Mini Lecture 8.3
The Algebra of Functions

Learning Objectives:

1. Find the domain of a function.
2. Use the algebra of functions to combine functions and determine domains.

Examples:
State the domain of each function.

1. a. $f(x)=3x-1$ b. $g(x)=\dfrac{4x}{x-2}$ c. $h(x)=x+\dfrac{2}{6-x}$ d. $p(x)=\dfrac{1}{x+5}+\dfrac{7}{x-9}$

2. Let $f(x)=x^2-2x$ and $g(x)=x+3$. Find the following;

   a. $(f+g)(x)$ b. the domain of $f+g$ c. $(f+g)(-2)$

3. Let $f(x)=\dfrac{5}{x+2}$ and $g(x)=\dfrac{6}{x-1}$. Find the following;

   a. $(f+g)(x)$ b. The domain of $f+g$

4. Let $f(x)=x^2+1$ and $g(x)+x=3$. Find the following;

   a. $(f+g)(x)$ b. $(f+g)(-2)$ c. $(f-g)(x)$

   d. $(f-g)(0)$ e. $\left(\dfrac{f}{g}\right)(-2)$

Teaching Notes:

- Students need to be reminded that division by zero is undefined. The value of "*x*" cannot be anything that would make the denominator of a fraction zero.
- Students often exclude values from the domain that would make the numerator zero, warn against this.
- Show students why the radicand of a square root function must be greater than or equal to zero. This is a good place to use the graphing calculator so students can "see" what happens.

Answers: 1. a. $(-\infty,\infty)$ b. $(-\infty,2)$ or $(2,\infty)$ c. $(-\infty,6)$ or $(6,\infty)$
d. $(-\infty,-5)$ or $(-5,9)$ or $(9,\infty)$ 2. a. x^2-x+3 b. $(-\infty,\infty)$ c. 3 4. a. $\dfrac{5}{x+2}+\dfrac{6}{x-1}$
b. $(-\infty,-2)$ or $(-2,1)$ or $(1,\infty)$ 4. a. x^2+x-2 b. 0 c. x^2-x+4 d. 4 e. -1

Mini Lecture 8.4

Composite and Inverse Functions

Learning Objectives:

1. Form composite functions.
2. Verify inverse functions.
3. Find the inverse of a function.
4. Use the horizontal line test to determine if a function has an inverse function.
5. Use the graph of a one-to-one function to graph its inverse function.

Examples:

1. Given $f(x) = x - 1$ and $g(x) = x^2 - 2$, find each of the following composite functions.
   a. $(f \circ g)(x)$ b. $(g \circ f)(x)$

2. Given $f(x) = \dfrac{x-1}{2}$ and $g(x) = 2x + 1$, show that each function is the inverse of the other.
   a. $f(g(x))$ b. $g(f(x))$

3. Find the inverse of each given function:
   a. $f(x) = 2x - 1$ b. $g(x) = 4y$

4. If the points (4, 2), (6, 3) and (8, 4) are on the graph of *f*, give three points on the graph of f^{-1}.

Teaching Notes:

- $f \circ g$ and $g \circ f$ are not the same.
- In the notation, f^{-1}, -1 is not an exponent. f^{-1} represents the inverse function of *x*.
- $f(f^{-1}(x)) = x$ and $f^{-1}(f(x)) = x$.
- If *f* and *g* are inverses, then $f(g(x)) = x$ and $g(f(x)) = x$.
- Use the horizontal line test for inverse function.
- Only one-to-one functions have inverse functions.

Answers: 1. a. $x^2 - 3$ b. $x^2 - 2x - 1$ 2. a. $f(g(x)) = f(2x+1) = \dfrac{2x+1-1}{2} = \dfrac{2x}{2} = x$

b. $g(f(x)) = g\left(\dfrac{x-1}{2}\right) = 2\left(\dfrac{x-1}{2}\right) + 1 = x - 1 + 1 = x$ 3. a. $f^{-1} = \dfrac{x+1}{2}$ b. $g^{-1} = \dfrac{x}{4}$ or $\dfrac{1}{4}x$

4. (2, 4), (3, 6), (4, 8)

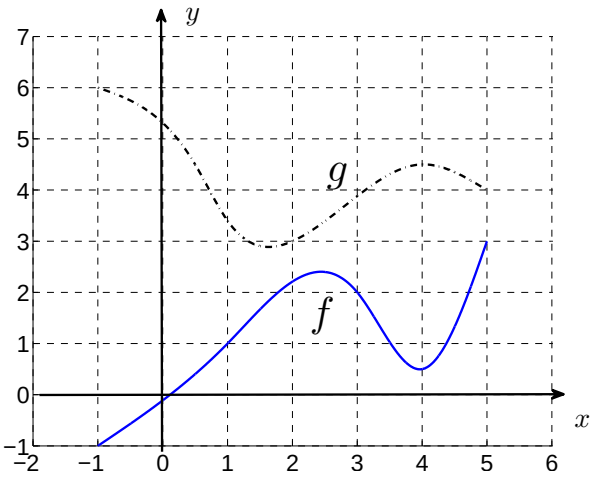
The graph below depicts functions f and g. The entire graph of both functions is shown in the figure.

1. **Use the graph to find the indicated functional values.**
   (a) $(f+g)(3)$

   (b) $(f-g)(-1)$

   (c) $\dfrac{f}{g}(5)$

   (d) Find the domain and range of f



2. **Use the graph to find the indicated functional values.**
   (a) $f(g(2))$

   (b) $f(g(-1))$

   (c) $g(f(3))$

   (d) $g(g(3))$