Mini-Lecture 7.1

Radicals and Radical Functions

Learning Objectives:

1. Find square roots.
2. Approximate roots.
3. Find cube roots.
4. Find nth roots.
5. Find $\sqrt[n]{a^n}$ where a is a real number.
6. Graph square and cube root functions.

Examples:

1. Find each square root. Assume that all variables represent non-negative real numbers.

   a) $\sqrt{25}$ b) $\sqrt{\frac{1}{9}}$ c) $\sqrt{0.04}$ d) $-\sqrt{49}$

   e) $\sqrt{x^2}$ f) $\sqrt{4x^4}$ g) $\sqrt{16x^{10}}$ h) $-\sqrt{100x^{36}}$

2. Approximate each square root to three decimal places.

   a) $\sqrt{11}$ b) $\sqrt{37}$ c) $\sqrt{113}$ d) $\sqrt{205}$

3. Find each cube root. Assume that all variables represent non-negative real numbers.

   a) $\sqrt[3]{8}$ b) $\sqrt[3]{\frac{1}{64}}$ c) $\sqrt[3]{x^6}$ d) $\sqrt[3]{-64x^9y^{12}}$

4. Find each nth root.

   a) $\sqrt[4]{16}$ b) $-\sqrt[4]{81}$ c) $\sqrt[4]{-81}$ d) $\sqrt[5]{-32x^{20}}$

   e) $\sqrt[4]{x^{16}}$ f) $\sqrt[5]{32}$ g) $\sqrt[4]{256x^{12}y^8}$

5. Simplify. Assume that the variables represent any real number.

   a) $\sqrt{(-6)^2}$ b) $\sqrt[3]{(-27)^3}$ c) $\sqrt{16x^2}$ d) $\sqrt[4]{(x-1)^4}$

6. If $f(x) = \sqrt[3]{x} + 2$, solve as indicated.

   a) Find $f(0)$. b) Find $f(-8)$. c) Find the domain. d) Graph $f(x)$.

Teaching Notes:

- Some students think $\sqrt{4} = +2 \text{ or } -2$. Be sure to define *principal square root* early.
- Some students find higher-order radicals confusing at first.
- Many students are unsure when the absolute value symbol is needed in objective 4.
- Refer students to the ***Finding*** $\boldsymbol{\sqrt[n]{a^n}}$ chart in the text.

Answers: (graphing answers at end of mini-lectures) 1a) 5; b) $\frac{1}{3}$; c) 0.2; d) -7; e) x; f) $2x^2$; g) $4x^5$; h) $-10x^{18}$; 2a) 3.317; b) 6.083; c) 10.630; d) 14.318; 3a) 2; b) $\frac{1}{4}$; c) x^2; d) $-4x^3y^4$; 4a) 2; b) -3; c) not a real number; d) x^4; e) 2; f) $4x^3y^2$; 5a) 6; b) -27; c) $4|x|$; d) $|x-1|$; 6a) 2; b) 0; c) all real numbers

Copyright © 2013 Pearson Education, Inc.

Mini-Lecture 7.2

Rational Exponents

Learning Objectives:

1. Understand the meaning of $a^{\frac{1}{n}}$.
2. Understand the meaning of $a^{\frac{m}{n}}$.
3. Understand the meaning of $a^{-\frac{m}{n}}$.
4. Use rules for exponents to simplify expressions that contain rational exponents.
5. Use rational exponents to simplify radical expressions.

Examples:

1. Use radical notation to rewrite each expression. Simplify if possible.

   a) $25^{\frac{1}{2}}$ b) $8^{\frac{1}{3}}$ c) $\left(\frac{1}{49}\right)^{\frac{1}{2}}$ d) $(-8)^{\frac{1}{3}}$ e) $(16x^6)^{\frac{1}{2}}$

2. Simplify if possible. Write final answers with positive exponents.

   a) $81^{\frac{3}{4}}$ b) $(-8)^{\frac{2}{3}}$ c) $(32x^5)^{\frac{2}{5}}$ d) $(-16)^{\frac{3}{2}}$

3. Simplify if possible. Write final answers with positive exponents.

   a) $8^{-\frac{2}{3}}$ b) $(-64)^{-\frac{4}{3}}$ c) $\dfrac{1}{x^{-\frac{2}{3}}}$ d) $\dfrac{3}{4x^{-\frac{5}{9}}}$

4. Use the properties of exponents to simplify each expression. Write with positive exponents.

   a) $x^{\frac{4}{3}}x^{\frac{5}{3}}$ b) $y^{\frac{5}{3}}y^{-\frac{1}{3}}$ c) $\dfrac{x^{\frac{3}{5}}}{x^{\frac{1}{10}}}$ d) $\left(81^{\frac{1}{4}}x^{\frac{2}{3}}\right)^3$

   e) $\dfrac{a^{\frac{3}{4}}a^{-\frac{1}{2}}}{a^{\frac{4}{3}}}$ f) $\dfrac{x^{\frac{10}{3}}}{\left(x^4\right)^{\frac{1}{3}}}$ g) $\dfrac{\left(3x^{\frac{1}{5}}\right)^4}{x^{\frac{3}{10}}}$ h) $\dfrac{\left(a^{-3}b^2\right)^{\frac{1}{8}}}{\left(a^{-2}b\right)^{-\frac{1}{4}}}$

5. Use rational exponents to simplify each radical. Assume that all variables represent positive real numbers.

   a) $\sqrt[12]{a^4}$ b) $\sqrt[4]{25}$ c) $\sqrt[4]{64x^2}$ d) $\sqrt[12]{a^6b^6}$

   Use rational exponents to write as a single radical expression.

   e) $\sqrt[3]{x}\cdot\sqrt{x}$ f) $\dfrac{\sqrt[8]{y}}{\sqrt[9]{y}}$ g) $\sqrt[12]{x}\cdot\sqrt[3]{x^2}$ h) $\sqrt[5]{2x}\cdot\sqrt[3]{y}$

Teaching Notes:

- Most students think rational exponents are easy once they see that the denominator is the root and the numerator is the power.
- Refer students to the ***Definition of*** $a^{\frac{1}{n}}$ / $a^{\frac{m}{n}}$ / $a^{-\frac{m}{n}}$ and ***Summary of Exponent Rules*** charts in text.

Answers: 1a) $\sqrt{25}=5$; b) $\sqrt[3]{8}=2$; c) $\sqrt{\frac{1}{49}}=\frac{1}{7}$; d) $\sqrt[3]{-8}=-2$; e) $\sqrt{16x^6}=4x^3$; 2a) 27; b) 4; c) $4x^2$; d) -64; 3a) $\frac{1}{4}$; b) $\frac{1}{256}$; c) $x^{\frac{2}{3}}$; d) $\frac{3x^{\frac{5}{9}}}{4}$; 4a) x^3; b) $y^{\frac{4}{3}}$; c) $x^{\frac{1}{2}}$; d) $27x^2$; e) $\frac{1}{a^{\frac{13}{12}}}$; f) x^2; g) $81x^{\frac{1}{2}}$; h) $\frac{b^{\frac{1}{2}}}{a^{\frac{7}{8}}}$; 5a) $\sqrt[3]{a}$; b) $\sqrt{5}$; c) $2\sqrt{2x}$; d) $\sqrt{ab}$; e) $\sqrt[6]{x^5}$; f) $\sqrt[72]{y}$; g) $\sqrt[4]{x^3}$; h) $\sqrt[15]{8x^3y^5}$

Copyright © 2013 Pearson Education, Inc.

Mini-Lecture 7.3

Simplifying Radical Expressions

Learning Objectives:

1. Use the product rule for radicals.
2. Use the quotient rule for radicals.
3. Simplify radicals.
4. Use the distance and midpoint formulas.

Examples:

1. Use the product rule to multiply. Assume that all variables represent positive real numbers.

   a) $\sqrt{5}\cdot\sqrt{2}$ b) $\sqrt[3]{7}\cdot\sqrt[3]{9}$ c) $\sqrt{5x}\cdot\sqrt{3y}$ d) $\sqrt[4]{5x^3}\cdot\sqrt[4]{4}$

2. Use the quotient rule to simplify. Assume that all variables represent positive real numbers.

   a) $\sqrt{\frac{9}{64}}$ b) $\sqrt[4]{\frac{x}{16y^4}}$ c) $\sqrt[3]{\frac{2}{8x^9}}$ d) $\sqrt{\frac{x^{12}}{25y^8}}$ e) $-\sqrt[3]{\frac{125x}{y^9}}$

3. Simplify. Assume that all variables represent positive real numbers.

   a) $\sqrt{20}$ b) $\sqrt{48}$ c) $\sqrt{16x^2}$ d) $\sqrt{16x^3}$

   e) $\sqrt{90x^7y^8}$ f) $\sqrt[3]{54}$ g) $\sqrt[3]{x^4}$ h) $\sqrt[3]{-24x^8y^{10}}$

   i) $\sqrt[5]{-32x^4y^{10}}$ j) $\frac{\sqrt{80}}{\sqrt{4}}$ k) $\frac{\sqrt[3]{81}}{\sqrt[3]{3}}$ l) $\frac{\sqrt{x^7y^3}}{\sqrt{xy}}$

   m) $\frac{\sqrt[3]{40x^5y^9}}{\sqrt[3]{5x^2}}$ n) $\frac{\sqrt{50x^2}}{-5\sqrt{25x^{-2}}}$ o) $\frac{\sqrt[5]{729x^9y^3}}{\sqrt[5]{3x^2y^{-7}}}$

4. Find the distance between each pair of points.

   a) $(2,3)$; $(-2,6)$ b) $(5,-7)$; $(2,-1)$ c) $\left(3\sqrt{5},2\right)$; $\left(7\sqrt{5},3\right)$

   Find the midpoint of each line segment whose endpoints are given.

   d) $(2,4)$; $(4,3)$ e) $\left(-\frac{3}{4},-1\right)$; $\left(-\frac{3}{2},-1\right)$ f) $\left(2\sqrt{5},-5\sqrt{5}\right)$; $\left(5\sqrt{5},-2\sqrt{5}\right)$

Teaching Notes:

- Some students have trouble simplifying roots with non-perfect squares inside. Encourage them to write numbers as the product of the highest possible perfect square with another number.
- Some students need a lot of practice simplifying radicals with no variables before attempting those with variables.
- Remind students that the root divides the exponent for variables within radicals.
- Refer students to the ***Product/Quotient Rules for Radicals, Distance Formula***, and ***Midpoint Formula*** charts.

Answers: 1a) $\sqrt{10}$; b) $\sqrt[3]{63}$; c) $\sqrt{15xy}$; d) $\sqrt[4]{20x^3}$; 2a) $\frac{3}{8}$; b) $\frac{\sqrt[4]{x}}{2y}$; c) $\frac{\sqrt[3]{2}}{2x^3}$; d) $\frac{x^6}{5y^4}$; e) $-\frac{5\sqrt[3]{x}}{y^3}$; 3a) $2\sqrt{5}$; b) $4\sqrt{3}$; c) $4x$; d) $4x\sqrt{x}$; e) $3x^3y^4\sqrt{10x}$; f) $3\sqrt[3]{2}$; g) $x\sqrt[3]{x}$; h) $-2x^2y^3\sqrt[3]{3x^2y}$; i) $-2y^2\sqrt[5]{x^4}$; j) $2\sqrt{5}$; k) 3; l) x^3y; m) $2xy^3$; n) $\frac{x^2\sqrt{2}}{-5}$; o) $3xy^2\sqrt[5]{x^2}$; 4a) 5 units; b) $\sqrt{45}\approx 6.708$ units; c) 9 units; d) $(3,\frac{7}{2})$; e) $(-\frac{9}{8},-1)$; f) $\left(\frac{7\sqrt{5}}{2},-\frac{7\sqrt{5}}{2}\right)$

Copyright © 2013 Pearson Education, Inc.

Mini-Lecture 7.4

Adding, Subtracting, and Multiplying Radical Expressions

Learning Objectives:

1. Add or subtract radical expressions.
2. Multiply radical expressions.

Examples:

1. Add or subtract as indicated. Assume that all variables represent positive real numbers.

a) $\sqrt{63}-\sqrt{7}$ b) $-3\sqrt{200}-5\sqrt{8}+9\sqrt{98}$ c) $\sqrt{300x^3}-x\sqrt{12x}$

d) $\sqrt[3]{8x}-\sqrt[3]{27x}$ e) $7\sqrt[3]{x^3y^{13}}+5xy\sqrt[3]{8y^{10}}$ f) $\dfrac{2\sqrt{2}}{3}+\dfrac{3\sqrt{2}}{5}$

g) $\dfrac{2x\sqrt{11}}{5}+\sqrt{\dfrac{11x^2}{100}}$ h) $10\sqrt[4]{x^7}-2x\sqrt[4]{x^3}$ i) $\sqrt{\dfrac{20}{x^2}}+\sqrt{\dfrac{5}{4x^2}}$

2. Multiply. Then simplify if possible. Assume that all variables represent positive real numbers.

a) $\sqrt{6}\left(\sqrt{5}+\sqrt{7}\right)$ b) $\sqrt{7}\left(\sqrt{11}+\sqrt{7}\right)$ c) $\left(\sqrt{7}-\sqrt{2}\right)^2$

d) $\sqrt{2x}\left(\sqrt{2}-\sqrt{x}\right)$ e) $\left(6\sqrt{y}+z\right)\left(3\sqrt{y}-1\right)$ f) $\left(\sqrt[3]{x}+5\right)\left(\sqrt[3]{x}+2\right)$

g) $\left(5\sqrt{3}+9\right)\left(6\sqrt{3}-4\right)$ h) $\left(\sqrt{x-4}+3\right)^2$ i) $\left(\sqrt[3]{x}+7\right)\left(\sqrt[3]{x}-7\sqrt{x}+2\right)$

Teaching Notes:

- Most students find objective 1 easy once they realize that adding/subtracting like radicals is analogous to adding/subtracting like terms.
- Some students are not sure how to handle a coefficient in front of a radical once the radical is simplified.
- Many students distribute the exponent in examples *2c)* and *2h)*.

Answers: 1a) $2\sqrt{7}$ *; b)* $23\sqrt{2}$ *; c)* $8x\sqrt{3x}$ *; d)* $-\sqrt[3]{x}$ *; e)* $17xy^4\sqrt[3]{y}$ *; f)* $\dfrac{19\sqrt{2}}{15}$ *; g)* $\dfrac{x\sqrt{11}}{2}$ *; h)* $8x\sqrt[4]{x^3}$ *; i)* $\dfrac{5\sqrt{5}}{2x}$ *;*
2a) $\sqrt{30}+\sqrt{42}$ *; b)* $\sqrt{77}+7$ *; c)* $9-2\sqrt{14}$ *; d)* $2\sqrt{x}-x\sqrt{2}$ *; e)* $18y+(3z-6)\sqrt{y}-z$ *; f)* $\sqrt[3]{x^2}+7\sqrt[3]{x}+10$ *;*
g) $54+34\sqrt{3}$ *; h)* $x+5+6\sqrt{x-4}$ *; i)* $\sqrt[3]{x^2}-7\sqrt[6]{x^5}+9\sqrt[3]{x}-49\sqrt{x}+14$

Copyright © 2013 Pearson Education, Inc.

Mini-Lecture 7.5

Rationalizing Denominators and Numerators of Radical Expressions

Learning Objectives:

1. Rationalize denominators.
2. Rationalize denominators having two terms.
3. Rationalize numerators.

Examples:

1. Rationalize each denominator. Assume that all variables represent positive real numbers.

a) $\frac{3}{\sqrt{5}}$ b) $\sqrt{\frac{1}{7}}$ c) $\frac{6}{\sqrt[3]{4}}$ d) $\frac{5}{\sqrt{18x}}$

e) $-\frac{7\sqrt{3}}{\sqrt{11}}$ f) $\sqrt{\frac{23a}{2b}}$ g) $\frac{\sqrt[3]{10x}}{\sqrt[3]{5y^4}}$ h) $\sqrt[4]{\frac{81}{49x^{19}}}$

2. Rationalize each denominator. Assume that all variables represent positive real numbers.

a) $\frac{2}{\sqrt{5}-4}$ b) $\frac{-6}{\sqrt{y}+3}$ c) $\frac{\sqrt{2}+\sqrt{4}}{\sqrt{3}+\sqrt{2}}$ d) $\frac{3\sqrt{x}-2}{3\sqrt{x}-\sqrt{y}}$

3. Rationalize each numerator. Assume that all variables represent positive real numbers.

a) $\sqrt{\frac{5}{2}}$ b) $\frac{\sqrt{2x^7}}{8}$ c) $\frac{\sqrt[3]{6x^2}}{\sqrt[3]{5y}}$ d) $\sqrt{\frac{16x^5y}{4z}}$

e) $\frac{\sqrt{13}+1}{2}$ f) $\frac{3-\sqrt{11}}{-4}$ g) $\frac{\sqrt{x}-1}{\sqrt{x}+1}$ h) $\frac{\sqrt{x}+2\sqrt{y}}{3\sqrt{x}}$

Teaching Notes:

- Some students need to see a few examples of why $\sqrt{a}\cdot\sqrt{a}=a$ before applying it to rationalizing a denominator.
- Most students can rationalize denominators easily for square roots.
- Some students have trouble figuring out what to multiply by when rationalizing higher roots and need a step-by-step procedure.

Answers: 1a) $\frac{3\sqrt{5}}{5}$; b) $\frac{\sqrt{7}}{7}$; c) $3\sqrt[3]{2}$; d) $\frac{5\sqrt{2x}}{6x}$; e) $-\frac{7\sqrt{33}}{11}$; f) $\frac{\sqrt{46ab}}{2b}$; g) $\frac{\sqrt[3]{2xy^2}}{y^2}$; h) $\frac{3\sqrt[4]{49x}}{7x^5}$;
2a) $-\frac{2(\sqrt{5}+4)}{11}$; b) $\frac{-6\sqrt{y}+18}{y-9}$; c) $\sqrt{6}-2+2\sqrt{3}-2\sqrt{2}$; d) $\frac{9x+3\sqrt{xy}-6\sqrt{x}-2\sqrt{y}}{9x-y}$; 3a) $\frac{5}{\sqrt{10}}$; b) $\frac{x^4}{4\sqrt{2x}}$;
c) $\frac{6x}{\sqrt[3]{180xy}}$; d) $\frac{2x^3y}{\sqrt{xyz}}$; e) $\frac{6}{\sqrt{13}-1}$; f) $\frac{1}{2(3+\sqrt{11})}$; g) $\frac{x-1}{x+2\sqrt{x}+1}$; h) $\frac{x-4y}{3x-6\sqrt{xy}}$

Copyright © 2013 Pearson Education, Inc.

Mini-Lecture 7.6

Radical Equations and Problem Solving

Learning Objectives:

1. Solve equations that contain radical expressions.
2. Use the Pythagorean Theorem to model problems.

Examples:

1. Solve. Check your solutions.

a) $\sqrt{4x} = 2$ b) $\sqrt{x+1} = 7$ c) $\sqrt{3x} = -6$

d) $\sqrt{5x+6} + 2 = 8$ e) $\sqrt[3]{6x} = -4$ f) $\sqrt[3]{3x+4} - 4 = 0$

Solve. Check your solutions.

g) $\sqrt{4x+1} = 3 + \sqrt{x-2}$ h) $\sqrt{x+20} - \sqrt{x-4} = 4$ i) $\sqrt{x} + 3 = \sqrt{x+21}$

j) $\sqrt{4x-3} = \sqrt{x+6}$ k) $\sqrt{x+1} - \sqrt{x-1} = 2$ l) $\sqrt[3]{7x-2} = \sqrt[3]{x+8}$

2. Solve.

a) ***Triangle*** A triangle has sides of length 12 m and 16 m. Find the length of the hypotenuse.
b) ***Triangle*** A triangle has a hypotenuse of length 25 cm and one leg of length 15 cm. Find the length of the other leg.
c) ***Kite*** A kite is secured to a rope that is tied to the ground. A breeze blows the kite so that the rope is taught while the kite is directly above a flagpole that is 30 ft from where the rope is staked down. Find the altitude of the kite if the rope is 110 ft long.
d) ***Voltage*** The maximum number of volts, E, that can be placed across a resistor is given by $E = \sqrt{PR}$, where P is the power in watts and R is resistance in ohms. If a 2 watt resistor can have at most 40 volts of electricity across it, find the number of ohms of resistance of this resistor.

Teaching Notes:

- Show students a simple example of an extraneous solution, such as: $x = 3 \rightarrow x^2 = 9 \rightarrow x = \pm 3 \rightarrow x = -3$ is extraneous.
- Some students have a lot of trouble with objective 2.
- Encourage students to draw a diagram whenever possible for applied problems.
- Refer students to the ***Power Rule, Solving a Radical Equation,*** and ***Pythagorean Theorem*** charts in the text.

Answers: 1a) 1; b) 48; c) $\varnothing$; d) 6; e) $-\frac{32}{3}$; f) 20; g) 6;2; h) 5; i) 4; j) 3; k) $\varnothing$; l) $\frac{5}{3}$; 2a) 20 m; b) 20 cm; c) 105.83 ft; d) 800 ohms of resistance

Copyright © 2013 Pearson Education, Inc.

Mini-Lecture 7.7

Complex Numbers

Learning Objectives:

1. Write square roots of negative numbers in the form *bi*.
2. Add or subtract complex numbers.
3. Multiply complex numbers.
4. Divide complex numbers.
5. Raise *i* to powers.

Examples:

1. Write using *i* notation.

   a) $\sqrt{-9}$ b) $\sqrt{-18}$ c) $-\sqrt{4}$ d) $5\sqrt{-20}$

   Write using *i* notation. Then multiply or divide as indicated.

   e) $\sqrt{-3}\cdot\sqrt{-7}$ f) $\sqrt{25}\cdot\sqrt{-1}$ g) $\sqrt{4}\cdot\sqrt{-64}$ h) $\dfrac{\sqrt{81}}{\sqrt{-6}}$

2. Add or subtract as indicated. Write your answers in $a+bi$ form.

   a) $(3-5i)+(2+4i)$ b) $(8-i)-(2-3i)$ c) $7-(9+3i)$

3. Multiply. Write your answers in $a+bi$ form.

   a) $6i\cdot 8i$ b) $-3i\cdot 5i$ c) $2i(4-9i)$

   d) $(2+i)(1+4i)$ e) $(\sqrt{2}-2i)(\sqrt{2}+2i)$ f) $(3-2i)^2$

4. Divide. Write your answers in $a+bi$ form.

   a) $\dfrac{2}{i}$ b) $\dfrac{3}{7i}$ c) $\dfrac{6}{2+3i}$ d) $\dfrac{3+2i}{4-3i}$

5. Find each power of *i*.

   a) i^3 b) i^4 c) i^5 d) i^6 e) i^{27} f) $(-2i)^5$

Teaching Notes:

- Most students find objectives 1 and 2 fairly straightforward.
- Encourage students to keep their work neat and organized to avoid errors with objectives 3 and 4.
- Some students have more success with problems *4c)* and *4d)* if they multiply the complex conjugates off to the side and then put the final result within the problem as they solve it.
- Refer to students to the ***Sum or Difference of Complex Numbers*** and ***Complex Conjugates*** charts.

Answers: 1a) 3i; b) $3i\sqrt{2}$; c) -2; d) $10i\sqrt{5}$; e) $-\sqrt{21}$; f) 5i; g) 16i; h) $-\frac{3}{2}i\sqrt{6}$; 2a) 5-i; b) 6+2i; c) -2-3i; 3a) -48; b) 15; c) 18+8i; d) -2+9i; e) 6; f) 5-12i; 4a) -2i; b) $-\frac{3}{7}i$; c) $\frac{12}{13}-\frac{18}{13}i$; d) $\frac{6}{25}+\frac{17}{25}i$; 5a) –i; b) 1; c) i; d) -1; e) –i; f) -32i