Mini-Lecture 8.1

Solving Quadratic Equations by Completing the Square

Learning Objectives:

1. Use the square root property to solve quadratic equations.
2. Solve quadratic equations by completing the square.
3. Use quadratic equations to solve problems.

Examples:

1. Use the square root property to solve each equation.

   a) $x^2 = 9$ b) $x^2 = 20$ c) $2x^2 + 72 = 0$ d) $4x^2 = 16$

   e) $(x-5)^2 = 25$ f) $(x+3)^2 = 11$ g) $(4x+1)^2 = 36$ h) $(5x-3)^2 = 48$

2. Solve each equation by completing the square.

   a) $x^2 + 4x = -3$ b) $x^2 - 2x = 35$ c) $x^2 + 20x + 30 = 0$

   d) $2x^2 - 5x = 3$ e) $2x^2 + 11x = -12$ f) $2x^2 + 5x - 3 = 0$

   g) $6x^2 + 10x + 2 = 0$ h) $4x^2 - 16x + 80 = 0$ i) $x^2 + x = -1$

3. The distance, $s(t)$, in feet traveled by a freely falling object is given by the function $s(t) = 16t^2$, where t is time in seconds. How long would it take for an object to fall to the ground from 576 feet high?

Teaching Notes:

- Many students forget the +/– when using the square root property.
- Most students are confused by completing the square at first and need to see many examples.
- Refer students to the ***Solving a Quadratic Equation in x by Completing the Square*** chart in text.

Answers: 1a) $\{3,-3\}$; b) $\{2\sqrt{5},-2\sqrt{5}\}$; c) $\{6i,-6i\}$; d) $\{2,-2\}$; e) $\{10,0\}$; f) $\{-3+\sqrt{11},-3-\sqrt{11}\}$; g) $\left\{\frac{5}{4},-\frac{7}{4}\right\}$; h) $\left\{\frac{3+4\sqrt{3}}{5},\frac{3-4\sqrt{3}}{5}\right\}$; 2a) $\{-3,-1\}$; b) $\{7,-5\}$; c) $\{-10+\sqrt{70},-10-\sqrt{70}\}$; d) $\left\{3,-\frac{1}{2}\right\}$; e) $\left\{-\frac{3}{2},-4\right\}$; f) $\left\{\frac{1}{2},-3\right\}$; g) $\left\{\frac{-5+\sqrt{13}}{6},\frac{-5-\sqrt{13}}{6}\right\}$; h) $\{2+4i,2-4i\}$; i) $\left\{-\frac{1}{2}+i\frac{\sqrt{3}}{2},-\frac{1}{2}-i\frac{\sqrt{3}}{2}\right\}$; 3) t=6 seconds

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Mini-Lecture 8.2
Solving Quadratic Equations by the Quadratic Formula

Learning Objectives:

1. Solve quadratic equations by using the quadratic formula.
2. Determine the number and type of solutions of a quadratic equation by using the discriminant.
3. Solve problems modeled by quadratic equations.

Examples:

1. Use the quadratic formula to solve each equation.
   a) $x^2+5x+6=0$ b) $x^2+4x-7=0$ c) $3x^2-9x=-2$
   d) $5x^2=-10x-3$ e) $5x^2=-8$ f) $9+3x(x-2)=8$
   g) $\frac{x^2}{18}+x+\frac{35}{9}=0$ h) $(x+8)(2x-9)=2(x-1)-72$

2. Use the discriminant to determine the number and types of solutions of each equation.
   a) $4x^2-8x+4=0$ b) $6x^2=2x-5$
   c) $x^2+8x+7=0$ d) $10-5x^2=6x+5$

3. Solve each equation by completing the square.
   a) ***Geometry*** A rectangular sign has an area of 21 square yards. Its length is 6 yards more than its width. Find the dimensions of the sign.
   b) ***Geometry*** The hypotenuse of a right triangle is 7 feet long. One leg of the triangle is 5 feet longer than the other leg. Find the perimeter of the triangle.
   c) ***Revenue*** The revenue for a small company is given by the quadratic function $r(t)=14t^2+16t+860$, where t is the number of years since 1998 and $r(t)$ is in thousands of dollars. Find the year in which the company's revenue will be \$1,290,000. Round to the nearest whole year.

Teaching Notes:

- Encourage students to memorize the quadratic formula.
- Many students reduce final answers incorrectly. For example: $\frac{4\pm\sqrt{5}}{8}\to\frac{1\pm\sqrt{5}}{2}$.
- Some students prefer to always use the quadratic formula because it has no restrictions on when it can be used. Encourage them to master the other methods, which are often quicker and easier to apply.
- Refer students to the ***Discriminant*** chart in the text.

Answers: *1a)* $\{-3,-2\}$; *b)* $\{-2+\sqrt{11},-2-\sqrt{11}\}$; *c)* $\left\{\frac{9+\sqrt{57}}{6},\frac{9-\sqrt{57}}{6}\right\}$; *d)* $\left\{\frac{-5+\sqrt{10}}{5},\frac{-5-\sqrt{10}}{5}\right\}$; *e)* $\left\{\frac{2i\sqrt{10}}{5},\frac{-2i\sqrt{10}}{5}\right\}$; *f)* $\left\{\frac{3+\sqrt{6}}{3},\frac{3-\sqrt{6}}{3}\right\}$; *g)* $\{-9+\sqrt{11},-9-\sqrt{11}\}$; *h)* $\left\{-\frac{1}{2},-2\right\}$; *2a) one real solution; b) two complex but not real solutions; c) two real solutions; d) two real solutions; 3a)* $-3+\sqrt{30}$ *yds by* $3+\sqrt{30}$ *yds; b)* $7+\sqrt{73}$ *ft; c) 2003*

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Mini-Lecture 8.3

Solving Equations by Using Quadratic Methods

Learning Objectives:

1. Solve various equations that are quadratic in form.
2. Solve problems that lead to quadratic equations.

Examples:

1. Solve.

a) $x = 8 + 2\sqrt{x}$ b) $8y = \sqrt{1-12y}$ c) $\sqrt{22x+11} = x+6$

d) $\frac{6}{x} + \frac{1}{x-3} = 1$ e) $\frac{12}{x^2} = \frac{6}{x+8}$ f) $x^4 = 25$

g) $x^4 + 2x^2 - 24 = 0$ h) $2x^4 - 13x^2 - 45 = 0$ i) $x^{\frac{2}{3}} + 2x^{\frac{1}{3}} - 8 = 0$

j) $3x^{\frac{2}{3}} - 2x^{\frac{1}{3}} - 8 = 0$ k) $(3x-4)^2 - 9(3x-4) + 18 = 0$ l) $2 + \frac{5}{2x-1} = \frac{-2}{(2x-1)^2}$

2. Solve.

a) ***Number*** The product of a number and 8 less than the number is –15. Find the number.
b) ***Geometry*** The hypotenuse of a right triangle is 9 feet long. One leg of the triangle is 3 feet longer than the other leg. Find the perimeter of the triangle.
c) ***Combined Rate*** Two pipes can be used to fill a pool. Working together, the two pipes can fill the pool in 6 hrs. The larger pipe can fill the pool in 3 hours less than the smaller pipe can alone. Find the time to the nearest tenth of an hour it takes for the smaller pipe working alone to fill the pool.

Teaching Notes:

- Most students find this section difficult due to the various number of solutions that are possible.
- Remind students to check for extraneous solutions.
- Encourage students to draw a diagram or make a chart when solving applied problems.
- Refer students to the ***Solving a Quadratic Equation*** chart in the text.

Answers: 1a) 16; b) $\frac{1}{16}$; c) 5; d) $\{5+\sqrt{7}, 5-\sqrt{7}\}$; e) $\{1+\sqrt{17}, 1-\sqrt{17}\}$; f) $\{\sqrt{5}, -\sqrt{5}, i\sqrt{5}, -i\sqrt{5}\}$; g) $\{2, -2, i\sqrt{6}, -i\sqrt{6}\}$; h) $\left\{3, -3, \frac{\sqrt{10}}{2}i, -\frac{\sqrt{10}}{2}i\right\}$; i) {8,-64}; j) $\left\{8, -\frac{64}{27}\right\}$; k) $\left\{\frac{7}{3}, \frac{10}{3}\right\}$; l) $\left\{\frac{1}{4}, -\frac{1}{2}\right\}$; 2a) 3 or 5; b) 21.37 ft; c) 13.7 hrs

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Mini-Lecture 8.4

Nonlinear Inequalities in One Variable

Learning Objectives:

1. Solve polynomial inequalities of degree 2 or greater.
2. Solve inequalities that contain rational expressions with variables in the denominator.

Examples:

1. Solve.

a) $(x+2)(x+3)>0$

b) $(x+2)(x+3)\le 0$

c) $x^2-9x+18\ge 0$

d) $5x^2-4x\ge 9$

e) $x(x+6)(x-2)<0$

f) $(x+1)(x-3)(x-6)>0$

g) $(x^2-36)(x^2-4)\le 0$

h) $16x^3+48x^2-25x-75>0$

2. Solve.

a) $\dfrac{x+5}{x-3}<0$

b) $\dfrac{x-7}{x-2}>0$

c) $\dfrac{4}{y-3}\le 0$

d) $\dfrac{-3}{y+4}\ge 3$

e) $\dfrac{(x+5)(x-5)}{x}<0$

f) $\dfrac{(3-x)(x-1)}{(x-2)^2}\ge 0$

g) $\dfrac{4x}{x+6}<x$

h) $\dfrac{(x-3)^2}{x^2-25}>0$

Teaching Notes:

- Many students understand the concepts of this section better if they are shown a graph of the quadratic function in *1a)* and *b)* and can see where the parabola is above or below the x-axis.
- Some students are confused by how to pick test points. Remind them that they can pick any convenient point except for the critical points that define regions.
- Encourage students to make a region chart as in the textbook examples.
- Refer students to the ***Solving a Polynomial Inequality*** and ***Solving a Rational Inequality*** charts in the text.

Answers: 1a) $(-\infty,-\infty3)\cup(-2,\infty)$; b) $[-3,-2]$; c) $(-\infty,3]\cup[6,\infty)$; d) $(-\infty,-1]\cup\left[\frac{9}{5},\infty\right)$; e) $(-\infty,-6)\cup(0,2)$; f) $(-1,3)\cup(6,\infty)$; g) $[-6,-2]\cup[2,6]$; h) $\left(-3,-\frac{5}{4}\right)\cup\left(\frac{5}{4},\infty\right)$; 2a) $(-5,3)$; b) $(-\infty,2)\cup(7,\infty)$; c) $(-\infty,3)$; d) $[-5,-4)$; e) $(-\infty,-5)\cup(0,5)$; f) $[1,2)\cup(2,3]$; g) $(-6,-2)\cup(0,\infty)$; h) $(-\infty,-5)\cup(5,\infty)$

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Mini-Lecture 8.5

Quadratic Functions and Their Graphs

Learning Objectives:

1. Graph quadratic functions of the form $f(x) = x^2 + k$.
2. Graph quadratic functions of the form $f(x) = (x-h)^2$.
3. Graph quadratic functions of the form $f(x) = (x-h)^2 + k$.
4. Graph quadratic functions of the form $f(x) = ax^2$.
5. Graph quadratic functions of the form $f(x) = a(x-h)^2 + k$.

Examples:

1. Graph each quadratic function. Label the vertex and sketch and label the axis of symmetry.

   a) $f(x) = x^2$ b) $f(x) = x^2 + 2$ c) $f(x) = x^2 - 3$

2. Graph each quadratic function. Label the vertex and sketch and label the axis of symmetry.

   a) $f(x) = (x-2)^2$ b) $f(x) = (x+3)^2$

3. Graph each quadratic function. Label the vertex and sketch and label the axis of symmetry.

   a) $f(x) = (x-2)^2 + 1$ b) $f(x) = (x+1)^2 - 3$

4. Graph each quadratic function. Label the vertex and sketch and label the axis of symmetry.

   a) $f(x) = 2x^2$ b) $f(x) = \frac{1}{2}x^2$

   c) $f(x) = -x^2$ d) $f(x) = -3x^2$

5. Graph each quadratic function. Label the vertex and sketch and label the axis of symmetry.

   a) $f(x) = 2(x+1)^2$ b) $f(x) = \frac{1}{2}(x-2)^2 + 1$

   c) $f(x) = -2(x-3)^2 + 4$ d) $f(x) = \frac{1}{3}(x+3)^2 - 2$

Teaching Notes:

- Most students find vertical shifts easy to understand.
- Some students are confused by the direction of a horizontal shift.
- Many students are uncertain of how to quickly determine the vertex until they have seen the graphs in objective 5 and can visualize how the vertex is (h, k).
- Refer students to the many graphing charts in the text.

Answers: (graphing answers at end of mini-lectures) 1a) (0,0), x=0; b) (0,2), x=0; c) (0,-3), x=0; 2a) (2,0), x=2; b) (-3,0), x=-3; 3a) (2,1), x=2; b) (-1,-3), x=-1; 4a) (0,0), x=0; b) (0,0), x=0; c) (0,0), x=0; d) (0,0), x=0; 5a) (-1,0), x=-1; b) (2,1), x=2; c) (3,4), x=3; d) (-3,-2), x=-3

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Mini-Lecture 8.6

Further Graphing of Quadratic Functions

Learning Objectives:

1. Write quadratic functions in the form $y = a(x-h)^2 + k$.
2. Derive a formula for finding the vertex of a parabola.
3. Find the minimum or maximum value of a quadratic function.

Examples:

1. Find the vertex of the graph of each quadratic function by completing the square.

   a) $f(x) = x^2 + 6x + 9$ b) $f(x) = -2x^2 + 4x + 6$ c) $f(x) = x^2 + x + 6$

2. Find the vertex of the graph of each quadratic function. Determine whether the graph opens upward or downward, find any intercepts, and graph the function.

   a) $f(x) = x^2 + 5x + 4$ b) $f(x) = -x^2 + 2x + 8$

   c) $f(x) = -2x^2 + 8x$ d) $f(x) = x^2 - 4x + 4$

   e) $f(x) = 2x^2 + 4x + \frac{5}{2}$ f) $f(x) = \frac{1}{4}x^2 + 2x + \frac{9}{4}$

3. Solve.

   a) ***Number*** Find two numbers whose sum is 44 and whose product is as large as possible.

   b) ***Geometry*** The length and width of a rectangle must have a sum of 94 feet. Find the dimensions of the rectangle whose area is as large as possible.

   c) ***Projectile*** An arrow is fired into the air with an initial velocity of 96 feet per second. The height in feet of the arrow t seconds after it was shot into the air is given by the function $h(x) = -16t^2 + 96t$. Find the maximum height of the arrow.

Teaching Notes:

- Most students need to be reminded of the completing the square procedure.
- Most students are comfortable using the vertex formula but some are confused at first by why the calculated value must be substituted back into the quadratic function.
- Remind students to always check that their graph opens in the expected direction.

Answers: (graphing answers at end of mini-lectures) 1a) (-3,0); b) (1,8); c) $\left(-\frac{1}{2},\frac{23}{4}\right)$*; 2a)* $\left(-\frac{5}{2},-\frac{9}{4}\right)$*, opens up, x-int (-4,0), (-1,0), y-int (0,4); b) (1,9), opens down, x-int (-2,0), (4,0), y-int (0,8); c) (2,8), opens down, x-int (0,0), (4,0), y-int (0,0); d) (2,0), opens up, x-int (2,0), y-int (0,4); e)* $\left(-1,\frac{1}{2}\right)$*, opens up, no x-int, y-int* $\left(0,\frac{5}{2}\right)$*; f)* $\left(-4,-\frac{7}{4}\right)$*, opens up, x-int* $\left(-4+\sqrt{7},0\right),\left(-4-\sqrt{7},0\right)$*, y-int* $\left(0,\frac{9}{4}\right)$*; 3a) 22 and 22; b) 47 ft by 47 ft; c) 144 ft*

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