Mini-Lecture 5.1

Exponents and Scientific Notation

Learning Objectives:

1. Use the product rule for exponents.
2. Evaluate expressions raised to the zero power.
3. Use the quotient rule for exponents.
4. Evaluate expressions raised to the negative *n*th power.
5. Convert between scientific notation and standard notation.

Examples:

1. Use the product rule to simplify each expression.
   a) $2^3 \cdot 2^4$ b) $m \cdot m^9 \cdot m^7$ c) $(-6xy)(6y)$ d) $(-3a^3b^2)(-5a^3b)$

2. Evaluate or simplify each expression.
   a) 2^0 b) -5^0 c) $(-10)^0$ d) $(2x+1)^0$

3. Use the quotient rule to simplify each expression.
   a) $\dfrac{x^8}{x^3}$ b) $-\dfrac{10y^{11}}{2y^7}$ c) $\dfrac{15x^6y^5}{9xy^3}$ d) $\dfrac{36a^2b^3c^{12}}{-4abc^9}$

4. Simplify and write using positive exponents only.
   a) 2^{-4} b) $(-3)^{-2}$ c) $\dfrac{y^{-3}}{y^6}$ d) $2a^{-3}$

   e) $\dfrac{x^{-5}x^4}{x^{-2}}$ f) $\dfrac{12ab^{-3}}{4a^{-3}b^3}$ g) $\dfrac{20x^{-8}yz^{-13}}{2xyz}$ h) $(3a^3b)(-2a^{-4}b^{-2})$

5. Write each number in scientific notation or in standard notation.
   a) 645,000 b) 0.005621 c) 3.6×10^{-4} d) 9.5×10^5

Teaching Notes:

- Students need a lot of repetition and practice in order to master these objectives.
- Students often move constants along with a variable that has a negative exponent. For example, in 4d) a common answer is $2a^{-3} = 1/(2a^3)$.
- Refer students to the exponent rule charts and the ***Writing a Number in Scientific Notation*** chart in the text.

Answers: 1a) 128; b) m^{17}; c) $-36xy^2$; d) $15a^6b^3$; 2a) 1; b) −1; c) 1; d) 1; 3a) x^5; b) $-5y^4$; c) $\dfrac{5x^5y^2}{3}$; d) $-9ab^2c^3$; 4a) $\dfrac{1}{16}$; b) $\dfrac{1}{9}$; c) $\dfrac{1}{y^9}$; d) $\dfrac{2}{a^3}$; e) x; f) $\dfrac{3a^4}{b^6}$; g) $\dfrac{10}{x^9z^{14}}$; 5a) $6.45\text{x}10^5$; b) $5.621\text{x}10^{-3}$; c) 0.00036; d) 950,000

Mini-Lecture 5.2

More Work with Exponents and Scientific Notation

Learning Objectives:

1. Use the power rules for exponents.
2. Use all exponent rules and definitions to simplify exponential expressions.
3. Compute using scientific notation.

Examples:

1. Simplify using the product rules for exponents. Write each answer using positive exponents only.

   a) $(x^3)^2$ b) $(x^2y^3)^2$ c) $\left(\frac{x^2}{y^3}\right)^2$ d) $(m^3)^{-4}$

   e) $(2x^2yz^3)^2$ f) $\left(\frac{3x^4}{y^{-2}}\right)^5$ g) $(4x^{-5}y^3z^0)^{-3}$ h) $(-2^{-3}y^{-3})^{-4}$

2. Simplify using exponent rules and definitions. Write each answer using positive exponents only.

   a) $\left(\frac{a^{-3}b^{-4}}{c^{-9}}\right)^{-2}$ b) $(-4x^2)^3$ c) $\left(\frac{n^6}{2m^{-3}}\right)^{-5}$ d) $\frac{7^{-2}x^{-2}y^{10}}{x^3y^{-4}}$

   e) $(-2x^0y^2)^{-3}$ f) $x^3(x^3by)^{-3}$ g) $\left(\frac{2z^{-3}}{y}\right)\left(\frac{7y^{-5}}{z^{-2}}\right)^{-1}$ h) $(3x^4y^2)^{-3}(2x^8y^3)$

3. Perform each indicated operation using the properties of exponents. Write each answer in scientific notation.

   a) $(4.9\times10^{-9})(6\times10^{7})$ b) $(4\times10^{-6})^5$ c) $\frac{1.2\times10^{8}}{3\times10^{-4}}$

Teaching Notes:

- Some students are confused by when to add exponents versus when to multiply exponents.
- Encourage students to write the exponent rules on an index card to view while doing homework.
- Refer students to the ***Summary of Rules for Exponents*** chart in the text.

Answers: 1a) x^6; b) x^4y^6; c) $\frac{x^4}{y^6}$; d) $\frac{1}{m^{12}}$; e) $4x^4y^2z^6$; f) $243x^{20}y^{10}$; g) $\frac{x^{15}}{64y^9}$; h) $4096y^{12}$; 2a) $\frac{a^6b^8}{c^{18}}$; b) $-64x^6$; c) $\frac{32}{m^{15}n^{30}}$; d) $\frac{y^{14}}{49x^5}$; e) $-\frac{1}{8y^6}$; f) $\frac{1}{b^3x^6y^3}$; g) $\frac{2y^4}{7z^5}$; h) $\frac{2}{27x^4y^3}$; 3a) $2.94\text{x}10^{-1}$; b) $1.024\text{x}10^{-27}$; c) $4.0\text{x}10^{11}$

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Mini-Lecture 5.3

Polynomials and Polynomial Functions

Learning Objectives:

1. Identify term, constant, polynomial, monomial, binomial, trinomial, and the degree of a term and of a polynomial.
2. Define polynomial functions.
3. Review combining like terms.
4. Add polynomials.
5. Subtract polynomials.
6. Recognize the graph of a polynomial function from the degree of the polynomial.

Examples:

1. Find the degree of each polynomial, state how many terms it has, and indicate whether it's a monomial, binomial, or trinomial.

   a) $3x$ b) $9x^2$ c) $-2x^3+5$ d) x^2y^2-4x+3

2. Define a polynomial function.

3. Simplify each polynomial by combining like terms.

   a) $2x+3x$ b) $10y-8y$ c) $xy+3x-2xy$

   d) $-x+2x-6x^2-3x^2$ e) $-9y+8y+2y^5$ f) $-2xy^2+3x-x+8xy^2-\frac{3}{5}$

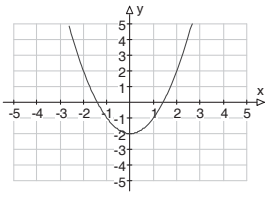
4. Add the polynomials.

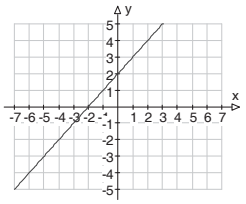
   a) $(-3y^2-2y+5)+(2y+7)$ b) $\left(2x^2-3x\right)+\left(-6x^2-7x\right)$ c) $\begin{array}{r} 5x^2+3x-2 \\ +\left(7x^2-5x-3\right) \\ \hline \end{array}$

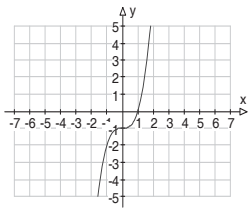
5. Subtract the polynomials.

   a) $\begin{array}{r} -6x^2-3x+9 \\ -\quad (7x+10) \\ \hline \end{array}$ b) $\begin{array}{r} 4x^2-3x-15 \\ -\left(5x^2-3x-22\right) \\ \hline \end{array}$ c) $(2x-2)-(-x-2)$

6. Match each equation with its graph.

1) 

2) 

3) 

a) $y=x^3-1$ _____

b) $y=x^2-2$ _____

c) $y=x+2$ _____

Teaching Notes:

- Most students find these polynomial operations easy.
- Tell students that identifying the degree of a polynomial is important for later work with factoring and solving equations.
- Remind students that this section is a review of distributing and collecting like terms.
- Some students forget to distribute the minus sign when lining up vertically.

Answers: 1a) 1,1 monomial; b) 2,1, monomial; c) 3,2, binomial; d) 4,3, trinomial; 3a) 5x; b) 2y; c) –xy+3x; d) x-9x^2; e) –y+2y^5; f) $6xy^2+2x-\frac{3}{5}$; 4a) -3y^2+12; b) -4x^2-10x; c) 12x^2-2x-5; 5a) -6x^2-10x-1; b) –x^2+7; c) 3x; 6a) graph 3; b) graph 1; c) graph 2

Mini-Lecture 5.4

Multiplying Polynomials

Learning Objectives:

1. Multiply two polynomials.
2. Multiply binomials.
3. Square binomials.
4. Multiply the sum and difference of two terms.
5. Multiply three or more polynomials.
6. Evaluate polynomial functions.

Examples:

1. Multiply.

   a) $(2x)(4x)$ b) $(-6a^2)(5a^3)$ c) $(4.1\,x\,y^2z^{10})(6\,xy^5z)$ d) $2x(3x-4)$

   e) $-3y(5xy+2x)$ f) $-2b^2z(2z^2a+baz-b)$ g) $(x+3)(2x^2-x+5)$

2. Multiply.

   a) $(x+3)(3x-4)$ b) $(x+2)(x+3)$ c) $\begin{array}{r} 4y-3 \\ \times\ 2y-2 \\ \hline \end{array}$

   d) $(x+6)(x+6)$ e) $(3x^2-4y^2)(x^2-6y^2)$ f) $\left(3y-\frac{1}{4}\right)\left(4y-\frac{1}{6}\right)$

3. Multiply.

   a) $(x+2)^2$ b) $(x-4)^2$ c) $(x+7)^2$

4. Multiply.

   a) $(x+5)(x-5)$ b) $(2xy-3b)(2xy+3b)$ c) $\left(5x-\frac{1}{2}\right)\left(5x+\frac{1}{2}\right)$ d) $\left[6-(2b-2)\right]^2$

5. Multiply.

   a) $(x+3)(x-2)(2x-1)$ b) $(y-2)^4$ c) $(x-y)(x+y)(x+y)$

6. If $f(x)=x^2-2x$, find the following.

   a) $f(a)$ b) $f(c)$ c) $f(a+b)$ d) $f(a-2)$

Teaching Notes:

- Encourage students to multiply binomials with FOIL mentally whenever possible. This will make factoring easier for them in future sections.
- Many students distribute the exponent when squaring a binomial, even after repeated reminders to multiply the binomial by itself.
- Refer students to the ***Square of a Binomial*** and ***Product of the Sum and Difference of Two Terms*** charts in the text.

Answers: 1a) $8x^2$; b) $-30a^5$; c) $24.6x^2y^7z^{11}$; d) $6x^2-8x$; e) $-15xy^2-6xy$; f) $-4ab^2z^3-2ab^3z^2+2b^3z$; g) $2x^3+5x^2+2x+15$;

2a) $3x^2+5x-12$; b) x^2+5x+6; c) $8y^2-14y+6$; d) $x^2+12x+36$; e) $3x^4-22x^2y^2+24y^4$; f) $12y^2-\frac{3}{2}y+\frac{1}{24}$;

3a) x^2+4x+4; b) $x^2-8x+16$; c) $x^2+14x+49$; 4a) x^2-25; b) $4x^2y^2-9b^2$; c) $25x^2-\frac{1}{4}$; d) $64-32b+4b^2$;

5a) $2x^3+x^2-13x+6$; b) $y^4-8y^3+24y^2-32y+16$; c) $x^3-x^2y-xy^2-y^3$; 6a) a^2-2a; b) c^2-2c; c) $a^2+2ab+b^2-2a-2b$; d) a^2-6a+8

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Mini-Lecture 5.5

The Greatest Common Factor and Factoring by Grouping

Learning Objectives:

1. Identify the GCF.
2. Factor out the GCF of a polynomial's terms.
3. Factor polynomials by grouping.

Examples:

1. Find the greatest common factor of each list of monomials.

   a) $4, 24$ b) $15x, 20$ c) $15x^2, 20x$ d) $9x^2y, 27xy^2$

2. Factor out the greatest common factor.

   a) $16x - 12$ b) $28x + 28$ c) $5z - 25xz^4$

   d) $18x + 9x^2 - 6x^3$ e) $18a^3b - 12ab + 9ab^2 - 12a^2b$ f) $3x(y-5) + (y-5)$

3. Factor each polynomial by grouping.

   a) $xy + y + 5x + 5$ b) $2y - 12 - xy + 6x$

   c) $xy + 9x - 7y - 63$ d) $5xy - 10x + 7y - 14$

4. Factor each polynomial.

   a) $16x^3 - 12x$ b) $-27xy^3 + 18x^4y$ c) $8a^2b^2c - 12ab^2c - 8ac + 6a$

   d) $9y(z+2) - 4(z+2)$ e) $4xy - 8x + 7y - 14$ f) $x^3 + 5x^2 + x + 5$

Teaching Notes:

- Remind students to check their factoring answers by multiplication.
- Some students need to rewrite the coefficients in problem 2 in factored form in order to see the greatest common factor.
- Some students omit the 1 in the answer to problem 2b).
- Many students are confused at first by factor by grouping problems where a negative sign must be factored out of the second grouping, as in problem 3b).

Answers: 1a) 4; b) 5; c) 5x; d) 9xy; 2a) 4(4x-3); b) 28(x+1); c) 5z(1-5xz^3); d) 3x(6+3x-2x^2); e) 3ab(6a^2-4+3b-4a); f) (y-5)(3x+1); 3a) (x+1)(y+5); b) (y-6)(2-x); c) (y+9)(x-7); d) (y-2)(5x+7); 4a) 4x(4x^2-3); b) 9xy(-3y^2+2x^3); c) 2a(4ab^{2}c-6b^{2}c-4c+3); d) (9y-4)(z+2); e) (y-2)(4y+7); f) (x+5)(x^2+1)

Mini-Lecture 5.6

Factoring Trinomials

Learning Objectives:

1. Factor trinomials of the form $x^2 + bx + c$.
2. Factor trinomials of the form $ax^2 + bx + c$.
   a. Method 1 - Trial and Check
   b. Method 2 - Grouping
3. Factor by substitution.

Examples:

1. Factor each trinomial.

   a) $x^2 + 3x + 2$ b) $x^2 + 6x + 8$ c) $x^2 - 6x + 8$ d) $x^2 + x - 2$

   e) $x^2 - x - 2$ f) $x^2 - 3x - 10$ g) $2x^2 + 4x - 48$ h) $3x^2 - 3x - 18$

   i) $x^2 + 15x + 16$ j) $x^2y^2 - 6xy^2 + 8y^2$ k) $x^5 + 4x^4 - 5x^3$

2. Factor each trinomial using the trial and check method.

   a) $4y^2 + 12y + 9$ b) $8x^2 - 18x + 9$ c) $6x^2 + 5x - 6$

   d) $7x^2 - 31x - 20$ e) $6x^2 + 27x - 15$ f) $6x^2y^2 - 7xy^2 - 20y^2$

   Factor each trinomial using the grouping method.

   g) $10x^2 - 7x - 33$ h) $20x^2 + 23x + 6$ i) $3x^2 - 8x - 11$

3. Use substitution to factor each trinomial completely.

   a) $x^4 - 5x^2 - 6$ b) $9x^6 + 6x^3 - 8$ c) $(a+4)^2 + 7(a+4) + 12$

Teaching Notes:

- Some students can factor trinomials very quickly using the trial and check method.
- Some students become very frustrated with the trial and check method and appreciate seeing the grouping method because it provides a recipe that works for any non-prime polynomial.
- Remind students to always try to factor a GCF first.
- Refer to the end of section exercises for mixed practice.
- Refer students to the ***Factoring a Trinomial of the Form $ax^2 + bx + c$*** and ***Factoring a Trinomial of the Form $ax^2 + bx + c$ by Grouping*** charts in the text.

Answers: 1a) (x+2)(x+1); b) (x+4)(x+2); c) (x-4)(x-2); d) (x+2)(x-1); e) (x-2)(x+1); f) (x-5)(x+2); g) 2(x+6)(x-4); h) 3(x-3)(x+2); i) prime; j) y^2(x-4)(x-2); k) x^3(x-4)(x-1); 2a) (2y+3)(2y+3); b) (4x-3)(2x-3); c) (3x-2)(2x+3); d) (7x+4)(x-5); e) 3(2x-1)(x+5); f) y^2(2x-5)(3x+4); g) (5x-11)(2x+3); h) (4x+3)(5x+2); i) (3x-11)(x+1); 3a) (x^2-6)(x^2+1); b) ($3x^3$+4)($3x^3$-2); c) (a+8)(a+7)

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Mini-Lecture 5.7

Factoring by Special Products

Learning Objectives:

1. Factor a perfect square trinomial.
2. Factor the difference of two squares.
3. Factor the sum or difference of two cubes.

Examples:

1. Factor completely or state if the polynomial is prime.

   a) $x^2 + 4x + 4$ b) $x^2 - 12x + 36$ c) $9x^2 + 6x + 1$

   d) $3x^2 - 12x + 12$ e) $25x^2y^3 - 10xy^3 - y^3$ f) $x^2 + 39xy + 40y^2$

2. Factor completely.

   a) $x^2 - 49$ b) $y^2 - 81$ c) $\frac{1}{16} - 25z^2$

   d) $(x+3)^2 - 64$ e) $3x^2 - 75$ f) $x^2 + 10x + 25 - x^4$

3. Factor completely.

   a) $x^3 + 8$ b) $x^3 + 1$ c) $y^3 - 27$ d) $64 - x^3$

   e) $p^3 + 8q^3$ f) $x^3y^2 + 125y^2$ g) $a^3b^2 - 27b^2$ h) $54y^3 + 250$

4 Factor completely.

   a) $64 - x^2$ b) $x^3 + 16x^2 + 64x$ c) $1000y^3 - 1$

   d) $x^2 - 6xy + 9y^2$ e) $18x^2 - 98$ f) $(2x+3)^2 - 64$

Teaching Notes:

- Encourage students to always check if the first and last terms of a trinomial are perfect squares. If they are, then perfect square trinomial factoring may apply.
- Some students understand the difference of a square formula better if 2a) and 2b) are also done using trinomial factoring with a $0x$ middle term.
- Some students find the sum and difference of cubes formulas confusing at first and need to see many examples.
- Remind students to factor out a GCF whenever possible.

Answers: 1a) $(x+2)^2$; b) $(x-6)^2$; c) $(3x+1)^2$; d) $3(x-2)^2$; e) $y^3(5x-1)^2$; f) prime; 2a) $(x+7)(x-7)$; b) $(y+9)(y-9)$; c) $\left(\frac{1}{4}+5z\right)\left(\frac{1}{4}-5z\right)$; d) $(x+11)(x-5)$; e) $3(x+5)(x-5)$; f) $(x+5+x^2)(x+5-x^2)$; 3a) $(x+2)(x^2-2x+4)$; b) $(x+1)(x^2-x+1)$; c) $(y-3)(y^2+3y+9)$; d) $(4-x)(16+4x+x^2)$; e) $(p+2q)(p^2-2pq+4q^2)$; f) $y^2(x+5)(x^2-5x+25)$; g) $b^2(a-3)(a^2+3a+9)$; h) $2(3y+5)(9y^2-15y+25)$; 4a) $(8+x)(8-x)$; b) $x(x+8)^2$; c) $(10y-1)(100y^2+10y+1)$; d) $(x-3y)^2$; e) $2(3x+7)(3x-7)$; f) $(2x+11)(2x-5)$

Mini-Lecture 5.8

Solving Equations by Factoring and Problem Solving

Learning Objectives:

1. Solve polynomial equations by factoring.
2. Solve problems that can be modeled by polynomial equations.
3. Find the x-intercepts of a polynomial function.

Examples:

1. Solve each equation.

   a) $x^2 - 11x + 30 = 0$ b) $6x^2 + 13x + 6 = 0$ c) $x^2 + 3x = 70$

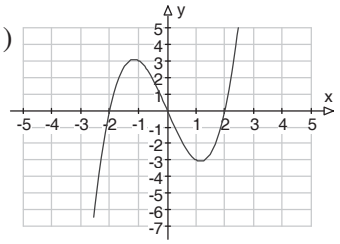
   d) $x(3x + 4) = 4$ e) $x(x - 8) = x^2 + 5x$ f) $\frac{x^2}{56} + \frac{1}{8} = \frac{x}{7}$

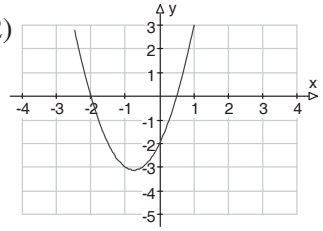
   g) $(3x + 2)(x - 9)(5x - 1) = 0$ h) $x^3 = 25x$ i) $x^3 + 7x^2 = 18x$

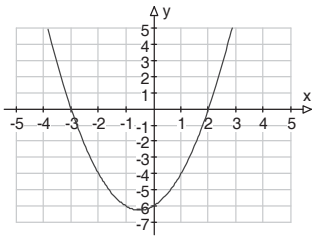
   j) $x^5 = 64x^3$ k) $x^3 - x = -3x^2 + 3$

2. Solve.
   a) One number exceeds another number by 6 and the product of the two numbers is 72. Find the numbers.
   b) A certain rectangle's length is 3 feet longer than its width. If the area of the rectangle is 70 square feet, find its dimensions.
   c) One leg of a right triangle is 14 inches longer than the smaller leg, and the hypotenuse is 16 inches longer than the smaller leg. Find the lengths of the sides of the triangle.

3. Match each polynomial function with its graph.

1) 

2) 

3) 

a) $f(x) = (x - 2)(x + 3)$ b) $f(x) = (2x - 1)(x + 2)$ c) $h(x) = x(x + 2)(x - 2)$

Teaching Notes:

- Remind students to always put the equation in standard form before factoring.
- Some students try to use the zero-factor property before the equation is in standard form. For example in 2c): $x^2 + 3x = 70 \rightarrow x(x + 3) = 70 \rightarrow x = 70,\ x + 3 = 70$ … etc.
- Many students find the applied problems difficult and need to see more examples.
- Remind students to check whether their answers are reasonable for applied problems.
- Refer students to the ***Solving a Polynomial Equation by Factoring*** chart in the text.

Answers: 1a) {6,5}; b) $\left\{-\frac{2}{3}, -\frac{3}{2}\right\}$; c) {-10,7}; d) $\left\{-2, \frac{2}{3}\right\}$; e) {0}; f) {1,7}; g) $\left\{-\frac{2}{3}, \frac{1}{5}, 9\right\}$; h) {-5,0,5}; i) {-9,0,2}; j) {-8,0,8}; k) {-3,-1,1}; 2a) 6 and 12 or -12 and -6; b) 10 ft by 7 ft; c) 10 in, 24 in, 26 in; 3a) graph 3; b) graph 2; c) graph 1

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