac{5x - 3}{3 - 2x}$ to lowest terms.

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Solution:

First degree polynomials have form $ax + b$ for real numbers a and b with a not equal to zero. First degree polynomials are always prime, unless the numbers a and b have a greatest common factor. So, the given expression is prime (not factorable), since both first degree polynomials do not have a common constant that can be divided out of both numerator and denominator. Therefore, the given rational expression is in lowest terms.

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Solution: $$\frac{16y^3-250}{12y^2-26y-10}=\frac{\not{2}\cdot(8y^3-125)}{\not{2}\cdot(6y^2-13y-5)}=\frac{(2y)^3-5^3}{6y^2+2y-15y-5}$$

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$$\frac{16y^3-250}{12y^2-26y-10} = \frac{\cancel{2}\cdot(8y^3-125)}{\cancel{2}\cdot(6y^2-13y-5)} = \frac{(2y)^3-5^3}{6y^2+2y-15y-5}$$

$$= \frac{(2y-5)(4y^2+10y+25)}{(6y^2+2y)+(-15y-5)} = \frac{(2y-5)(4y^2+10y+25)}{2y\cdot(3y+2)+(-5)\cdot(3y+2)} = \frac{\cancel{(2y-5)}(4y^2+10y+25)}{(3y+2)\cdot\cancel{(2y-5)}}$$

$$= \frac{(4y^2+10y+25)}{(3y+2)}$$

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Solution: $$\frac{3a^3+3}{6a^2-6a+6} = \frac{3(a^3+1)}{6(a^2-a+1)} = \frac{3(a+1)\cancel{(a^2-a+1)}}{6\cancel{(a^2-a+1)}}$$

$$= \frac{3(a+1)}{6} = \frac{3(a+1)}{3\cdot 2} = \frac{\cancel{3}(a+1)}{\cancel{3}\cdot 2} = \frac{(a+1)}{2}$$

Try This One! Reduce $\dfrac{x^2 - 3x + ax - 3a}{x^2 - ax - 3x + 3a}$ to lowest terms.

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Solution:

$$\frac{x^2-3x+ax-3a}{x^2-ax-3x+3a} = \frac{(x^2-3x)+(ax-3a)}{(x^2-ax)+(-3x+3a)} = \frac{x(x-3)+a(x-3)}{x(x-a)+(-3)(x-a)}$$

$$= \frac{(x+a)(x-3)}{(x-a)(x-3)} = \frac{(x+a)\cancel{(x-3)}}{(x-a)\cancel{(x-3)}} = \frac{(x+a)}{(x-a)}$$

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Rational Functions

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A rational function is any function that can be written in the form

$$f(x) = \frac{p(x)}{q(x)}$$

where $p(x)$ and $q(x)$ are polynomial functions and $q(x) \neq 0$.

Try This One! Find $f(0)$ and $f(-2)$ for $f(x) = \dfrac{3-x}{x-5}$.

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The Domain of a Rational Functions

Definition

The domain of any rational function is the set of x values represented by all real numbers except the value(s) of x which make the denominator zero upon substitution. That is, for

$$f(x) = \frac{p(x)}{q(x)}$$

where $p(x)$ and $q(x)$ are polynomial functions and $q(x) \neq 0$, the domain of $f(x)$ is all real numbers except those which make $q(x) = 0$.

Try This One! what is the domain of $f(x) = \dfrac{3-x}{x-5}$? of $f(x) = \dfrac{3-x}{(x-5)(x-3)}$

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