Mini-Lecture 6.1

Rational Functions and Multiplying and Dividing Rational Expressions

Learning Objectives:

1. Find the domain of a rational expression.
2. Simplify rational expressions.
3. Multiply rational expressions.
4. Divide rational expressions.
5. Use rational functions in applications.

Examples:

1. Find the domain of each rational expression.

   a) $f(x)=\dfrac{2+3x}{4}$ b) $g(x)=-\dfrac{6x+x^2}{5x}$ c) $h(x)=\dfrac{2-5x}{-7+x}$

   d) $p(x)=\dfrac{-9}{7x+5}$ e) $f(x)=\dfrac{x}{2x^2+x-3}$

2. Simplify each rational expression.

   a) $\dfrac{2x^2-6x}{2x}$ b) $\dfrac{x^2-81}{9+x}$ c) $\dfrac{x^2-10x+25}{x-5}$

   d) $\dfrac{x-8}{8-x}$ e) $\dfrac{y^2+5y+6}{y^2+10y+21}$ f) $\dfrac{y^3-64}{4y-16}$

3. Multiply and simplify.

   a) $\dfrac{3x-3}{x}\cdot\dfrac{8x^2}{5x-5}$ b) $\dfrac{24xy^2}{x^2-49}\cdot\dfrac{3x-21}{8x^2y^2}$ c) $\dfrac{x^2+7x+10}{x^2+8x+15}\cdot\dfrac{x^2+3x}{x^2-7x-18}$

4. Divide and simplify.

   a) $\dfrac{4x^2}{5}\div\dfrac{x^3}{40}$ b) $\dfrac{x^2+5x-6}{x^2+9x+18}\div\dfrac{x^2-1}{x^2+7x+12}$ c) $\dfrac{x^2-4x}{x^3-64}\div\dfrac{2x}{2x^2+8x+32}$

5. Use rational functions in applications.
   A company's cost per book for printing *x* particular books is given by the rational functions $C(x)=\dfrac{0.8x+5000}{x}$. Find the cost per book for printing a) 300 books b) 3000 books

Teaching Notes:

- Many students need a review of simplifying, multiplying and dividing numerical fractions before attempting algebraic ones.
- Many students have trouble with problems where the factors in the numerator and denominator have opposite signs.
- Refer to the end-of-section exercises for applied problems.
- Refer students to the ***Simplifying/Multiplying/Dividing Rational Expressions*** chart in the text.

Answers: 1a) {x|x is a real number}; b) {x|x is a real number and x≠0}; c) {x|x is a real number and x≠7}; d) {x|x is a real number and $x\neq-\frac{5}{7}$*}; e) {x|x is a real number and* $x\neq-\frac{3}{2}$ *;x≠1}; 2a) x-3; b) x-9; c) x-5; d) -1; e)* $\frac{y+2}{y+7}$*; f)* $\frac{y^2+4y+16}{4}$*; 3a)* $\frac{24x}{5}$*; b)* $\frac{9}{x(x+7)}$*; c)* $\frac{x}{x-9}$*; 4a)* $\frac{32}{5x}$*; b)* $\frac{x+4}{x+1}$*; c) 1; 5a) $17.47; b) $2.47*

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Mini-Lecture 6.2

Adding and Subtracting Rational Expressions

Learning Objectives:

1. Add or subtract rational expressions with a common denominator.
2. Identify the Least Common Denominator (LCD) of two or more rational expressions.
3. Add or subtract rational expressions with unlike denominators.

Examples:

1. Add or subtract as indicated.

   a) $\frac{3}{x}+\frac{8}{x}$ b) $\frac{x^2}{x+3}-\frac{9}{x+3}$ c) $\frac{8x-5}{x^2+6x+8}+\frac{7-7x}{x^2+6x+8}$

2. Find the LCD of the rational expressions in each list.

   a) $\frac{3}{11},\frac{2}{7x}$ b) $\frac{6}{7y},\frac{3}{y^2}$

   c) $\frac{4}{x-3},\frac{9}{x+3}$ d) $\frac{6}{x^2-y^2},\frac{5}{x^2+2xy+y^2},\frac{1}{8}$

3. Add or subtract as indicated. If possible, simplify your answer.

   a) $\frac{5}{6y}-\frac{9}{5y}$ b) $\frac{7}{x^2}+\frac{3}{x}$ c) $\frac{6}{r}+\frac{8}{r-2}$

   d) $\frac{1}{x-4}-\frac{1}{4-x}$ e) $\frac{x+3}{x^2+4x-12}+\frac{3x+2}{x^2+14x+48}$

   f) $\frac{7x}{x+1}+\frac{8}{x-1}-\frac{14}{x^2-1}$ g) $\frac{10}{x^2+5x}+\frac{6}{x}-\frac{2}{x+5}$

Teaching Notes:

- Most students need a review of adding, subtracting, and finding LCDs of numerical fractions before attempting algebraic ones.
- Many students find this section difficult.
- Some students need to see more examples for objective 2. Extra time spent here is well worth it and pays off with greater success in objective 3.
- Refer students to the ***Adding or Subtracting Rational Expressions with Common/Different Denominators*** and ***Finding the Least Common Denominator*** charts in the text.

Answers: 1a) $\frac{11}{x}$; b) x-3; c) $\frac{1}{x+4}$; 2a) 77x; b) $7y^2$; c) (x-3)(x+3); d) $8(x+y)^2(x-y)$; 3a) $\frac{-29}{30y}$; b) $\frac{7+3x}{x^2}$; c) $\frac{14r-12}{r(r-2)}$; d) $\frac{2}{x-4}$; e) $\frac{4x^2+7x+20}{(x-2)(x+6)(x+8)}$; f) $\frac{7x-6}{x-1}$; g) $\frac{4x+40}{x(x+5)}$

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Mini-Lecture 6.3

Simplifying Complex Fractions

Learning Objectives:

1. Simplify complex fractions by simplifying the numerator and denominator and then dividing.
2. Simplify complex fractions by multiplying by a common denominator.
3. Simplify expressions with negative exponents.

Examples:

1. Simplify each complex fraction by simplifying the numerator and denominator and then dividing.

a) $\dfrac{3+\frac{1}{8}}{4-\frac{5}{8}}$ b) $\dfrac{\frac{x}{x+4}}{\frac{4}{x+4}}$ c) $\dfrac{\frac{9}{a}+9}{\frac{9}{a}-9}$

d) $\dfrac{\frac{16x^2-25y^2}{xy}}{\frac{4}{y}-\frac{5}{x}}$ e) $\dfrac{\frac{2}{x}+\frac{9}{x^2}}{\frac{4}{x^2}-\frac{81}{x}}$ f) $\dfrac{\frac{4}{5-x}+\frac{5}{x-5}}{\frac{2}{x}+\frac{3}{x-5}}$

g) $\dfrac{\frac{4}{x+5}}{\frac{1}{x-5}-\frac{2}{x^2-25}}$ h) $\dfrac{\frac{3}{x+5}+\frac{9}{x+7}}{\frac{2x+11}{x^2+12x+35}}$

2. Simplify selected problems from *1a)* through *1h)* by multiplying the least common denominator.

3. Simplify.

a) $\dfrac{x^{-2}+y^{-1}}{x^{-3}}$ b) $\dfrac{2x^{-1}+5y^{-1}}{-7x^{-2}-3y^{-2}}$ c) $\dfrac{-6x^{-1}+(6y)^{-1}}{x^{-2}}$

Teaching Notes:

- Stronger students tend to prefer using the multiply by LCD method.
- Many students need to be reminded of how to deal with negative exponents before attempting objective 3.
- Refer students to the ***Simplifying a Complex Fraction: Method 1/Method 2*** charts in the text.

Answers: 1a) $\frac{25}{27}$; *b)* $\frac{x}{4}$; *c)* $\frac{1+a}{1-a}$; *d)* $4x+5y$; *e)* $\frac{2x+9}{4-81x}$; *f)* $\frac{x}{5x-10}$; *g)* $\frac{4x-20}{x+3}$; *h) 6; 2a-h) same as 1a-h);* *3a)* $\frac{xy+x^3}{y}$; *b)* $\frac{2xy^2+5x^2y}{-7y^2-3x^2}$; *c)* $\frac{-36xy+x^2}{6y}$

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Mini-Lecture 6.4

Dividing Polynomials: Long Division and Synthetic Division

Learning Objectives:

1. Divide a polynomial by a monomial.
2. Divide by a polynomial.
3. Use synthetic division to divide a polynomial by a binomial.
4. Use the remainder theorem to evaluate polynomials.

Examples:

1. Divide.

   a) $8x^4 - 4x^3$ by $4x^2$ b) $\dfrac{3x^3y + 9x^2y^2 - 3xy^3}{3xy}$ c) $\dfrac{8x^5y + 32x^4y - 16x^3y^2}{-4x^4y}$

2. Divide.

   a) $\left(x^2 + 12x + 35\right) \div \left(x + 5\right)$ b) $\left(5x^2 - 17x + 14\right) \div \left(x - 2\right)$

   c) $\left(-4x^3 - 8x^2 + 7x - 1\right) \div \left(2x - 1\right)$ d) $\left(20x + 12x^2 + 3\right) \div \left(-6x - 1\right)$

3. Use synthetic division to divide.

   a) $\dfrac{x^2 - 4x - 45}{x + 5}$ b) $\dfrac{2x^2 - 9x - 35}{x - 7}$

   c) $\dfrac{-2x^3 - 6x^2 + 14x + 24}{x + 4}$ d) $\dfrac{x^4 + 16}{x - 2}$

4. Use the remainder theorem to find $P(c)$.

   a) $P(x) = x^2 + 3x - 8;\ c = 2$ b) $P(x) = 5x^4 + 3x^2 - 2x + 12;\ c = -2$

Teaching Notes:

- Remind students to check their answers by multiplying.
- Encourage students to write the intermediate step $\frac{a+b}{c} = \frac{a}{c} + \frac{b}{c}$ when dividing by a monomial.
- Most students understand dividing by a binomial better if a numerical long division problem is shown in parallel.
- Most students understand the synthetic division process after a couple of examples.
- Most students prefer synthetic division over long division.

Answers: 1a) $2x^2-x$; b) $x^2+3xy-y^2$; c) $-2x-8+\frac{4y}{x}$; 2a) $x+7$; b) $5x-7$; c) $-2x^2-5x+1$; d) $-2x-3$; 3a) $x-9$; b) $2x+5$; c) $-2x^2+2x+6$; d) $x^3+2x^2+4x+8+\frac{32}{x-2}$; 4a) 2; b) 108

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Mini-Lecture 6.5

Solving Equations Containing Rational Expressions

Learning Objectives:

1. Solve equations containing rational expressions.

Examples:

1. Solve each equation and check the solution.

a) $\frac{2}{5}y - \frac{1}{3}y = 5$

b) $\frac{3y+6}{5} = 1 + \frac{3}{4}y$

c) $\frac{14}{x} = 5 - \frac{1}{x}$

d) $\frac{x-5}{x+2} = \frac{12}{x+2}$

e) $1 + \frac{1}{x} = \frac{20}{x^2}$

f) $\frac{4x+1}{2x-5} = \frac{6x-1}{3x-6}$

g) $\frac{4}{3x} - \frac{1}{x+1} = \frac{1}{2x^2+2x}$

h) $\frac{x+6}{x^2+5x+4} - \frac{6}{x^2+2x+1} = \frac{x-6}{x^2+5x+4}$

i) $\frac{x}{x-5} - 2 = \frac{5}{x-5}$

j) $\frac{1}{x+5} + \frac{2}{x+3} = \frac{-2}{x^2+8x+15}$

Teaching Notes:

- Remind students to always determine the values not allowed for x before solving a rational expression.
- Many students are confused by the concept of an extraneous solution. Show them a simple example such as:
  $x = 3 \rightarrow x \cdot x = 3 \cdot x \rightarrow x^2 = 3x \rightarrow x^2 - 3x = 0 \rightarrow x = 0, 3;\ x = 0$ is extraneous.
- Some students prefer to make equivalent fractions with a common denominator, and then set the numerators equal to each other.
- Refer students to the ***Solving an Equation Containing Rational Expressions*** chart in the text.

Answers: 1a) {75}; b) $\left\{\frac{4}{3}\right\}$*; c) {3}; d) {17}; e) {-5;4}; f) {1}; g)* $\left\{-\frac{5}{2}\right\}$*; h) {2}; i)* $\varnothing$*; j)* $\varnothing$

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Mini-Lecture 6.6

Rational Equations and Problem Solving

Learning Objectives:

1. Solve an equation containing rational expressions for a specified variable.
2. Solve number problems by writing equations containing rational expressions.
3. Solve problems modeled by proportions.
4. Solve problems about work.
5. Solve problems about distance, rate, and time.

Examples:

1. Solve each equation for the specific variable.

   a) $\frac{PV}{T} = \frac{pv}{t}$ for V b) $\frac{1}{a} + \frac{1}{b} = \frac{1}{c}$ for c c) $P = \frac{A}{1+rt}$ for r

   d) $A = \frac{1}{2}h(B+b)$ for B e) $F = \frac{-GMm}{r^2}$ for M f) $S = \frac{a_1 - a_n r}{1-r}$ for a_1

2. Solve.

   a) ***Number*** Two times the reciprocal of a number equals 4 times the reciprocal of 5. Find the number.

   b) ***Proportion*** The ratio of the weight of an object on Earth to an object on Planet X is 4 to 9. If a person weighs 230 pounds on Earth, find his weight on Planet X. Round to the nearest whole number.

   c) ***Work*** One pump can drain a pool in 9 minutes. When a second pump is also used, the pool only takes 6 minutes to drain. How long would it take the second pump to drain the pool if it were the only pump in use?

   d) ***Rate*** Alex can run 5 miles per hour on level ground on a still day. One windy day he runs 11 miles with the wind, and in the same amount of time runs 4 miles against the wind. What is the rate of the wind?

Teaching Notes:

- Many students find this section difficult.
- Most students need to set up a chart to solve work and rate problems. Refer them to the textbook examples for samples.
- Encourage students to check whether their solutions seem reasonable.
- Refer students to the ***Solving an Equation for a Specified Variable*** chart in the text.

Answers: 1a) $V = \frac{pvT}{tP}$; *b)* $c = \frac{ab}{b+a}$; *c)* $r = \frac{A-P}{Pt}$; *d)* $B = \frac{2A-bh}{h}$; *e)* $M = -\frac{Fr^2}{Gm}$; *f)* $a_1 = S(1-r) + a_n r$; *2a)* $\frac{5}{2}$; *b) 518 pounds; c) 18 minutes; d)* $2\frac{1}{3}$ *mph*

Mini-Lecture 6.7

Variation and Problem Solving

Learning Objectives:

1. Solve problems involving direct variation.
2. Solve problems involving inverse variation.
3. Solve problems involving joint variation.
4. Solve problems involving combined variation.

Examples:

1. Find the constant of variation and the ***direct*** variation equation for each situation. Then solve as indicated.

   a) $y = 4$ when $x = 3$. Find y when $x = 9$.
   b) The amount of gas that a helicopter uses is directly proportional to the number of hours spent flying. The helicopter flies for 3 hours and uses 18 gallons of fuel. Find the number of gallons of fuel that the helicopter uses to fly for 5 hours.

2. Find the constant of variation and the ***inverse*** variation equation for each situation. Then solve as indicated.

   a) $y = 4$ when $x = 3$. Find y when $x = 6$.
   b) The amount of time it takes a swimmer to swim a race is inversely proportional to the swimmer's speed. A swimmer finishes a race in 50 seconds with a speed of 3 feet per second. Find the speed if it takes 25 seconds to finish the race.

3. Find the constant of variation and the ***joint*** or the ***combined*** variation equation for each situation. Then solve as indicated.

   a) r varies jointly as the square of s and the square of t. $r = 12$ when $s = 1$ and $t = 2$.
   b) x is directly proportional to y and inversely proportional to the cube of z. $x = 3$ when $y = 3$ and $z = 2$.
   c) The volume V of a given mass of gas varies directly as the temperature T and inversely as the pressure P. A measuring device is calibrated to give $V = 300\ in^3$ when $T = 250^\circ$ and $P = 10\ lb/in^2$. What is the volume on this device when the temperature is 370° and the pressure is $20\ lb/in^2$?

Teaching Notes:

- Most students will understand the concepts of direct and inverse variation better if real-life examples are discussed in a qualitative way for problem 1.
- Some students are confused by solving for the constant of variation and then using that constant in the original equation and solving for a different variable.

Answers: 1a) $k = \frac{4}{3}, y = \frac{4}{3}x, y = 12$; b) k=6, g=6h, 30 gallons of fuel; 2a) $k = 12, y = \frac{12}{x}, y = 2$; b) $k=150,\ t=\frac{150}{s}$, 6 feet per second; 3a) $k=3,\ r=3s^2t^2$; b) $k=8,\ x = \frac{8y}{z^3}$; c) $k=12,\ V = \frac{12T}{P}$, 222 in^3

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