Mini-Lecture 9.1

The Algebra of Functions; Composite Functions

Learning Objectives:

1. Add, subtract, multiply, and divide functions.
2. Construct composite functions.

Examples:

1. For $f(x) = 2x^2 - x$ and $g(x) = 3 - 5x$, find the following.

   a) $(f + g)(x)$ b) $(g - f)(x)$ c) $(f + g)(2)$ d) $(g - f)(-2)$

   For $f(x) = 2x$, $g(x) = \sqrt{4x+5}$, and $h(x) = 2x^3 + 6x^2 + 2x$, find the following.

   e) $(f \cdot g)(x)$ f) $\left(\frac{h}{f}\right)(x)$ g) $(f \cdot g)(-1)$ h) $\left(\frac{h}{f}\right)(-3)$

2. For $f(x) = \sqrt{x-2}$ and $g(x) = 3x - 1$, find the following.

   a) $(f \circ g)(x)$ b) $(g \circ f)(x)$ c) $(f \circ g)(4)$ d) $(g \circ f)(4)$

   For $f(x) = x^2 + 3$, $g(x) = -6x$, and $h(x) = \sqrt{x-3}$, write the given $F(x)$ as a composition of *f*, *g*, or *h*.

   e) $F(x) = 36x^2 + 3$ f) $F(x) = -6x^2 - 18$ g) $F(x) = x$

   Find $f(x)$ and $g(x)$ so that the given function $h(x) = (f \circ g)(x)$.

   h) $h(x) = |x - 2|$ i) $h(x) = \frac{1}{3x+5}$ j) $h(x) = \sqrt{x-2} + 4$

Teaching Notes:

- Most students do not have trouble with the objectives.
- Some students are very confused by the concept of and mechanics of a composition function.
- Point out to students that in most situations, $(f \circ g)(x)$ and $(g \circ f)(x)$ are different.
- Refer students to the ***Algebra of Functions*** and ***Composition of Functions*** charts in the text.

Answers: 1a) $2x^2-6x+3$; b) $-2x^2-4x+3$; c) -1; d) 3; e) $2x\sqrt{4x+5}$; f) x^2+3x+1; g) -2; h) 1; 2a) $\sqrt{3x-3}$; b) $3\sqrt{x-2}-1$; c) 3; d) $3\sqrt{2}-1$; e) $(f \circ g)(x)$; f) $(g \circ f)(x)$; g) $(f \circ h)(x)$; h) $f(x)=|x|$; $g(x)=x-2$; i) $f(x)=\frac{1}{x}$; $g(x)=3x+5$; j) $f(x)=\sqrt{x}+4$; $g(x)=x-2$

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Mini-Lecture 9.2

Inverse Functions

Learning Objectives:

1. Determine whether a function is a one-to-one function.
2. Use the horizontal line test to decide whether a function is a one-to-one function.
3. Find the inverse of a function.
4. Find the equation of the inverse of a function.
5. Graph functions and their inverses.
6. Determine whether two functions are inverses of each other.

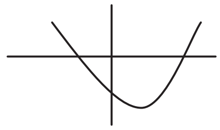
Examples:

1. Determine whether each function is a one-to-one function.

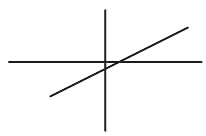
   a) $B = \{ (6,2), (9,9), (1,4), (-1,5) \}$ b) $C = \{ (8,-1), (9,-1), (11,7), (12,2) \}$

2. Use the horizontal line test to determine whether the graph of each function is the graph of a one-to-one function.

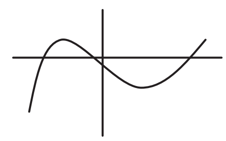
   a)

   b)

   c)

3. Find the inverse of each function.

   a) $A = \{ (1,2), (-1,3), (-3,4) \}$ b) $C = \{ (6,2), (9,9), (1,4), (-1,5) \}$

4. Find the inverse of each one-to-one function.

   a) $f(x) = 2x + 3$ b) $f(x) = \dfrac{4x-5}{3}$ c) $f(x) = \sqrt[3]{x+9}$ d) $f(x) = \dfrac{6}{7-x}$

5. Find the inverse of each function and graph the function and its inverse on the same set of axes. Graph the line $y = x$ as a dashed line.

   a) $R = \{ (-9,6), (-6,9), (3,4) \}$ b) $f(x) = 3x + 5$ c) $f(x) = x^3 - 2$

6. Determine whether functions are inverses of each other.

   a) If $f(x) = 3x + 2$, show that $f^{-1}(x) = \dfrac{x-2}{3}$. b) If $f(x) = 2x - 7$, show that $f^{-1}(x) = \dfrac{x+7}{2}$.

Teaching Notes:

- Tell students early on that f^{-1} means the inverse function of the function *f*, it does not mean $\dfrac{1}{f}$.
- Many students reduce final answers incorrectly. For example: $\dfrac{4 \pm \sqrt{5}}{8} \to \dfrac{1 \pm \sqrt{5}}{2}$.
- Most students understand the concept of an inverse better if they are told that $f^{-1}(x)$ "undoes" whatever $f(x)$ did to x, i.e. $f^{-1}(f(x)) = x$.
- Remind students to always check that their graphs of *f* and f^{-1} are symmetric about $y = x$.
- Refer students to the ***Horizontal Line Test*** and ***Finding the Inverse of a One-to-One Function f(x)*** charts in the text.

<u>Answers</u>: *(graphing answers at end of mini-lectures) 1a) one-to-one; b) not one-to-one; 2a) not one-to-one; b) one-to-one; c) not one-to-one; 3a)* A^{-1}*={(2,1), (3,-1), (4,-3)}; b)* C^{-1}*={(2,6), (9,9), (4,1), (5,-1)};*
4a) $f^{-1}(x) = \dfrac{x-3}{2}$; *b* $f^{-1}(x) = \dfrac{3x+5}{4}$; *c)* $f^{1}(x)=x^3-9$*; d)* $f^{-1}(x) = 7 - \dfrac{6}{x}$; *5a)* R^{-1}*={(6,-9), (9,-6), (4,3)};*
b) $f^{-1}(x) = \dfrac{x-5}{3}$; *c)* $f^{-1}(x) = \sqrt[3]{x+2}$; *6a) & 6b)* $(f \circ f^{-1})(x) = x$

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Mini-Lecture 9.3
Exponential Functions

Learning Objectives:

1. Graph exponential functions.
2. Solve equations of the form $b^x = b^y$.
3. Solve problems modeled by exponential equations.

Examples:

1. Graph each exponential function.

   a) $y = 3^x$ b) $y = 3^x + 1$ c) $y = 3^x - 2$

   d) $y = \left(\frac{1}{2}\right)^x$ e) $y = \left(\frac{1}{2}\right)^x - 3$ f) $y = \left(\frac{1}{2}\right)^x + 2$

   g) $y = -2^x$ h) $y = -\left(\frac{1}{4}\right)^x$ i) $y = 3^{x+1}$

2. Solve.

   a) $3^x = 9$ b) $4^x = 1$ c) $5^x = 125$ d) $\left(\frac{1}{3}\right)^x = 27$

   e) $4^x = 32$ f) $4^{x+2} = 64$ g) $\frac{1}{8} = 2^{3x}$ h) $2^{5-3x} = \frac{1}{16}$

3. Solve.

   a) The rabbit population in a forest grows at the rate of 4% monthly. If there are 220 rabbits in August, find how many rabbits should be expected by next August. Use $y = 220\,(2.7)^{0.04t}$. Round to the nearest whole number.

   b) Jared borrows $3750 at a rate of 10.5% compounded monthly. Find how much Jared owes at the end of 3 years. Round to the nearest cent. Use $A = P\left(1 + \frac{r}{n}\right)^{nt}$.

Teaching Notes:

- Many students understand the graphs better if the first few are done by plotting points instead of by using shifting ideas.
- Some students find the problems in objective 2 confusing at first and need to be given a step-by-step process for solving them.

Answers: (graphing answers at end of mini-lectures) 2a) 2; b) 0; c) 3; d) -3; e) $\frac{5}{2}$; f) 1; g) -1; h) 3;

3a) 354 rabbits; b) $5,131.44

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Mini-Lecture 9.4

Exponential Growth and Decay Functions

Learning Objectives:

1. Model exponential growth.
2. Model exponential decay.

Examples:

1. In 2005, a city named Margatroid had a population of 48,250 and was consistently increasing by 7.5% per year. If this yearly increase continues, predict the city's population in 2017. Round to the nearest whole.

2. A certain bacteria multiplies such that its population increases by 30% every minute. If a sample of this bacteria contains 16 such organisms and the sample is left unchecked, how many bacteria will be in the sample in a half-hour? Round to the nearest whole.

3. A singles golf tournament has 1024 participants. After each round, half the players are eliminated. How many players are left after 7 rounds?

4. The number of animals of a certain species in a rain forest is estimated to be 950. If this number decreases by 6% each year, find the number of animals in this species in 8 years. Round to the nearest whole.

5. A rare isotope of a nuclear material is very unstable, decaying at a rate of 12% each second. Find how much isotope remains 10 seconds after 6 grams of the isotope is created. Round to the nearest tenth of a gram.

6. Suppose a radioactive material has a half-life of 30 years. How much of a 1750-gram sample remains after 171 years? Round to the nearest tenth of a gram.

Teaching Notes:

- Tables can be helpful visual models of both growth and decay. This is especially true of problems that use specific calendar years as reference points, because they help the student visually match a calendar year to a time interval.
- Remind students to be careful when working with rates given as percents. The decimal form of the percent needs to be used in the growth and decay formulas; if they use percent values (i.e. substituting 10 for r in a 10% growth problem), they will end up with wildly incorrect answers.
- Sports tournaments are a good and easy source of examples of exponential decay, such as the NCAA March Madness Tournament (after the play-in games are completed). Other examples include the NBA and NHL playoffs, as well as the FIFA World Cup Knockout Stage.
- Point out to students that the value of r in a half-life problem will always be 0.50. The challenge will be to find the proper value of x.
- Point out to students that the concept of "half-life" is used to estimate the age of materials that bear certain elements. One of the most well-known examples of this is "carbon dating," which, among other things, is used to estimate the ages of organic remains found through archaeological digs. Encourage students to consult their science teachers for further information on half-lives and carbon dating.

Answers: 1) 114,921 people; 2) 41,920 bacteria; 3) 8 players; 4) 579 animals; 5) 1.7 g; 6) 33.7 g

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Mini-Lecture 9.5

Logarithmic Functions

Learning Objectives:

1. Write exponential equations with logarithmic notation and write logarithmic equations with exponential notation.
2. Solve logarithmic equations by using exponential notation.
3. Identify and graph logarithmic functions.

Examples:

1. Write each as an exponential equation.

   a) $\log_7 49 = 2$ b) $\log_2 16 = 4$ c) $\log_5 \frac{1}{125} = -3$ d) $\log_3 \frac{1}{9} = -2$

   e) $\log_e x = 5$ f) $\log_e \frac{1}{e} = -1$ g) $\log_{11} \sqrt{11} = \frac{1}{2}$ h) $\log_{0.5} 0.125 = 3$

   Write each as a logarithmic equation.

   i) $5^2 = 25$ j) $2^5 = 32$ k) $10^{-2} = 0.01$ l) $10^{\frac{1}{3}} = \sqrt[3]{10}$

2. Solve.

   a) $\log_2 16 = x$ b) $\log_x 64 = 3$ c) $\log_2 \frac{1}{32} = x$ d) $\log_{25} x = \frac{1}{2}$

   e) $\log_{\frac{3}{4}} x = 2$ f) $\log_x 81 = 4$ g) $\log_7 7^{-2} = x$ h) $9^{\log_9 8} = x$

3. Graph each logarithmic function.

   a) $y = \log_2 x$ b) $f(x) = \log_{\frac{1}{2}} x$ c) $f(x) = \log_{10} x$

Teaching Notes:

- Many students have trouble understanding the concept of a logarithm.
- Tell students early on that a logarithm is an exponent.
- Remind students frequently that the domain of $y = \log_b x$ is $x > 0$.
- Refer students to the ***Logarithmic Definition*** chart in the text.

Answers: (graphing answers at end of mini-lectures) 1a) $7^2=49$; b) $2^4=16$; c) $5^{-3} = \frac{1}{125}$; d) $3^{-2} = \frac{1}{9}$; e) $e^5=x$; f) $e^{-1} = \frac{1}{e}$; g) $11^{\frac{1}{2}} = \sqrt{11}$; h) $0.5^3=0.125$; i) $\log_5 25 = 2$; j) $\log_2 32 = 5$; k) $\log_{10} 0.01 = -2$; l) $\log_{10} \sqrt[3]{10} = \frac{1}{3}$; 2a) 4; b) 4; c) -5; d) 5; e) $\frac{9}{16}$; f) 3; g) -2; h) 8

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Mini-Lecture 9.6

Properties of Logarithms

Learning Objectives:

1. Use the product property of logarithms.
2. Use the quotient property of logarithms.
3. Use the power property of logarithms.
4. Use the properties of logarithms together.

Examples:

1. Write each sum as a single logarithm. Assume that variables represent positive numbers.

   a) $\log_3 5 + \log_3 8$ b) $\log_4 y^3 + \log_4 (y-9)$ c) $\log_2 6 + \log_2 (x+1) + \log_2 4$

2. Write each difference as a single logarithm. Assume that variables represent positive numbers.

   a) $\log_3 13 - \log_3 2$ b) $\log_5 x - \log_5 (y+1)$ c) $\log_7 (x^2+2) - \log_7 (x^2+5)$

3. Use the power property to rewrite each expression.

   a) $\log_2 x^3$ b) $\log_9 5^{-3}$ c) $\log_4 \sqrt{x}$ d) $\log_5 \sqrt[3]{y}$

4. Write each as a single logarithm. Assume that variables represent positive numbers.

   a) $\log_5 3 + \log_5 x^2$ b) $4\log_6 x + 5\log_6 y$

   c) $\log_4 14 + \log_4 2 - \log_4 7$ d) $2\log_3 x + \frac{1}{3}\log_3 x - 2\log_3 (x+1)$

   Write each expression as a sum or difference of logarithms. Assume that variables represent positive numbers.

   e) $\log_6 \frac{4x}{3}$ f) $\log_b \sqrt{6x}$ g) $\log_5 x^4(x+2)$ h) $\log_5 \frac{(x+3)^2}{x}$

   If $\log_b 3 \approx 0.5$ and $\log_b 5 \approx 0.7$, evaluate each expression.

   i) $\log_b \left(\frac{3}{5}\right)$ j) $\log_b 9$ k) $\log_b \sqrt[3]{5}$

Teaching Notes:

- Most students do not have trouble applying the properties of logarithms separately.
- Some students have trouble with objective 4, where all of the properties are combined and need to see many examples.
- Encourage students to write the three properties of logarithms on an index card for easy reference.
- Refer students to the ***Product/Quotient/Power Properties of Logarithms*** charts in the text.

Answers: 1a) $\log_3 40$; b) $\log_4(y^4 - 9y^3)$; c) $\log_2(24x+24)$; 2a) $\log_3\left(\frac{13}{2}\right)$; b) $\log_5\left(\frac{x}{y+1}\right)$; c) $\log_7\left(\frac{x^2+2}{x^2+5}\right)$;

3a) $3\log_2 x$; b) $-3\log_9 5$; c) $\frac{1}{2}\log_4 x$; d) $\frac{1}{3}\log_5 y$; 4a) $\log_5 3x^2$; b) $\log_6 x^4y^5$; c) $\log_4 4$; d) $\log_3\left(\frac{x^{\frac{7}{3}}}{(x+1)^2}\right)$;

e) $\log_6 4 + \log_6 x - \log_6 3$; f) $\frac{1}{2}(\log_b 6x + \log_b x)$; g) $4\log_5 x + \log_5 (x+2)$; h) $2\log_5(x+3) - \log_5 x$; i) -0.2; j) 1; k) 0.23

Mini-Lecture 9.7

Common Logarithms, Natural Logarithms, and Change of Base

Learning Objectives:

1. Identify common logarithms and approximate them by calculator.
2. Evaluate common logarithms of powers of 10.
3. Identify natural logarithms and approximate them by calculator.
4. Evaluate natural logarithms of powers of *e*.
5. Use the change of base formula.

Examples:

1. Use a calculator to approximate each logarithm to four decimal places.

   a) $\log 10$ b) $\log 23.1$ c) $\log 45{,}600$ d) $\log 0.369$

2. Find the exact value of each logarithm.

   a) $\log 1000$ b) $\log \frac{1}{100}$ c) $\log 0.001$ d) $\log \sqrt[3]{10}$

3. Identify natural logarithms.

   a) $\ln e$ b) $\ln 9.82$ c) $\ln 132{,}000$ d) $\ln 0.015$

4. Evaluate natural logarithms of powers of *e*.

   a) $\ln e^3$ b) $\ln \sqrt[6]{e}$ c) $\ln e^{2.1}$ d) $\ln 1$

5. Approximate each logarithm to four decimal places.

   a) $\log_3 6$ b) $\log_5 9$ c) $\log_6 \frac{1}{3}$ d) $\log_{\frac{1}{2}} 3$

Teaching Notes:

- Some students need help with calculator strokes.
- Most students find the change of base formula very non-intuitive and need to try many examples to become comfortable with it.
- Many students wonder where the change of base formula comes from. Tell them they will be able to derive it in the next textbook section.
- Most students understand objective 4 after a few examples.

Answers: 1a) 1; b) 1.3636; c) 4.6590; d) -0.4330; 2a) 3; b) -2; c) -3; d) $\frac{1}{3}$; 3a) 1; b) 2.2844; c) 11.7906; d) -4.1997; 4a) 3; b) $\frac{1}{6}$; c) 2.1; d) 0; 5a) 1.6309; b) 1.3652; c) -0.6131; d) -1.5850

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