Mini-Lecture 10.1

The Parabola and the Circle

Learning Objectives:

1. Graph parabolas of the form $x = a(y-k)^2 + h$ and $y = a(x-h)^2 + k$.
2. Graph circles of the form $(x-h)^2 + (y-k)^2 = r^2$.
3. Find the center and the radius of a circle, given its equation.
4. Write the equation of a circle given its center and radius.

Examples:

1. The graph of each equation is a parabola. Find the vertex of the parabola, and then graph it.

   a) $y = x^2$ b) $y = x^2 + 2$ c) $y = (x-2)^2$ d) $y = -2(x+1)^2 - 1$

   e) $x = y^2$ f) $x = \frac{1}{2}y^2$ g) $x = -3y^2$ h) $x = (y-2)^2 + 1$

   i) $x = -2(y+3)^2 - 3$ j) $y = x^2 - 4x + 1$ k) $x = -2y^2 + 12y - 8$

2. Graph circles of the form $(x-h)^2 + (y-k)^2 = r^2$ by determining the center and radius.

   a) $x^2 + y^2 = 9$ b) $(x-2)^2 + y^2 = 16$ c) $(x+3)^2 + (y-4)^2 = 25$

3. Write an equation of the circle with the given center and radius.

   a) $(3, 5)\,;\,2$ b) $(-2, 4)\,;\,\sqrt{3}$ c) *the origin* ; $5\sqrt{2}$

4. Rewrite each equation in standard form, and determine the center and radius of each circle.

   a) $x^2 + y^2 + 6x - 4y = 23$ b) $x^2 + y^2 - 12x - 2y - 27 = 0$

Teaching Notes:

- Most students need to be reminded of how to graph vertical parabolas.
- Some students find horizontal parabolas very confusing.
- Encourage students to identify the axis of symmetry when graphing parabolas and to plot a couple of points to the right and to the left of the axis of symmetry.
- Most students understand the circle equation once they see how it results from the distance formula.
- Many students need to be reminded of the procedure for completing the square.
- Refer students to the ***Parabolas*** and ***Circle*** charts in the text.

Answers: (graphing answers at end of mini-lectures) 1a) (0,0); b) (0,2); c) (2,0); d) (-1,-1); e) (0,0); f) (0,0); g) (0,0); h) (1,2); i) (-3,-3); j) (2,-3); k) (10,3); 2a) (0,0), r=3; b) (2,0), r=4; c) (-3,4), r=5; 3a) $(x-3)^2+(y-5)^2=4$; b) $(x+2)^2+(y-4)^2=3$; c) $x^2+y^2=50$; 4a) $(x+3)^2+(y-2)^2=36$, center (-3,2), r=6; b) $(x-6)^2+(y-1)^2=64$, center (6,1), r=8

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Mini-Lecture 10.2
The Ellipse and the Hyperbola

Learning Objectives:

1. Define and graph an ellipse.
2. Define and graph a hyperbola.

Examples:

1. Graph each ellipse.

a) $\frac{x^2}{9}+\frac{y^2}{16}=1$ b) $\frac{x^2}{25}+\frac{y^2}{4}=1$ c) $\frac{x^2}{16}+y^2=1$

d) $25x^2+4y^2=100$ e) $\frac{(x+3)^2}{36}+\frac{(y-2)^2}{16}=1$ f) $\frac{(x+2)^2}{25}+\frac{(y+4)^2}{9}=1$

2. Graph each hyperbola.

a) $\frac{x^2}{4}-\frac{y^2}{4}=1$ b) $\frac{y^2}{4}-\frac{x^2}{4}=1$ c) $\frac{x^2}{25}-\frac{y^2}{9}=1$

d) $4y^2-x^2=16$ e) $25x^2-4y^2=100$

3. Identify each equation as that of an ellipse or a hyperbola, then sketch the graph.

a) $\frac{x^2}{25}=1-y^2$ b) $4x^2-25y^2=100$ c) $4(x+3)^2+9(y-3)^2=36$

Teaching Notes:

- Some students understand the graphs better if the domains of *1a)* and *2a)* are discussed before they are graphed.
- Encourage students to memorize the standard forms of the equations of an ellipse or hyperbola centered at the origin. Then the equation for an ellipse centered at (*h, k)* can easily be remembered using graph-shifting ideas.
- Most students need to see many examples of hyperbola graphs in order to master this section.
- Students view the asymptotes as less mysterious if they are shown how a hyperbola equation behaves for very large x (or y) values. For example:
  $\frac{y^2}{b^2}-\frac{x^2}{a^2}=1 \rightarrow y=\pm\frac{b}{a}\sqrt{x^2+a^2} \rightarrow$ as x gets large $\rightarrow$ $y=\pm\frac{b}{a}x$
- Refer students to the ***Ellipse with Center (0,0)*** and ***Hyperbola with Center (0,0)*** charts in this section, and the ***Conic Sections*** chart in the Integrated Review at the end of this section.

Answers: (graphing answers at end of mini-lectures) 3a) $\frac{x^2}{25}+y^2=1$*, ellipse; b)* $\frac{x^2}{25}-\frac{y^2}{4}=1$*, hyperbola;*

c) $\frac{(x+3)^2}{9}+\frac{(y-3)^2}{4}=1$*, ellipse*

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Mini-Lecture 10.3

Solving Nonlinear Systems of Equations

Learning Objectives:

1. Solve a nonlinear system by substitution.
2. Solve a nonlinear system by elimination.

Examples:

1. Solve each nonlinear system of equations by substitution.

a) $\begin{aligned} x^2 + y^2 &= 25 \\ x + y &= 7 \end{aligned}$ b) $\begin{aligned} y &= x^2 - 4x + 4 \\ x + y &= 14 \end{aligned}$ c) $\begin{aligned} x + y &= -3 \\ y^2 - x^2 &= 3 \end{aligned}$

2. Solve each nonlinear system of equations by elimination.

a) $\begin{aligned} x^2 + y^2 &= 52 \\ x^2 - y^2 &= 20 \end{aligned}$ b) $\begin{aligned} y &= x^2 + 2 \\ y &= -x^2 + 8 \end{aligned}$ c) $\begin{aligned} x^2 + y^2 &= 25 \\ y &= \frac{1}{5}x^2 - 5 \end{aligned}$

Teaching Notes:

- Most students understand this section better if they make a rough sketch of each system before trying to solve it.
- Encourage students to check if their intersection points agree with what the sketch suggested for the number of and the rough positions of intersection points.
- Encourage students to write all of the standard form equations for conic sections on an index card for easy reference.
- Most students have a preferred method of solving systems, either substitution or elimination. Encourage them to master both methods so that they can choose the method that is most appropriate for each situation.

Answers: 1a) {(4,3),(3,4)}; b) {(5,9),(-2,16)}; c) {(-1,-2)}; 2a) {(6,4),(6,-4),(-6,4),(-6,-4)}; b) $\{(\sqrt{3},5),(-\sqrt{3},5)\}$; *c) {(0,-5),(5,0),(-5,0)}*

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