ch10 review solns for sections 10.5, 10.6 and 10.7

55. $\sqrt{3}\left(2\sqrt{6}+4\sqrt{15}\right)=2\sqrt{18}+4\sqrt{45}$
$=2\sqrt{9\cdot 2}+4\sqrt{9\cdot 5}$
$=2\cdot 3\sqrt{2}+4\cdot 3\sqrt{5}$
$=6\sqrt{2}+12\sqrt{5}$

56. $\sqrt[3]{5}\left(\sqrt[3]{50}-\sqrt[3]{2}\right)=\sqrt[3]{250}-\sqrt[3]{10}$
$=\sqrt[3]{125\cdot 2}-\sqrt[3]{10}$
$=5\sqrt[3]{2}-\sqrt[3]{10}$

57. $\left(\sqrt{7}-3\sqrt{5}\right)\left(\sqrt{7}+6\sqrt{5}\right)$
$=7+6\sqrt{35}-3\sqrt{35}-18\cdot 5$
$=7+3\sqrt{35}-90$
$=3\sqrt{35}-83$ or $-83+3\sqrt{35}$

58. $\left(\sqrt{x}-\sqrt{11}\right)\left(\sqrt{y}-\sqrt{11}\right)$
$=\sqrt{xy}-\sqrt{11x}-\sqrt{11y}+11$

59. $\left(\sqrt{5}+\sqrt{8}\right)^2=5+2\cdot\sqrt{5}\cdot\sqrt{8}+8$
$=13+2\sqrt{40}$
$=13+2\sqrt{4\cdot 10}$
$=13+2\cdot 2\sqrt{10}$
$=13+4\sqrt{10}$

60. $\left(2\sqrt{3}-\sqrt{10}\right)^2$
$=4\cdot 3-2\cdot 2\sqrt{3}\cdot\sqrt{10}+10$
$=12-4\sqrt{30}+10=22-4\sqrt{30}$

61. $\left(\sqrt{7}+\sqrt{13}\right)\left(\sqrt{7}-\sqrt{13}\right)$
$=\left(\sqrt{7}\right)^2-\left(\sqrt{13}\right)^2=7-13=-6$

62. $\left(7-3\sqrt{5}\right)\left(7+3\sqrt{5}\right)=7^2-\left(3\sqrt{5}\right)^2$
$=49-9\cdot 5$
$=49-45=4$

63. $\frac{4}{\sqrt{6}}=\frac{4}{\sqrt{6}}\cdot\frac{\sqrt{6}}{\sqrt{6}}=\frac{4\sqrt{6}}{6}=\frac{2\sqrt{6}}{3}$

64. $\sqrt{\frac{2}{7}}=\frac{\sqrt{2}}{\sqrt{7}}=\frac{\sqrt{2}}{\sqrt{7}}\cdot\frac{\sqrt{7}}{\sqrt{7}}=\frac{\sqrt{14}}{7}$

65. $\frac{12}{\sqrt[3]{9}}=\frac{12}{\sqrt[3]{3^2}}\cdot\frac{\sqrt[3]{3}}{\sqrt[3]{3}}=\frac{12\sqrt[3]{3}}{\sqrt[3]{3^3}}$

$=\frac{12\sqrt[3]{3}}{3}=4\sqrt[3]{3}$

66. $\sqrt{\frac{2x}{5y}}=\frac{\sqrt{2x}}{\sqrt{5y}}\cdot\frac{\sqrt{5y}}{\sqrt{5y}}=\frac{\sqrt{10xy}}{\sqrt{5^2y^2}}=\frac{\sqrt{10xy}}{5y}$

67. $\frac{14}{\sqrt[3]{2x^2}}=\frac{14}{\sqrt[3]{2x^2}}\cdot\frac{\sqrt[3]{2^2x}}{\sqrt[3]{2^2x}}=\frac{14\sqrt[3]{2^2x}}{\sqrt[3]{2^3x^3}}$

$=\frac{14\sqrt[3]{4x}}{2x}=\frac{7\sqrt[3]{4x}}{x}$

68. $\sqrt[4]{\frac{7}{3x}}=\frac{\sqrt[4]{7}}{\sqrt[4]{3x}}=\frac{\sqrt[4]{7}}{\sqrt[4]{3x}}\cdot\frac{\sqrt[4]{3^3x^3}}{\sqrt[4]{3^3x^3}}$

$=\frac{\sqrt[4]{7\cdot 3^3x^3}}{\sqrt[4]{3^4x^4}}=\frac{\sqrt[4]{7\cdot 27x^3}}{3x}$

$=\frac{\sqrt[4]{189x^3}}{3x}$

69. $\frac{5}{\sqrt[5]{32x^4y}}=\frac{5}{\sqrt[5]{2^5x^4y}}\cdot\frac{\sqrt[5]{xy^4}}{\sqrt[5]{xy^4}}$

$=\frac{5\sqrt[5]{xy^4}}{\sqrt[5]{2^5x^5y^5}}=\frac{5\sqrt[5]{xy^4}}{2xy}$

70. $\frac{6}{\sqrt{3}-1}=\frac{6}{\sqrt{3}-1}\cdot\frac{\sqrt{3}+1}{\sqrt{3}+1}$

$=\frac{6\left(\sqrt{3}+1\right)}{\left(\sqrt{3}\right)^2-1^2}=\frac{6\left(\sqrt{3}+1\right)}{3-1}$

$=\frac{6\left(\sqrt{3}+1\right)}{2}=3\left(\sqrt{3}+1\right)$

$=3\sqrt{3}+3$

71. $\frac{\sqrt{7}}{\sqrt{5}+\sqrt{3}}=\frac{\sqrt{7}}{\sqrt{5}+\sqrt{3}}\cdot\frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}-\sqrt{3}}$

$=\frac{\sqrt{35}-\sqrt{21}}{\left(\sqrt{5}\right)^2-\left(\sqrt{3}\right)^2}$

$=\frac{\sqrt{35}-\sqrt{21}}{5-3}=\frac{\sqrt{35}-\sqrt{21}}{2}$

72. $\frac{10}{2\sqrt{5}-3\sqrt{2}}$

$=\frac{10}{2\sqrt{5}-3\sqrt{2}}\cdot\frac{2\sqrt{5}+3\sqrt{2}}{2\sqrt{5}+3\sqrt{2}}$

$=\frac{10\left(2\sqrt{5}+3\sqrt{2}\right)}{\left(2\sqrt{5}\right)^2-\left(3\sqrt{2}\right)^2}=\frac{10\left(2\sqrt{5}+3\sqrt{2}\right)}{4\cdot 5-9\cdot 2}$

$=\frac{10\left(2\sqrt{5}+3\sqrt{2}\right)}{20-18}=\frac{10\left(2\sqrt{5}+3\sqrt{2}\right)}{2}$

$=5\left(2\sqrt{5}+3\sqrt{2}\right)=10\sqrt{5}+15\sqrt{2}$

73. $\frac{\sqrt{x}+5}{\sqrt{x}-3}=\frac{\sqrt{x}+5}{\sqrt{x}-3}\cdot\frac{\sqrt{x}+3}{\sqrt{x}+3}$

$=\frac{x+3\sqrt{x}+5\sqrt{x}+15}{\left(\sqrt{x}\right)^2-3^2}$

$=\frac{x+8\sqrt{x}+15}{x-9}$

74. $\frac{\sqrt{7}+\sqrt{3}}{\sqrt{7}-\sqrt{3}}=\frac{\sqrt{7}+\sqrt{3}}{\sqrt{7}-\sqrt{3}}\cdot\frac{\sqrt{7}+\sqrt{3}}{\sqrt{7}+\sqrt{3}}$

$=\frac{7+2\cdot\sqrt{7}\cdot\sqrt{3}+3}{\left(\sqrt{7}\right)^2-\left(\sqrt{3}\right)^2}$

$=\frac{10+2\sqrt{21}}{7-3}=\frac{10+2\sqrt{21}}{4}$

$=\frac{2\left(5+\sqrt{21}\right)}{4}=\frac{5+\sqrt{21}}{2}$

75. $\frac{2\sqrt{3}+\sqrt{6}}{2\sqrt{6}+\sqrt{3}} = \frac{2\sqrt{3}+\sqrt{6}}{2\sqrt{6}+\sqrt{3}} \cdot \frac{2\sqrt{6}-\sqrt{3}}{2\sqrt{6}-\sqrt{3}}$

$= \frac{4\sqrt{18}-2\cdot 3+2\cdot 6-\sqrt{18}}{\left(2\sqrt{6}\right)^2-\left(\sqrt{3}\right)^2}$

$= \frac{3\sqrt{18}-6+12}{4\cdot 6-3} = \frac{3\sqrt{9\cdot 2}+6}{24-3}$

$= \frac{3\cdot 3\sqrt{2}+6}{21} = \frac{9\sqrt{2}+6}{21}$

$= \frac{3\left(3\sqrt{2}+2\right)}{21} = \frac{3\sqrt{2}+2}{7}$

76. $\sqrt{\frac{2}{7}} = \frac{\sqrt{2}}{\sqrt{7}} = \frac{\sqrt{2}}{\sqrt{7}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{2}{\sqrt{14}}$

77. $\frac{\sqrt[3]{3x}}{\sqrt[3]{y}} = \frac{\sqrt[3]{3x}}{\sqrt[3]{y}} \cdot \frac{\sqrt[3]{3^2x^2}}{\sqrt[3]{3^2x^2}}$

$= \frac{\sqrt[3]{3^3x^3}}{\sqrt[3]{3^2x^2y}} = \frac{3x}{\sqrt[3]{9x^2y}}$

78. $\frac{\sqrt{7}}{\sqrt{5}+\sqrt{3}} = \frac{\sqrt{7}}{\sqrt{5}+\sqrt{3}} \cdot \frac{\sqrt{7}}{\sqrt{7}}$

$= \frac{7}{\sqrt{35}+\sqrt{21}}$

79. $\frac{\sqrt{7}+\sqrt{3}}{\sqrt{7}-\sqrt{3}}$

$= \frac{\sqrt{7}+\sqrt{3}}{\sqrt{7}-\sqrt{3}} \cdot \frac{\sqrt{7}-\sqrt{3}}{\sqrt{7}-\sqrt{3}}$

$= \frac{\left(\sqrt{7}\right)^2-\left(\sqrt{3}\right)^2}{7-2\sqrt{7}\sqrt{3}+3} = \frac{7-3}{10-2\sqrt{21}}$

$= \frac{4}{10-2\sqrt{21}} = \frac{4}{2\left(5-\sqrt{21}\right)}$

$= \frac{2}{5-\sqrt{21}}$

80. $\sqrt{2x+4} = 6$

$\left(\sqrt{2x+4}\right)^2 = 6^2$

$2x+4 = 36$

$2x = 32$

$x = 16$

The solution checks. The solution set is $\{16\}$.

81. $\sqrt{x-5}+9 = 4$

$\sqrt{x-5} = -5$

The square root of a number is always nonnegative. The solution set is $\varnothing$ or $\{\ \}$.

82. $\sqrt{2x-3}+x = 3$

$\sqrt{2x-3} = 3-x$

$\left(\sqrt{2x-3}\right)^2 = (3-x)^2$

$2x-3 = 9-6x+x^2$

$0 = 12-8x+x^2$

$0 = x^2-8x+12$

$0 = (x-6)(x-2)$

$x-6=0 \qquad x-2=0$

$x=6 \qquad x=2$

6 is an extraneous solution. The solution set is $\{2\}$.

83. $\sqrt{x-4}+\sqrt{x+1} = 5$

$\sqrt{x-4} = 5-\sqrt{x+1}$

$\left(\sqrt{x-4}\right)^2 = \left(5-\sqrt{x+1}\right)^2$

$x-4 = 25-10\sqrt{x+1}+x+1$

$-30 = -10\sqrt{x+1}$

$\frac{-30}{-10} = \frac{-10\sqrt{x+1}}{-10}$

$3 = \sqrt{x+1}$

$3^2 = \left(\sqrt{x+1}\right)^2$

$9 = x+1$

$8 = x$

The solution checks. The solution set is $\{8\}$.

84. $\left(x^2+6x\right)^{\frac{1}{3}}+2=0$

$$\left(x^2+6x\right)^{\frac{1}{3}}=-2$$
$$\sqrt[3]{x^2+6x}=-2$$
$$\left(\sqrt[3]{x^2+6x}\right)^3=(-2)^3$$
$$x^2+6x=-8$$
$$x^2+6x+8=0$$
$$(x+4)(x+2)=0$$
$$x+4=0 \qquad x+2=0$$
$$x=-4 \qquad x=-2$$

Both solutions check. The solution set is $\{-4,-2\}$.

85. a. $f(x)=-1.6\sqrt{x}+54$

$$f(20)=-1.6\sqrt{20}+54$$
$$\approx 46.8$$

In 2005 (20 years after 1985), 46.8% of freshmen women described their health as above average.; The rounded value is the same as the value displayed by the graph.

b. $f(x)=-1.6\sqrt{x}+54$

$$f(x)=-1.6\sqrt{x}+54$$
$$44.4=-1.6\sqrt{x}+54$$
$$-9.6=-1.6\sqrt{x}$$
$$\frac{-9.6}{-1.6}=\frac{-1.6\sqrt{x}}{-1.6}$$
$$6=\sqrt{x}$$
$$6^2=\left(\sqrt{x}\right)^2$$
$$36=x$$

According to the model, 44.4% of freshmen women describe their health as above average 36 years after 1985, or in 2021.

86. $20,000=5000\sqrt{100-x}$

$$\frac{20,000}{5000}=\frac{5000\sqrt{100-x}}{5000}$$
$$4=\sqrt{100-x}$$
$$4^2=\left(\sqrt{100-x}\right)^2$$
$$16=100-x$$
$$-84=-x$$
$$84=x$$

20,000 people in the group will survive to 84 years old.

87. $\sqrt{-81}=\sqrt{81\cdot -1}=\sqrt{81}\sqrt{-1}=9i$

88. $\sqrt{-63}=\sqrt{9\cdot 7\cdot -1}$

$$=\sqrt{9}\sqrt{7}\sqrt{-1}=3i\sqrt{7}$$

89. $-\sqrt{-8}=-\sqrt{4\cdot 2\cdot -1}$

$$=-\sqrt{4}\sqrt{2}\sqrt{-1}=-2i\sqrt{2}$$

90. $(7+12i)+(5-10i)$

$$=7+12i+5-10i=12+2i$$

91. $(8-3i)-(17-7i)=8-3i-17+7i$

$$=-9+4i$$

92. $4i(3i-2)=4i\cdot 3i-4i\cdot 2$

$$=12i^2-8i$$
$$=12(-1)-8i$$
$$=-12-8i$$

93. $(7-5i)(2+3i)=14+21i-10i-15i^2$

$$=14+11i-15(-1)$$
$$=14+11i+15$$
$$=29+11i$$

94. $(3-4i)^2=3^2-2\cdot 3\cdot 4i+(4i)^2$

$$=9-24i+16i^2$$
$$=9-24i+16(-1)$$
$$=9-24i-16$$
$$=-7-24i$$

95. $(7+8i)(7-8i)=7^2-(8i)^2$

$$=49-64i^2$$
$$=49-64(-1)$$
$$=49+64$$
$$=113 \text{ or } 113+0i$$

96. $\sqrt{-8}\cdot\sqrt{-3}=\sqrt{4\cdot 2\cdot -1}\cdot\sqrt{3\cdot -1}$

$$=2\sqrt{2}i\cdot\sqrt{3}i=2\sqrt{6}i^2$$
$$=2\sqrt{6}(-1)=-2\sqrt{6}$$
$$=-2\sqrt{6} \text{ or } -2\sqrt{6}+0i$$

97. $\dfrac{6}{5+i}=\dfrac{6}{5+i}\cdot\dfrac{5-i}{5-i}=\dfrac{30-6i}{25-i^2}$
$=\dfrac{30-6i}{25-(-1)}=\dfrac{30-6i}{25+1}$
$=\dfrac{30-6i}{26}=\dfrac{30}{26}-\dfrac{6}{26}i$
$=\dfrac{15}{13}-\dfrac{3}{13}i$

98. $\dfrac{3+4i}{4-2i}=\dfrac{3+4i}{4-2i}\cdot\dfrac{4+2i}{4+2i}$
$=\dfrac{12+6i+16i+8i^2}{16-4i^2}$
$=\dfrac{12+22i+8(-1)}{16-4(-1)}$
$=\dfrac{12+22i-8}{16+4}=\dfrac{4+22i}{20}$
$=\dfrac{4}{20}+\dfrac{22}{20}i=\dfrac{1}{5}+\dfrac{11}{10}i$

99. $\dfrac{5+i}{3i}=\dfrac{5+i}{3i}\cdot\dfrac{i}{i}=\dfrac{5i+i^2}{3i^2}$
$=\dfrac{5i+(-1)}{3(-1)}=\dfrac{5i-1}{-3}$
$=\dfrac{-1}{-3}+\dfrac{5}{-3}i=\dfrac{1}{3}-\dfrac{5}{3}i$

100. $i^{16}=\left(i^2\right)^8=(-1)^8=1$

101. $i^{23}=i^{22}\cdot i=\left(i^2\right)^{11}i=(-1)^{11}i=(-1)i=-i$

Chapter 10 Test

1. a. $f(-14)=\sqrt{8-2(-14)}$
$=\sqrt{8+28}=\sqrt{36}=6$

b. To find the domain, set the radicand greater than or equal to zero and solve the resulting inequality.
$8-2x\ge 0$
$-2x\ge -8$
$x\le 4$
The domain of f is $(-\infty,4]$.

2. $27^{-\frac{4}{3}}=\dfrac{1}{27^{\frac{4}{3}}}=\dfrac{1}{\left(\sqrt[3]{27}\right)^4}=\dfrac{1}{(3)^4}=\dfrac{1}{81}$

3. $\left(25x^{-\frac{1}{2}}y^{\frac{1}{4}}\right)^{\frac{1}{2}}=25^{\frac{1}{2}}x^{-\frac{1}{4}}y^{\frac{1}{8}}$
$=5x^{-\frac{1}{4}}y^{\frac{1}{8}}$
$=\dfrac{5y^{\frac{1}{8}}}{x^{\frac{1}{4}}}$

4. $\sqrt[8]{x^4}=\left(x^4\right)^{\frac{1}{8}}=x^{4\cdot\frac{1}{8}}=x^{\frac{1}{2}}=\sqrt{x}$

5. $\sqrt[4]{x}\cdot\sqrt[5]{x}=x^{\frac{1}{4}}\cdot x^{\frac{1}{5}}=x^{\frac{1}{4}+\frac{1}{5}}=x^{\frac{5}{20}+\frac{4}{20}}$
$=x^{\frac{9}{20}}=\sqrt[20]{x^9}$

6. $\sqrt{75x^2}=\sqrt{25\cdot 3x^2}=5|x|\sqrt{3}$

7. $\sqrt{x^2-10x+25}=\sqrt{(x-5)^2}$
$=|x-5|$

8. $\sqrt[3]{16x^4y^8}=\sqrt[3]{8\cdot 2\cdot x^3\cdot x\cdot y^6\cdot y^2}$
$=\sqrt[3]{8x^3y^6\cdot 2xy^2}$
$=2xy^2\sqrt[3]{2xy^2}$

9. $\sqrt[5]{-\dfrac{32}{x^{10}}}=\sqrt[5]{-\dfrac{2^5}{\left(x^2\right)^5}}=-\dfrac{2}{x^2}$

10. $\sqrt[3]{5x^2}\cdot\sqrt[3]{10y}=\sqrt[3]{50x^2y}$

11. $\sqrt[4]{8x^3y}\cdot\sqrt[4]{4xy^2}=\sqrt[4]{32x^4y^3}$
$=\sqrt[4]{16\cdot 2\cdot x^4\cdot y^3}$
$=\sqrt[4]{16x^4\cdot 2y^3}$
$=2x\sqrt[4]{2y^3}$