Section 2.4 Linear Inequalities in One Variable

Professor Tim Busken

Department of Mathematics

June 16, 2014

Learning Objectives:

- Solve a linear inequality in one variable and graph the solution set.
- Write solutions to inequalities using interval notation.
- Solve a compound inequality and graph the solution set.
- Solve application problems using inequalities.

- An equation states that two algebraic expressions are equal, while an inequality is a statement that indicates two algebraic expressions are not equal in a *particular* way.

- An equation states that two algebraic expressions are equal, while an inequality is a statement that indicates two algebraic expressions are not equal in a *particular* way.

- Inequalities are stated using one the following symbols:
    1. less than $<$,
    2. less than or equal to $\leq$,
    3. greater than $>$,
    4. or greater than or equal to $\geq$.

- An equation states that two algebraic expressions are equal, while an inequality is a statement that indicates two algebraic expressions are not equal in a *particular* way.

- Inequalities are stated using one the following symbols:
  1. less than $<$,
  2. less than or equal to $\leq$,
  3. greater than $>$,
  4. or greater than or equal to $\geq$.

Definition

Replacing the equal sign in the general linear equation $a \cdot x + b = c$ by any of the symbols $<, \leq, >$ or $\geq$ gives a **linear inequality in one variable**.

For example, $2 \cdot x - 1 \leq 0$ and $3x + 5 > 8$ are two different linear inequalities in a single variable, x.

Definition

The solution to any linear inequality is a SET of real numbers.

Definition

The solution to any linear inequality is a SET of real numbers.

For example, $\{x \mid x < -2\}$ is shorthand notation for the set of real numbers less than -2.

$$\{x \;\mid\; x < -2\}$$

$$x \ \underbrace{\text{such that}} \ \underbrace{x \text{ is any real number less than } -2}$$

Addition Property for Inequalities

For any three algebraic expressions A, B and C,

$$\text{If} \quad A < B$$

$$\text{then} \quad A + C < B + C$$

In words: Adding the same quantity to both sides of an inequality will not change the solution set.

We can use the Addn. Prop. to write ***equivalent inequalities***.

Example 1 Solve the inequality, $5x + 4 < 4x + 2$, then graph the solution.

Example 1 Solve the inequality, $5x + 4 < 4x + 2$, then graph the solution.

Solution: Try to get the variable terms on the left-hand side of the inequality, and the constant terms on the right-hand side.

Example 1 Solve the inequality, $5x + 4 < 4x + 2$, then graph the solution.

Solution: Try to get the variable terms on the left-hand side of the inequality, and the constant terms on the right-hand side.

$$5x + 4 < 4x + 2$$

$$5x + 4 + (-4) < 4x + 2 + (-4) \qquad \text{Addition Prop. of Inequalities}$$

$$5x + \left(4 + (-4)\right) < 4x + \left(2 + (-4)\right) \qquad \text{Associative Prop. of Addition}$$

$$5x + 0 < 4x + (-2) \qquad \text{Additive Inverse \& Closure Props.}$$

$$5x < 4x - 2 \qquad \text{Additive Identity \& the Defn. of Subtraction}$$

Example 1 Solve the inequality, $5x + 4 < 4x + 2$, then graph the solution.

Solution:

$$5x < 4x - 2$$

$$5x + (-4x) < 4x - 2 + (-4x)$$ Addition Prop. of Inequalities

$$5x + (-4x) < 4x + (-4x) - 2$$ Commutative Prop. of Addn.

$$\Big(5x + (-4x)\Big) < \Big(4x + (-4x)\Big) - 2$$ Associative Prop. of Addn.

$$(5 - 4) \cdot x < 0 - 2$$ Distributive & Additive Inverse Props.

$$1 \cdot x < -2$$ Closure & Additive Identity Props.

$$x < -2$$ Multiplicative Identity Prop.

Example 1 Solve the inequality, $5x + 4 < 4x + 2$, then graph the solution.

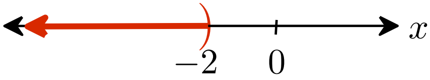
Example 1 Solve the inequality, $5x + 4 < 4x + 2$, then graph the solution.

Conclusion: The solution set of the given inequality is $\{x \mid x < -2\}$. This is called writing the solution using ***set notation*** (or set-builder notation).

Example 1 Solve the inequality, $5x + 4 < 4x + 2$, then graph the solution.

Conclusion: The solution set of the given inequality is $\{x \mid x < -2\}$. This is called writing the solution using ***set notation*** (or set-builder notation).

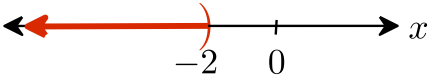
Graph: We can shade the number line to the left of -2 to give a graphical description of the solution set.



Example 1 Solve the inequality, $5x + 4 < 4x + 2$, then graph the solution.

Conclusion: The solution set of the given inequality is $\{x \mid x < -2\}$. This is called writing the solution using ***set notation*** (or set-builder notation).

Graph: We can shade the number line to the left of -2 to give a graphical description of the solution set.



We use a left-opening parenthesis at -2 to indicate that -2 is not part of the solution set.

Example 1 Solve the inequality, $5x + 4 < 4x + 2$, then graph the solution.

An alternate and more compact way of writing the solution set is

$$(-\infty, -2)$$

Example 1 Solve the inequality, $5x + 4 < 4x + 2$, then graph the solution.

An alternate and more compact way of writing the solution set is

$$(-\infty, -2)$$

This gives us 3 equivalent representations of the solution set to the original inequality:

Example 1 Solve the inequality, $5x + 4 < 4x + 2$, then graph the solution.

An alternate and more compact way of writing the solution set is

$$(-\infty, -2)$$

This gives us 3 equivalent representations of the solution set to the original inequality:

Set Notation
$\{x \mid x < -2\}$

Example 1 Solve the inequality, $5x + 4 < 4x + 2$, then graph the solution.

An alternate and more compact way of writing the solution set is

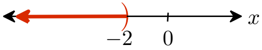
$$(-\infty, -2)$$

This gives us 3 equivalent representations of the solution set to the original inequality:

Set Notation

$\{x \mid x < -2\}$

Line Graph

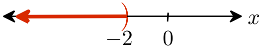


Example 1 Solve the inequality, $5x + 4 < 4x + 2$, then graph the solution.

An alternate and more compact way of writing the solution set is

$$(-\infty, -2)$$

This gives us 3 equivalent representations of the solution set to the original inequality:

Set Notation	Line Graph	Interval Notation
$\{x \mid x < -2\}$		$(-\infty, -2)$

Multiplication Property of Inequalities

For any three algebraic expressions A, B and C, where $C \neq 0$,

$$\begin{array}{rll} \text{If} & A < B, & \\ \text{then} & C \cdot A < C \cdot B & \text{if } C \text{ is positive } (C > 0) \\ \text{or} & C \cdot A > C \cdot B & \text{if } C \text{ is negative } (C < 0) \end{array}$$

Multiplication Property of Inequalities

For any three algebraic expressions A, B and C, where $C \neq 0$,

$$\begin{array}{rll} \text{If} & A < B, & \\ \text{then} & C \cdot A < C \cdot B & \text{if } C \text{ is positive } (C > 0) \\ \text{or} & C \cdot A > C \cdot B & \text{if } C \text{ is negative } (C < 0) \end{array}$$

In words: Multiplying both sides of an inequality by a positive quantity always produces an equivalent inequality. Multiplying both sides of an inequality by a negative number produces an equivalent inequality BUT it reverses the direction of the inequality symbol.

Example 2 Determine what set is the solution to $-2x - 3 \leq 3$

Example 2 Determine what set is the solution to $-2x - 3 \leq 3$

Solution:

$-2x - 3 \leq 3$	
$-2x - 3 + 3 < 3 + 3$	Addition Prop. of Inequalities
$-2x \leq 6$	Additive Inverse & Identity Props
$\left(-\frac{1}{2}\right) \cdot (-2x) \geq \left(-\frac{1}{2}\right) \cdot 6$	Multiplication Prop. of Inequalities
$x \geq -3$	Closure

Example 2 Determine what set is the solution to $-2x - 3 \leq 3$

Solution:

$-2x - 3 \leq 3$	
$-2x - 3 + 3 < 3 + 3$	Addition Prop. of Inequalities
$-2x \leq 6$	Additive Inverse & Identity Props
$\left(-\frac{1}{2}\right) \cdot (-2x) \geq \left(-\frac{1}{2}\right) \cdot 6$	Multiplication Prop. of Inequalities
$x \geq -3$	Closure

Set Notation

$\{x \mid x \geq -3\}$

Solution:

$$-2x-3 \leq 3$$

$$-2x-3+\color{red}{3} < 3+\color{red}{3}$$ Addition Prop. of Inequalities

$$-2x \leq 6$$ Additive Inverse & Identity Props

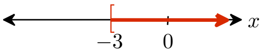
$$\left(-\frac{1}{2}\right)\cdot(-2x) \color{red}{\geq} \left(-\frac{1}{2}\right)\cdot 6$$ Multiplication Prop. of Inequalities

$$x \geq -3$$ Closure

Set Notation

$\{x \mid x \geq -3\}$

Line Graph



Example 2 Determine what set is the solution to $-2x-3 \leq 3$

Solution:

$$-2x-3 \leq 3$$

$$-2x-3+\color{red}{3} < 3+\color{red}{3}$$ Addition Prop. of Inequalities

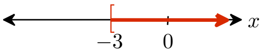
$$-2x \leq 6$$ Additive Inverse & Identity Props

$$\left(-\frac{1}{2}\right)\cdot(-2x) \color{red}{\geq} \left(-\frac{1}{2}\right)\cdot 6$$ Multiplication Prop. of Inequalities

$$x \geq -3$$ Closure

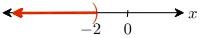
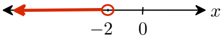
Set Notation
$\{x \mid x \geq -3\}$

Line Graph

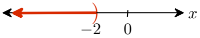
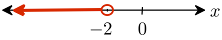
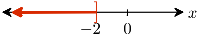
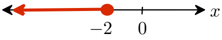


Interval Notation
$[-3, \infty)$

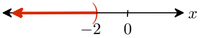
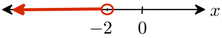
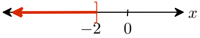
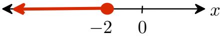
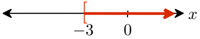
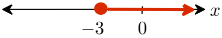
Interval Notation and Graphing

Inequality Notation	Interval Notation	Graph Using Parenthesis/Brackets	Graph using open and closed circles
$x < -2$	$(-\infty, -2)$		

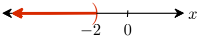
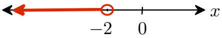
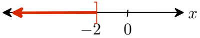
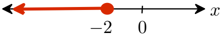
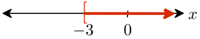
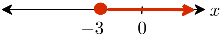
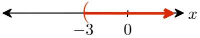
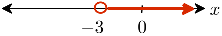
Interval Notation and Graphing

Inequality Notation	Interval Notation	Graph Using Parenthesis/Brackets	Graph using open and closed circles
$x < -2$	$(-\infty, -2)$		
$x \leq -2$	$(-\infty, -2]$		

Interval Notation and Graphing

Inequality Notation	Interval Notation	Graph Using Parenthesis/Brackets	Graph using open and closed circles
$x < -2$	$(-\infty, -2)$		
$x \leq -2$	$(-\infty, -2]$		
$x \geq -3$	$[-3, \infty)$		

Interval Notation and Graphing

Inequality Notation	Interval Notation	Graph Using Parenthesis/Brackets	Graph using open and closed circles
$x < -2$	$(-\infty, -2)$		
$x \leq -2$	$(-\infty, -2]$		
$x \geq -3$	$[-3, \infty)$		
$x > -3$	$(-3, \infty)$		

Classroom Example: Solve the following inequality.

- $3(2x + 5) \leq -3x$

Classroom Examples: Take the next five minutes to work these 6 problems. Graph the solution set to the given inequality, then write the solution set using interval notation.

- $x \leq -6$
- $x > 5$
- $x \geq -1$
- $x > 10$

Classroom Examples: Solve each inequality. Graph the solution set, then write the solution set using interval notation.

- $2x - 1 \leq -6$
- $-3x < 2x - 6$

Definition

A compound inequality is two or more simple inequalities {sets} joined by the terms 'and' or 'or' .

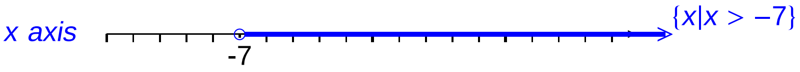
For Example, the set $\left\{ x \;\middle|\; 3x - 6 \leq -3 \quad \text{or} \quad 3x - 6 \geq 3 \right\}$ is a compound inequality.

The inequality statement $-7 < x < 7$ is to be read "x is in between -7 and 7."

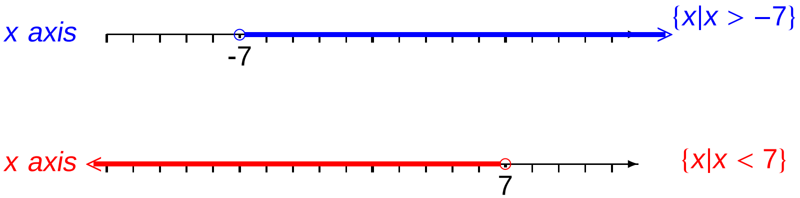
The inequality statement $-7 < x < 7$ is to be read "x is in between -7 and 7." The statement $-7 < x < 7$ is called a composite inequality because it is composed of the intersection of the sets described by $-7 < x$ AND $x < 7$.

The inequality statement $-7 < x < 7$ is to be read "x is in between -7 and 7." The statement $-7 < x < 7$ is called a composite inequality because it is composed of the intersection of the sets described by $-7 < x$ AND $x < 7$. We can use the coordinate line to illustrate each solution set as follows:

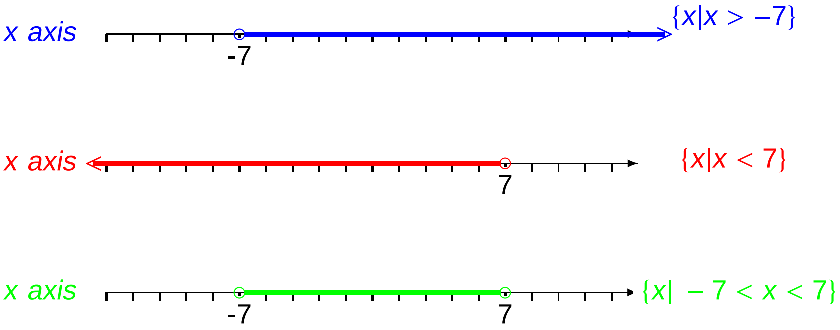
The inequality statement $-7 < x < 7$ is to be read "x is in between -7 and 7." The statement $-7 < x < 7$ is called a composite inequality because it is composed of the intersection of the sets described by $-7 < x$ AND $x < 7$. We can use the coordinate line to illustrate each solution set as follows:



The inequality statement $-7 < x < 7$ is to be read "x is in between -7 and 7." The statement $-7 < x < 7$ is called a composite inequality because it is composed of the intersection of the sets described by $-7 < x$ AND $x < 7$. We can use the coordinate line to illustrate each solution set as follows:



The inequality statement $-7 < x < 7$ is to be read "x is in between -7 and 7." The statement $-7 < x < 7$ is called a composite inequality because it is composed of the intersection of the sets described by $-7 < x$ AND $x < 7$. We can use the coordinate line to illustrate each solution set as follows:



Classroom Examples: Solve the following compound inequalities. Graph the solution set on a number line, then write the solution set using interval notation.

- $-7 \le 2x + 1 \le 7$
- $3x - 6 \le -3$ or $3x - 6 \ge 3$

Interval Notation and Graphing

Inequality Notation	Interval Notation	Graph Using Parenthesis/Brackets	Graph using open and closed circles
$-4 < x < 3$	$(-4, 3)$		

Interval Notation and Graphing

Inequality Notation	Interval Notation	Graph Using Parenthesis/Brackets	Graph using open and closed circles
$-4 < x < 3$	$(-4, 3)$		
$-4 \leq x \leq 3$	$[-4, 3]$		

Interval Notation and Graphing

Inequality Notation	Interval Notation	Graph Using Parenthesis/Brackets	Graph using open and closed circles
$-4 < x < 3$	$(-4, 3)$		
$-4 \leq x \leq 3$	$[-4, 3]$		
$-4 < x \leq 3$	$(-4, 3]$		

Interval Notation and Graphing

Inequality Notation	Interval Notation	Graph Using Parenthesis/Brackets	Graph using open and closed circles
$-4 < x < 3$	$(-4, 3)$		
$-4 \leq x \leq 3$	$[-4, 3]$		
$-4 < x \leq 3$	$(-4, 3]$		
$-4 \leq x < 3$	$[-4, 3)$