# Mini-Lecture 3.1

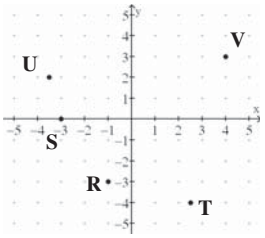
Graphing Equations

**Learning Objectives:**

1. Plot ordered pairs.
2. Determine whether an ordered pair of numbers is a solution to an equation in two variables.
3. Graph linear equations.
4. Graph nonlinear equations.

**Examples:**

1. Determine the ordered pairs, or plot the points. Name the quadrant in which each point lies.

   a) 


   b) (4,2) ; (−3,5) ; (−2,−4) ; (3,−4) ; (0,5) ; (−2.5,0)

2. Determine whether each ordered pair is a solution of the given equation.

   a) $x + y = 7$ ; (1,6), (−3,10)  b) $y = -3x + 2$ ; (0,2), (−2,10)  c) $4x - 3y = 1$ ; $\left(\frac{1}{2}, \frac{2}{3}\right)$, (0,1)

3. Graph each linear equation by finding any three ordered pairs that are solutions to the equation.

   a) $x + y = 2$  b) $2x - 4y = 8$  c) $y = \frac{2}{3}x + 3$  d) $x = 3$  e) $y = -2$

4. Graph each nonlinear equation by finding any 5 ordered pairs that are solutions to the equation.

   a) $y = 3x^2$  b) $y = x^2 - 2$  c) $y = x^3$

**Teaching Notes:**

- In problem 3, some students do not realize that they can choose any $x$ value at all and solve for $y$, or vice versa.
- Be sure to show students how to plot using *x*- and *y-intercept*s too.
- Refer to the end of section exercises for scatter diagram problems and word problems.
- Refer students to the ***Linear Equation in Two Variables*** and ***Finding x- and y-Intercepts*** charts in the text.

*Answers: (graphing answers at end of mini-lectures)  1a) R(−1,−3), S(−3,0), T(2.5,−4), U(−3.5,2), V(4,3); 2a) yes, yes;  b) yes, no; c) no, no*

# Mini-Lecture 3.2

Introduction to Functions

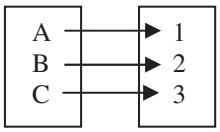
**Learning Objectives:**

1. Define relation, domain, and range.
2. Identify functions.
3. Use the vertical line test for functions.
4. Find the domain and range of a function.
5. Use function notation.

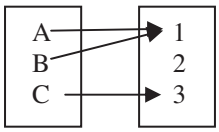
**Examples:**

1. Find the domain and range of each relation. Also determine whether the relation is a function.

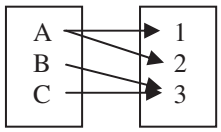
   a) Input: Output:

   b) Input: Output:

   c) Input: Output:

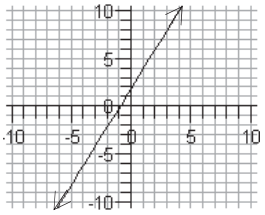
   d) $\{(1,4),(1,6)\}$ e) $\{(-2,-6),(0,-6)\}$ f) $\{(-6,-7),(-2,-5),\left(\frac{1}{2},\frac{2}{3}\right),(0.5,3)\}$

2. Determine whether each relation is also a function.

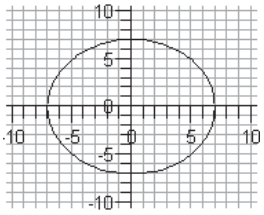
   a) $y = x + 3$ b) $y - x = 5$ c) $x = 3y^2$

3. Use the vertical line test to determine whether each graph is the graph of a function.

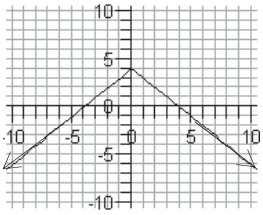
   a)

   b)

   c)

4. Refer to the graphs in problem 3 to answer this question.
   Find the domain and range of each relation.

5. For each function, find the indicated values.

   a) $f(x) = x - 2;$ find $f(3), f(-1)$ b) $g(x) = 3x^2 - 4x + 1;$ find $g(0), g(-2)$

**Teaching Notes:**

- For domain and range, students find it helpful to think of $x$ values as inputs, and $y$ values as outputs.
- Point out to students that equivalent domain or range elements that occur more than once only need to be listed once.
- Some students are very confused by function notation.
- Refer to the end of section exercises for application problems.
- Refer students to the ***Vertical Line Test*** chart in the text.

*Answers: 1a) domain {A,B,C}, range {1,2,3}, function; b) domain {A,B,C}, range {1,3}, function; c) domain {A,B,C}, range {1,2,3}, not a function; d) domain {1}, range {4,6}, not a function; e) domain {−2,0}, range {−6}, function; f) domain {−6,−2,0.5 }, range $\{-7,-5,\frac{2}{3},3\}$, not a function; 2a)function; b) function; c) not a function; 3a) function; b) not a function; c) function; 4a) domain $(-\infty,\infty)$, range $(-\infty,\infty)$; b) domain $(-7,7)$, range $(-7,7)$; c) domain $(-\infty,\infty)$, range $(-\infty,4)$; d) domain $(-\infty,\infty)$, range $(-\infty,\infty)$; e) domain $(-\infty,\infty)$, range {6}; f) domain {−7}, range $(-\infty,\infty)$; 5a) 1,−3; b) 1,21*

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# Mini-Lecture 3.3

Graphing Linear Functions

**Learning Objectives:**

1. Graph linear functions.
2. Graph linear functions by using intercepts.
3. Graph vertical and horizontal lines.

**Examples:**

1. Graph each linear function.

   a) $f(x) = x$ b) $f(x) = -2x + 1$ c) $f(x) = 2x - 3$

2. Find the intercepts and graph. Then write each equation using function notation.

   a) $4x + 3y = 12$ b) $y = -4x$ c) $x - y = 4$

3. Graph vertical and horizontal lines.

   a) $x = -5$ b) $y = -2$ c) $x - 4 = 0$

**Teaching Notes:**

- Remind students that any function can be written with or without function notation.
- Refer students to the ***Finding x- and y-Intercepts*** and ***Graphing Vertical and Horizontal Lines*** charts in the text.

*Answers: (graphing answers at end of mini-lectures) 2a) f(x) = −4/3x+4; b) f(x) = −4x; c) f(x) = x−4*

# Mini-Lecture 3.4

The Slope of a Line

**Learning Objectives:**

1. Find the slope of a line given two points on the line.
2. Find the slope of a line given the equation of the line.
3. Interpret the slope-intercept form in an application.
4. Find the slopes of horizontal and vertical lines.
5. Compare the slopes of parallel and perpendicular lines.

**Examples:**

1. Find the slope of the line given two points on the line.

   a) $(1, 5)$ , $(6, 11)$ b) $(3, 6)$ , $(-2, 9)$ c) $(3, -1)$ , $(4, -5)$

2. Find the slope and the *y-intercept* of each line.

   a) $y = x + 3$ b) $y = -4x - 1$ c) $-3x + y = 9$

   d) $x = 3.4$ e) $y = -\frac{1}{3}x$ f) $2x - 9y = 36$ g) $y - 8 = 0$

3. Solve.

   a) When a road-side service truck is called, the cost of the service is given by the linear function $y = 2x + 60$, where $y$ is in dollars and $x$ is the number of hours the car is worked on. Find and interpret the slope and *y-intercept* of the linear equation.

   b) The amount of water in a leaky water jug is given by the linear function $y = 117 - 10x$, where $y$ is in ounces and $x$ is in minutes. Find and interpret the slope and *y-intercept* of the linear function.

4. Find the slope of each line.

   a) $x = 3$ b) $x - 5 = 0$ c) $y = -4$

5. Determine whether each pair of lines is parallel, perpendicular, or neither.

   a) $y = 3x - 4$, $y = 3x + 2$ b) $-2x + 4y = 1$, $6x + 3y = 3$ c) $y = 3x + 4$, $y = -3x + 4$

**Teaching Notes:**

- Some students need to see many numeric examples of $m$ = rise/run shown on a graph before trying to use the slope formula.
- Many students make sign errors with the slope formula.
- Some students consistently put the change in $x$ instead of the change in $y$ in the numerator.
- Some students are confused by the slopes of horizontal and vertical lines.
- Some students understand objective 5 better if it is introduced using a discovery activity.
- Refer students to the ***Slope of a Line***, ***Slope-Intercept Form***, ***Parallel Lines***, and ***Perpendicular Lines*** charts in the text.

*Answers: 1a) $m = \frac{6}{5}$; b) $m = -\frac{3}{5}$; c) m=−4; 2a) m=1, (0,3); b) m=−4, (0,−1); c) m=3, (0,9); d) undefined, no y-intercept; e) $m = -\frac{1}{3}$, (0,0); f) $m = \frac{2}{9}$, (0,−4); g) m=0, (0,8); 3a) m=2,cost increases 2 dollars for every hour of work, (0,60)…there is a minimum basic charge of $60; b) m=−10…the jug loses 10 ounces per minute, (0,117)…the jug started with 117 ounces in it; 4a) undefined; b) undefined; c) zero; 5a) parallel; b) perpendicular; c) neither*




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# Mini-Lecture 3.5

Equations of Lines

**Learning Objectives:**

1. Graph a line using its slope and *y*-intercept.
2. Use the slope-intercept form to write the equation of a line.
3. Use the point-slope form to write the equation of a line.
4. Write equations of vertical and horizontal lines.
5. Write equations of parallel and perpendicular lines.

**Examples:**

1. Graph each linear equation using the slope and *y*-intercept.

   a) $y = 2x$ b) $y = 2x + 3$ c) $y = -2x + 1$

   d) $y = \frac{1}{2}x - 2$ e) $x + 2y = 6$ f) $3x - 2y = 12$

2. Use the slope-intercept form of a linear equation to write the equation of each line with the given slope and *y*-intercept.

   a) slope $-1$; *y*-intercept $(0,4)$ b) slope $\frac{1}{3}$; *y*-intercept $(0,-7)$ c) slope $-\frac{5}{2}$; *y*-intercept $(0,0)$

3. Write an equation of each line with the given slope and containing the given point. Write the final equation in slope-intercept form.

   a) slope 3; through $(6,2)$ b) slope $-\frac{2}{3}$; through $(1,-5)$ c) slope $\frac{3}{2}$; through $(-2,-7)$

   Write an equation of the line passing through the given points. Write the final equation in standard form.

   d) $(3,0)$ and $(5,4)$ e) $(8,-4)$ and $(5,5)$ f) $\left(-\frac{1}{2},\frac{1}{3}\right)$ and $\left(\frac{5}{2},-\frac{2}{3}\right)$

4. Write an equation of each line.

   a) vertical; through $(2,4)$ b) horizontal; through $(-1,-3)$

   c) undefined slope; through $(0,3)$ d) slope 0; through $(-6,4)$

5. Write an equation of each line. Write the equation in the form $x = a$, $y = b$, or $y = mx+b$.

   a) through $(0,3)$; parallel to $y = 2x - 1$ b) through $(1,4)$; parallel to $2x - 3y = 1$

   c) through $(0,-2)$; perpendicular to $y = -4x + 2$ d) through $(-6,4)$; perpendicular to $2x + 5y = 10$

**Teaching Notes:**

- Some students need a lot of practice using the slope to graph a line.
- Emphasize to students how the sign of the slope is built into the direction you go when using the slope to graph a line.
- Most students understand the point-slope form better if they see that it is just a re-arranging of the slope formula.
- Some students struggle with the fractions that arise when solving the problems in number 5.
- Refer students to the ***Point-Slope Form of the Equation of a Line*** chart in the text.

*Answers: (graphing answers at end of mini-lectures); 2a)* $y = -x + 4$; *b)* $y = \frac{1}{3}x - 7$; *c)* $y = -\frac{5}{2}x$; *3a)* $y=3x-16$; *b)* $y = -\frac{2}{3}x - \frac{13}{3}$; *c)* $y = \frac{3}{2}x - 4$; *d)* $2x-y=6$; *e)* $3x+y=20$; *f)* $2x+6y=1$; *4a)* $x=2$; *b)* $y=-3$; *c)* $x=0$; *d)* $y=4$; *5a)* $y=2x+3$; *b)* $y = \frac{2}{3}x + \frac{10}{3}$; *c)* $y = \frac{1}{4}x - 2$; *d)* $y = \frac{5}{2}x + 19$

# Mini-Lecture 3.6

Graphing Piecewise-Defined Functions and Shifting and Reflecting Graphs of Functions

**Learning Objectives:**

1. Graph piecewise-defined functions.
2. Vertical and horizontal shifts.
3. Reflect graphs.

**Examples:**

1. Graph each piecewise-defined function.

   a) $f(x) = \begin{cases} x & \text{if } x \le 0 \\ x+2 & \text{if } x > 0 \end{cases}$ b) $g(x) = \begin{cases} 4x+3 & \text{if } x \le 1 \\ \frac{1}{3}x - 2 & \text{if } x > 1 \end{cases}$

   Graph each piecewise-defined function. Use the graph to determine the domain and range.

   c) $g(x) = \begin{cases} x+2 & \text{if } x < 0 \\ -x+2 & \text{if } x \ge 0 \end{cases}$ d) $h(x) = \begin{cases} -2 & \text{if } x \le 0 \\ 2 & \text{if } x \ge 1 \end{cases}$

2. Sketch each pair of functions on one axis.

   a) $f(x) = x$, $g(x) = x + 2$ b) $f(x) = |x|$, $g(x) = |x| - 2$ c) $f(x) = |x|$, $g(x) = |x - 2|$

   d) $f(x) = |x|$, $g(x) = |x + 2|$ e) $f(x) = x^2$, $g(x) = (x-2)^2 + 1$ f) $f(x) = \sqrt{x}$, $g(x) = \sqrt{x+1} - 2$

3. Sketch each pair of functions on one axis.

   a) $f(x) = x$, $g(x) = -x$ b) $f(x) = |x|$, $g(x) = -|x|$

   c) $f(x) = \sqrt{x}$, $g(x) = -\sqrt{x-2}$ d) $f(x) = x^2$, $g(x) = -(x+2)^2 - 1$

**Teaching Notes:**

- Most students find vertical shifts easy to understand.
- Some students are confused by the directions of a horizontal shift.
- Objectives 2 and 3 can be covered in a more timely manner if students are broken into groups and each group is given one type of common graph to focus on. Then the class can discuss the results and generalize to arrive at the shifting and reflecting properties.
- Refer students to the ***Vertical Shifts, Horizontal Shifts,*** and ***Reflections About the x-axis*** charts in the text.

*Answers: (graphing answers at end of mini-lectures) 1c) domain $(-\infty, \infty)$, range $(-\infty, 2]$; d) domain $(-\infty, 0] \cup [1, \infty)$, range $\{-2, 2\}$*




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# Mini-Lecture 3.7

Graphing Linear Inequalities

**Learning Objectives:**

1. Graph linear inequalities.
2. Graph the intersection or union of two linear inequalities.

**Examples:**

1. Graph each inequality. Use a test point to check the solution region.

   a) $y < x$ b) $y \geq x + 2$ c) $y \leq -x - 3$

   d) $x + 2y > -2$ e) $-2x - 5y \geq 10$ f) $2x < -3y$

   g) $y > \frac{1}{2}x$ h) $y \leq 2$ i) $x \geq -2\frac{1}{3}$

2. Graph each union or intersection.

   a) The intersection of $x \leq 2$ and $y \geq -3$
   b) The union of $x \leq 2$ or $y \geq -3$
   c) The intersection of $x - y < 2$ and $x + y \geq 3$
   d) The union of $2x - 3y < 6$ or $2x + y \geq 3$

**Teaching Notes:**

- Most students who are good at graphing linear equations find this section easy.
- Remind students to always use a test point from the solution region, and not from the boundary line, to check their graph.
- Remind students to use a dashed line for < or > and a solid line for ≤ or ≥.
- Refer students to the ***Graphing a Linear Inequality in Two Variables*** chart in the text.

*Answers: (graphing answers at end of mini-lectures)*