Mini-Lecture 2.1

Linear Equations in One Variable

Learning Objectives:

1. Solve linear equations using properties of equality.
2. Solve linear equations that can be simplified by combining like terms.
3. Solve linear equations containing fractions or decimals.
4. Recognize identities and equations with no solution.

Examples:

1. Solve each equation and check.

   a) $x-5=7$ b) $x+3=15$ c) $-3x=15$ d) $\frac{x}{4}=3$

2. Solve each equation and check.

   a) $4x-2=6+3x$ b) $5y-4=10+3y$

   c) $3(2x+4)=9x-3$ d) $-2(3n-1)-n=-5(n-4)$

3. Solve each equation and check.

   a) $\frac{x}{3}+\frac{x}{2}=\frac{1}{4}$ b) $\frac{2x}{5}-\frac{x}{3}=5$ c) $\frac{2r}{5}-3=\frac{r}{10}$

   d) $\frac{28-4x}{3}=x$ e) $\frac{2y-6}{5}=1-2y$ f) $3.4(2x+5)=-0.2(2x+5)$

4. Solve each equation.

   a) $2(x+6)=12+2x$ b) $4(x+5)+3=5(x+2)-x$

Teaching Notes:

- Encourage students to check their solutions.
- Some students prefer to always end up with the variable on the left, while others prefer to always end up with a positive coefficient in front of the variable.
- Some students try to subtract the coefficient from a variable instead of dividing it off.
- Refer students to ***The Addition and Multiplication Properties of Equality*** and ***Solving a Linear Equation in One Variable*** charts in the text.

Answers: 1a) 12; b) 12; c) -5; d) 12; 2a) 8; b) 7; c) 5; d) -9; 3a) $\frac{3}{10}$*; b) 75; c) 10; d) 4; e)* $\frac{11}{12}$*; f) -2.5; 4a) {x|x is a real number}; b)* $\varnothing$

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Mini-Lecture 2.2

An Introduction to Problem Solving

Learning Objectives:

1. Write algebraic expressions that can be simplified.
2. Apply the steps for problem solving.

Examples:

1. Write the following as algebraic expressions. Then simplify.

   a) The sum of three consecutive integers if the first integer is x
   b) The perimeter of a rectangle with length x and width $x - 7$
   c) The total amount of money (in cents) in x quarters, $5x$ dimes, and $(3x\text{-}1)$ nickels

2. Solve using the General Strategy for Problem Solving.

   a) ***Number Problem*** One number is two times another number. The sum of the numbers is 90. What are the two numbers?

   b) ***Number Problem*** Three times the difference of a number and 5 is the same as 1 increased by five times the number plus twice the number.

   c) ***Age Problem*** Today Henry is 7 years older than twice his age of 23 years ago. Find Henry's age today.

   d) ***Car Rental*** A car rental agency advertised renting a luxury, full-size car for \$19.95 per day and \$0.29 per mile. If you rent this car for 5 days, how many whole miles can you drive if you only have \$200 to spend?

   e) ***Carpentry*** A 7-ft board is cut into 2 pieces so that one piece is 3 feet longer than 3 times the shorter piece. If the shorter piece is x feet long, find the lengths of both pieces.

   f) ***Unknown Sides*** A triangle has sides measuring $2.5x$ cm, $3x$ cm, and $(2x + 3)$ cm. It's perimeter measures 60 cm. Find the measures of the sides.

   g) ***Unknown Angles*** Two angles are complementary if their sum is 90°. If the measure of the first angle is $x°$, and the measure of the second angle is $(3x - 2)°$, find the measure of each angle.

   h) ***Lay-offs*** A major car manufacturer announced it would lay off 17,000 employees worldwide. This is equivalent to 20% of its work force. Find the size of the work force prior to lay-offs.

Teaching Notes:

- Many students have difficulty with word problems.
- Encourage students to draw and label diagrams when appropriate.
- Some students need to see several examples of consecutive or consecutive odd/even integers.
- Refer students to the ***General Strategy for Problem Solving*** chart in the text.

Answers: 1a) x+x+1+x+2=3x+3; b) 4x-14; c) 90x-5; 2a) 30, 60; b) -4; c) 39; d) 345 miles; e) 1 foot, 6 feet; f) 19 cm, 22.8 cm, 18.2 cm; g) 23°, 67°; h) 85,000 employees

Mini-Lecture 2.3

Formulas and Problem Solving

Learning Objectives:

1. Solve a formula for a specified variable.
2. Use formulas to solve problems.

Examples:

1. Solve each equation for the specified variable.

   a) $M = kt$ for t b) $C = 2\pi r$ for r c) $a^2 + b^2 = c^2$ for a^2

   d) $4x + 5y = 16$ for y e) $P = 2l + 2w$ for l f) $C = \frac{5}{9}(F - 32)$ for F

2. Solve. Round all dollar amounts to two decimal places.

   a) ***Volume*** Find the volume of a rectangular crate with dimensions 3 ft by 4 ft by 8 ft.

   b) ***Distance*** Sheranda drives at a constant 65 miles per hour. How far will she travel in 4 hours?

   c) ***Compound Interest*** Emmanuel puts $5010 at 9% compounded semiannually for 12 years. What is the value of his account at the end of the 12 years?

   d) ***Circle*** Crystal is making a cover for a round table that has a diameter of 46 inches. How much fabric will she need if she wants the cover to fit exactly, with no material hanging off? (Use 3.14 for π and round to two decimal places.)

   e) ***Office Rental*** An accountant rents office space. He is charged $2040 per month for a rectangular office that measures 17 ft by 20 ft. How much is he paying each month in rent per square foot?

   f) ***Temperature*** Michael's cousin Luke was visiting from Montreal during the summer. On a news report, Luke heard that the temperature in Montreal that day was 98°F. He was used to hearing temperature in degrees Celsius. What is 98°F in degrees Celsius?

   g) ***Triangle*** A triangular piece of wood needs to be varnished. The base of the triangle is 3 meters and the height is 13 meters. How many cans of varnish will be needed if each can covers 10 square meters?

Teaching Notes:

- Some students are very confused by solving for a variable when other variables are present.
- Many students benefit from seeing a parallel example with numbers instead of variables. For example, next to 1a) solve: $6 = 3t$.
- Encourage students to draw and label diagrams when appropriate.
- Refer students to the ***Solving Equations for a Specified Variable*** chart in the text.

Answers: 1a) $t = \frac{M}{k}$; *b)* $r = \frac{C}{2\pi}$; *c)* $a^2 = c^2 - b^2$; *d)* $y = \frac{16 - 4x}{5}$; *e)* $l = \frac{P - 2w}{2}$; *f)* $F = \frac{9}{5}C + 32$; *2a) 96 cubic feet; b) 260 miles; c) $14;408.83; d) 1;661.06 square inches; e) $6.00 per square foot; f) 36.67°C; g) 2 cans*

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Mini-Lecture 2.4

Linear Inequalities and Problem Solving

Learning Objectives:

1. Use interval notation.
2. Solve linear inequalities using the addition property of inequality.
3. Solve linear inequalities using the multiplication and the addition properties of inequality.
4. Solve problems that can be modeled by linear inequalities.

Examples:

1. Graph the solution set of each inequality on a number line and then write it in interval notation.

   a) $\{x \mid x > 3\}$ b) $\{x \mid x < -2\}$ c) $\{x \mid -4.2 \geq x\}$ d) $\{x \mid -3 < x \leq 0\}$

2. Solve. Graph the solution set and write it in interval notation.

   a) $x + 2 \leq 6$ b) $10x < 9x + 3$ c) $5x - 5 \geq 4x - 5$

3. Solve. Graph the solution set and write it in interval notation.

   a) $\frac{1}{2}x \geq 2$ b) $2x > -7.2$ c) $-3x \leq 6$

   d) $2(x+2) \geq x + 2$ e) $0.3(6x-1) < 1.4(x-3) - 0.1$ f) $\frac{5}{6} - \frac{3}{4} > \frac{x}{3}$

4. Solve. Show your answer as an inequality.
   A salesperson earns $2000 a month plus a commission of 20% of sales. Find the minimum amount of sales needed to receive a total income of at least $6000.

Teaching Notes:

- Some students are very confused by solving for a variable when other variables are present.
- Many students forget to reverse the direction of the inequality symbol when necessary.
- Some students prefer to move the variable in such a way that it has a positive coefficient if possible.
- Refer to the end-of-section exercises for application problems.
- Refer students to the ***Addition/Multiplication Property of Inequality*** and ***Linear Inequality in One Variable*** charts in the text.

Answers: (graphing answers at end of mini-lectures) 1a) $(3,\infty)$*; b)* $(-\infty,-2)$*; c)* $(-\infty,-4.2]$*; d)* $(-3,0]$*; 2a)* $(-\infty,4]$*; b)* $(-\infty,3)$*; c)* $[0,\infty)$*; 3a)* $[4,\infty)$*; b)* $(-3.6,\infty)$*; c)* $[-2,\infty)$*; d)* $x \leq -2$*; e)* $x < -10$*; f)* $x < \frac{1}{4}$*; 4)* $\{x \mid x \geq \$20{,}000\}$

Mini-Lecture 2.5

Compound Inequalities

Learning Objectives:

1. Find the intersection of two sets.
2. Solve compound inequalities containing *and*.
3. Find the union of two sets.
4. Solve compound inequalities containing *or*.

Examples:

1. If $A=\{x \mid x \text{ is an even integer}\}$, $B=\{x \mid x \text{ is an odd integer}\}$, $C=\{1,2,3,4\}$, and $D=\{3,4,5,6\}$, list the elements of each set.

   a) $C \cap D$ b) $B \cap C$ c) $A \cap B$

2. Solve each compound inequality by graphing the solution on a number line.

   a) $x \le 1$ and $x \ge -3$ b) $x < 1$ and $x > 4$ c) $x \ge -3$ and $x > 2$

   Solve each compound inequality. Write solutions in interval notation.

   d) $x+3 \ge 4$ and $5x-2 \ge 8$ e) $-5x < -15$ and $x-15 < -10$

   f) $-4 \le x+1 \le -2$ g) $-3 < \frac{2}{3}x - 1 < 1$ h) $-1 \le \frac{-3x+4}{5} \le 1$

3. If $A=\{x \mid x \text{ is an even integer}\}$, $B=\{x \mid x \text{ is an odd integer}\}$, $C=\{3,4,5,6\}$ and $D=\{4,5,6,7\}$, list the elements of each set.

   a) $B \cup C$ b) $C \cup D$ c) $A \cup D$

4. Solve each compound inequality by graphing the solution on a number line.

   a) $x \ge -3$ or $x \le 3$ b) $x < -1$ or $x < 1$ c) $x \ge -2$ or $x \le -3$

   Solve each compound inequality. Write solutions in interval notation.

   d) $-10x \le 20$ or $3x-4 \ge 2$ e) $x+8 < -1$ or $5x > -15$ f) $6(x-2) \ge -12$ or $4-x \le 10$

Teaching Notes:

- In problems 2a-c) and 4a-c), show students how each inequality can be graphed separately on its own number line. Then the solution graph is the intersection (or union) of the individual graphs.

Answers: (graphing answers at end of mini-lectures) 1a) {3,4}; b) {1,3}; c) $\varnothing$; 2a) [-3,1]; b) no solution; c) $(2,\infty)$; d) $[2,\infty)$; e) (3,5); f) [-5,-3]; g) (-3,3); h) $\left[-\frac{1}{3},3\right]$; 3a) {x|x is an odd integer, x=4, x=6}; b) {1,2,3,4,5,6}; c) {x|x is an even integer, x=3, x=5}; 4a) all real numbers; b) $(-\infty,1)$; c) $(-\infty,-3] \cup [-2,\infty)$; d) $[-2,\infty)$; e) $(-\infty,-9) \cup (-3,-\infty)$; f) $[-6,\infty)$

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