

Mini-Lecture 4.1

Solving Systems of Linear Equations in Two Variables

Learning Objectives:

1. Determine whether an ordered pair is a solution of a system of two linear equations.
2. Solve a system by graphing.
3. Solve a system by substitution.
4. Solve a system by elimination.

Examples:

1. Determine whether the given ordered pair is a solution of the system.

a) $\begin{cases} x + y = 4 \\ x - y = 2 \end{cases}$; (3,1) b) $\begin{cases} y = 4 \\ x = -3y \end{cases}$; (-6,4) c) $\begin{cases} 2x + y = 4 \\ -3x = 2y + 8 \end{cases}$; $\left(\frac{1}{2}, 3\right)$

2. Solve each system by graphing.

a) $\begin{cases} x + y = 4 \\ x - y = 2 \end{cases}$ b) $\begin{cases} 2x + 4y = 10 \\ 4x + 3y = 10 \end{cases}$ c) $\begin{cases} y = -x + 3 \\ 2x + 2y = -1 \end{cases}$

3. Use the substitution method to solve each system of equations.

a) $\begin{cases} x + y = 4 \\ x - y = 2 \end{cases}$ b) $\begin{cases} \frac{1}{4}x + \frac{1}{4}y = 2 \\ x - y = 2 \end{cases}$ c) $\begin{cases} y = -3x + 8 \\ 12x + 4y = 32 \end{cases}$

4. Use the elimination method to solve each system of equations.

a) $\begin{cases} x + y = 4 \\ x - y = 2 \end{cases}$ b) $\begin{cases} x - 6y = -9 \\ 8x - 6y = -30 \end{cases}$ c) $\begin{cases} x - 4y = -8 \\ -6x - 3y = -6 \end{cases}$

d) $\begin{cases} 3x + 6y = 3 \\ 2x + 9y = -8 \end{cases}$ e) $\begin{cases} 6x - 8y = 8 \\ 12x = 16y + 24 \end{cases}$ f) $\begin{cases} -6x - 4y = -2 \\ -12y = -6 + 18x \end{cases}$

Teaching Notes:

- Help students visualize a system by graphing examples of the three possible results: one solution, no solution, ∞ solutions.
- Some students have trouble with the substitution method when fractions are involved.
- Most students prefer the addition method.
- Encourage students to check final answers.
- Many students have trouble drawing the conclusion of “no solution” or “infinite solutions” from the non-graphing methods.
- Refer students to the *Possible Solutions to Systems of Two Linear Equations*, and *Solving a System of Two Equations Using the Substitution/Elimination Method* charts in the text.

Answers: (graphing answers at end of mini-lectures) 1a) yes; b) no; c) no; 2a) (3,1); b) (1,2); c) \emptyset ; 3a) (3,1); b) (5,3); c) $\{(x,y)|y=-3x+8\}$; 4a) (3,1); b) (-3,1); c) (0,2); d) (5,-2); e) \emptyset ; f) $\{(x,y)|-6x-4y=-2\}$

Mini-Lecture 4.2

Solving Systems of Linear Equations in Three Variables

Learning Objectives:

1. Solve a system of three linear equations in three variables.

Examples:

1. Solve each system.

$$\begin{array}{l} x + y + z = 3 \\ \text{a) } x - y + 2z = -1 \\ 4x + y + z = 15 \end{array}$$

$$\begin{array}{l} 5x + 3y + z = 25 \\ \text{b) } 3x - 3y - z = 7 \\ 4x + y + 4z = 14 \end{array}$$

$$\begin{array}{l} x + 4y + 2z = -7 \\ \text{c) } 5y + 4z = -15 \\ z = -5 \end{array}$$

$$\begin{array}{l} x - y + 4z = 3 \\ \text{d) } 5x + z = 0 \\ x + 3y + z = -9 \end{array}$$

$$\begin{array}{l} \frac{2}{3}x - \frac{1}{2}y + 2z = -18 \\ \text{e) } x - \frac{2}{3}y - \frac{1}{2}z = -12 \\ x - \frac{1}{2}y - z = -8 \end{array}$$

Teaching Notes:

- Students need to be extremely neat and organized to succeed with these.
- Most students prefer to use the elimination method repeatedly.
- Some students prefer to use the substitution method to eliminate the first variable whenever it is possible to do so without generating fractions.
- Most students have trouble visualizing these systems. Refer them to the figures of intersecting planes in the text.
- Refer students to the *Solving a System of Three Linear Equations by the Elimination Method* chart in the text.

Answers: 1a) $(4, 1, -2)$; b) $(4, 2, -1)$; c) $(-1, 1, -5)$; d) $(0, -3, 0)$; e) $(-6, 12, -4)$

Mini-Lecture 4.5

Systems of Linear Inequalities

Learning Objectives:

1. Graph a system of linear inequalities.

Examples:

1. Graph the solutions of each system of two linear inequalities.

a) $y \geq 2x - 4$
 $y \leq -x + 1$

b) $y \leq 2x - 1$
 $x + y > -4$

c) $y \leq 2x + 1$
 $y < -3x$

d) $x + 3y > -6$
 $y < -2$

e) $x \geq -2$
 $y \geq 6$

Graph the solutions of each system of three linear inequalities.

f) $x + y \geq 1$
 $x - y \geq 1$
 $x \leq 4$

g) $2x + 3y \geq 6$
 $x - y \leq 3$
 $y \leq 2$

h) $2x + 3y \leq 6$
 $x - y \geq 3$
 $x \geq 1$

Teaching Notes:

- Remind students to use a dashed line for $<$ or $>$ and a solid line for \leq or \geq .
- Encourage students to use different colors for each line.
- Encourage students to check their graphs using a test point from the solution region.
- Refer students to the *Graphing the Solutions of a System of Linear Inequalities* chart in the text.

Answers: (graphing answers at end of mini-lectures)