142. $\dfrac{1+i}{1+2i}+\dfrac{1-i}{1-2i}$

143. $\dfrac{8}{1+\dfrac{2}{i}}$

Review Exercises

144. Simplify:

$$\frac{\dfrac{x}{y^2}+\dfrac{1}{y}}{\dfrac{y}{x^2}+\dfrac{1}{x}}.$$

(Section 7.5, Example 3 or Example 6)

145. Solve for x: $\dfrac{1}{x}+\dfrac{1}{y}=\dfrac{1}{z}$.

(Section 7.6, Example 7)

146. If $f(x)=2x^2-x$ and $g(x)=x-6$, find $(g-f)(3)$. (Section 8.3, Example 4)

Preview Exercises

Exercises 147–149 will help you prepare for the material covered in the first section of the next chapter.

147. Solve by factoring: $2x^2+7x-4=0$.

148. Solve by factoring: $x^2=9$.

149. Use substitution to determine if $-\sqrt{6}$ is a solution of the quadratic equation $3x^2=18$.

GROUP PROJECT

CHAPTER 10

Group members should consult an almanac, newspaper, magazine, or the Internet and return to the group with as much data as possible that show phenomena that are continuing to grow over time, but whose growth is leveling off. Select the five data sets that you find most intriguing. Let x represent the number of years after the first year in each data set. Model the data by hand using

$$f(x)=a\sqrt{x}+b.$$

Use the first and last data points to find values for a and b. The first data point corresponds to $x=0$. Its second coordinate gives the value of b. To find a, substitute the second data point into $f(x)=a\sqrt{x}+b$, with the value that you obtained for b. Now solve the equation and find a. Substitute a and b into $f(x)=a\sqrt{x}+b$ to obtain a square root function that models each data set. Then use the function to make predictions about what might occur in the future. Are there circumstances that might affect the accuracy of the predictions? List some of these circumstances.

Chapter 10 Summary

Definitions and Concepts	Examples

Section 10.1 Radical Expressions and Functions

Definitions and Concepts	Examples
If $b^2=a$, then b is a square root of a. The principal square root of a, designated $\sqrt{a}$, is the nonnegative number satisfying $b^2=a$. The negative square root of a is written $-\sqrt{a}$. A square root of a negative number is not a real number. A radical function in x is a function defined by an expression containing a root of x. The domain of a square root function is the set of real numbers for which the radicand is nonnegative.	Let $f(x)=\sqrt{6-2x}$. $f(-15)=\sqrt{6-2(-15)}=\sqrt{6+30}=\sqrt{36}=6$ $f(5)=\sqrt{6-2\cdot5}=\sqrt{6-10}=\sqrt{-4}$, not a real number Domain of f: Set the radicand greater than or equal to zero. $6-2x\ge0$ $-2x\ge-6$ $x\le3$ Domain of $f=(-\infty,3]$

Definitions and Concepts	Examples

Section 10.1 Radical Expressions and Functions (continued)

The cube root of a real number a is written $\sqrt[3]{a}$.

$$\sqrt[3]{a} = b \quad \text{means that} \quad b^3 = a.$$

The nth root of a real number a is written $\sqrt[n]{a}$. The number n is the index. Every real number has one root when n is odd. The odd nth root of a, $\sqrt[n]{a}$, is the number b for which $b^n = a$. Every positive real number has two real roots when n is even. An even root of a negative number is not a real number.

If n is even, then $\sqrt[n]{a^n} = |a|$.

If n is odd, then $\sqrt[n]{a^n} = a$.

- $\sqrt[3]{-8} = -2$ because $(-2)^3 = -8$.
- $\sqrt[4]{-16}$ is not a real number.
- $\sqrt{x^2 - 14x + 49} = \sqrt{(x-7)^2} = |x - 7|$
- $\sqrt[3]{125(x+6)^3} = 5(x+6)$

Section 10.2 Rational Exponents

- $a^{\frac{1}{n}} = \sqrt[n]{a}$
- $a^{\frac{m}{n}} = (\sqrt[n]{a})^m$ or $\sqrt[n]{a^m}$
- $a^{-\frac{m}{n}} = \dfrac{1}{a^{\frac{m}{n}}}$

- $121^{\frac{1}{2}} = \sqrt{121} = 11$
- $64^{\frac{1}{3}} = \sqrt[3]{64} = 4$
- $27^{\frac{5}{3}} = (\sqrt[3]{27})^5 = 3^5 = 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 = 243$
- $16^{-\frac{3}{4}} = \dfrac{1}{16^{\frac{3}{4}}} = \dfrac{1}{(\sqrt[4]{16})^3} = \dfrac{1}{2^3} = \dfrac{1}{8}$
- $(\sqrt[3]{7xy})^4 = (7xy)^{\frac{4}{3}}$

Properties of integer exponents are true for rational exponents. An expression with rational exponents is simplified when no parentheses appear, no powers are raised to powers, each base occurs once, and no negative or zero exponents appear.

Simplify: $(8x^{\frac{1}{3}}y^{-\frac{1}{2}})^{\frac{1}{3}}$.

$$= 8^{\frac{1}{3}}(x^{\frac{1}{3}})^{\frac{1}{3}}(y^{-\frac{1}{2}})^{\frac{1}{3}}$$

$$= 2x^{\frac{1}{9}}y^{-\frac{1}{6}} = \frac{2x^{\frac{1}{9}}}{y^{\frac{1}{6}}}$$

Some radical expressions can be simplifed using rational exponents. Rewrite the expression using rational exponents, simplify, and rewrite in radical notation if rational exponents still appear.

- $\sqrt[9]{x^3} = x^{\frac{3}{9}} = x^{\frac{1}{3}} = \sqrt[3]{x}$
- $\sqrt[5]{x^2} \cdot \sqrt[4]{x} = x^{\frac{2}{5}} \cdot x^{\frac{1}{4}} = x^{\frac{2}{5}+\frac{1}{4}}$
  $= x^{\frac{8}{20}+\frac{5}{20}} = x^{\frac{13}{20}} = \sqrt[20]{x^{13}}$

Section 10.3 Multiplying and Simplifying Radical Expressions

The product rule for radicals can be used to multiply radicals

$$\sqrt[n]{a} \cdot \sqrt[n]{b} = \sqrt[n]{ab}.$$

$\sqrt[3]{7x} \cdot \sqrt[3]{10y^2} = \sqrt[3]{7x \cdot 10y^2} = \sqrt[3]{70xy^2}$

The product rule for radicals can be used to simplify radicals:

$$\sqrt[n]{ab} = \sqrt[n]{a} \cdot \sqrt[n]{b}.$$

A radical expression with index n is simplified when its radicand has no factors that are perfect nth powers. To simplify, write the radicand as the product of two factors, one of which is the greatest perfect nth power. Then use the product rule to take the nth root of each factor. If all variables in a radicand are positive, then

$$\sqrt[n]{a^n} = a.$$

Some radicals can be simplified after multiplication is performed.

- Simplify: $\sqrt[3]{54x^7y^{11}}$.
  $= \sqrt[3]{27 \cdot 2 \cdot x^6 \cdot x \cdot y^9 \cdot y^2}$
  $= \sqrt[3]{(27x^6y^9)(2xy^2)}$
  $= \sqrt[3]{27x^6y^9} \cdot \sqrt[3]{2xy^2} = 3x^2y^3\sqrt[3]{2xy^2}$
- Assuming positive variables, multiply and simplify:
  $\sqrt[4]{4x^2y} \cdot \sqrt[4]{4xy^3}$.
  $= \sqrt[4]{4x^2y \cdot 4xy^3} = \sqrt[4]{16x^3y^4}$
  $= \sqrt[4]{16y^4} \cdot \sqrt[4]{x^3} = 2y\sqrt[4]{x^3}$

Definitions and Concepts	Examples

Section 10.4 Adding, Subtracting, and Dividing Radical Expressions

Like radicals have the same indices and radicands. Like radicals can be added or subtracted using the distributive property. In some cases, radicals can be combined once they have been simplified.

$$4\sqrt{18} - 6\sqrt{50}$$
$$= 4\sqrt{9\cdot 2} - 6\sqrt{25\cdot 2} = 4\cdot 3\sqrt{2} - 6\cdot 5\sqrt{2}$$
$$= 12\sqrt{2} - 30\sqrt{2} = -18\sqrt{2}$$

The quotient rule for radicals can be used to simplify radicals:

$$\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}.$$

$$\sqrt[3]{-\frac{8}{x^{12}}} = \frac{\sqrt[3]{-8}}{\sqrt[3]{x^{12}}} = -\frac{2}{x^4}$$

$$\sqrt[3]{x^{12}} = (x^{12})^{\frac{1}{3}} = x^4$$

The quotient rule for radicals can be used to divide radicals with the same indices:

$$\frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\frac{a}{b}}.$$

Some radicals can be simplified after the division is performed.

Assuming a positive variable, divide and simplify:

$$\frac{\sqrt[4]{64x^5}}{\sqrt[4]{2x^{-2}}} = \sqrt[4]{32x^{5-(-2)}} = \sqrt[4]{32x^7}$$
$$= \sqrt[4]{16\cdot 2\cdot x^4\cdot x^3} = \sqrt[4]{16x^4}\cdot\sqrt[4]{2x^3}$$
$$= 2x\sqrt[4]{2x^3}.$$

Section 10.5 Multiplying with More Than One Term and Rationalizing Denominators

Radical expressions with more than one term are multiplied in much the same way that polynomials with more than one term are multiplied.

- $\sqrt{5}(2\sqrt{6} - \sqrt{3}) = 2\sqrt{30} - \sqrt{15}$
- $(4\sqrt{3} - 2\sqrt{2})(\sqrt{3} + \sqrt{2})$

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$$= 4\sqrt{3}\cdot\sqrt{3} + 4\sqrt{3}\cdot\sqrt{2} - 2\sqrt{2}\cdot\sqrt{3} - 2\sqrt{2}\cdot\sqrt{2}$$
$$= 4\cdot 3 + 4\sqrt{6} - 2\sqrt{6} - 2\cdot 2$$
$$= 12 + 4\sqrt{6} - 2\sqrt{6} - 4 = 8 + 2\sqrt{6}$$

Radical expressions that involve the sum and difference of the same two terms are called conjugates. Use

$$(A + B)(A - B) = A^2 - B^2$$

to multiply conjugates.

$$(8 + 2\sqrt{5})(8 - 2\sqrt{5})$$
$$= 8^2 - (2\sqrt{5})^2$$
$$= 64 - 4\cdot 5$$
$$= 64 - 20 = 44$$

The process of rewriting a radical expression as an equivalent expression without any radicals in the denominator is called rationalizing the denominator. When the denominator contains a single radical with an nth root, multiply the numerator and the denominator by a radical of index n that produces a perfect nth power in the denominator's radicand.

Rationalize the denominator: $\frac{7}{\sqrt[3]{2x}}$.

$$= \frac{7}{\sqrt[3]{2x}}\cdot\frac{\sqrt[3]{4x^2}}{\sqrt[3]{4x^2}} = \frac{7\sqrt[3]{4x^2}}{\sqrt[3]{8x^3}} = \frac{7\sqrt[3]{4x^2}}{2x}$$

Definitions and Concepts	Examples

Section 10.5 Multiplying with More Than One Term and Rationalizing Denominators (continued)

If the denominator contains two terms, rationalize the denominator by multiplying the numerator and the denominator by the conjugate of the denominator.

$$\frac{13}{5-\sqrt{3}} = \frac{13}{5-\sqrt{3}} \cdot \frac{5+\sqrt{3}}{5+\sqrt{3}} = \frac{13(5+\sqrt{3})}{5^2 - (\sqrt{3})^2} = \frac{13(5+\sqrt{3})}{25-3} = \frac{13(5+\sqrt{3})}{22}$$

Section 10.6 Radical Equations

A radical equation is an equation in which the variable occurs in a radicand.

Solving Radical Equations Containing *n*th Roots

1. Isolate one radical on one side of the equation.
2. Raise both sides to the nth power.
3. Solve the resulting equation.
4. Check proposed solutions in the original equation. Solutions of an equation to an even power that is radical-free, but not the original equation, are called extraneous solutions.

Solve: $\sqrt{6x+13} - 2x = 1$.

$\sqrt{6x+13} = 2x + 1$ — Isolate the radical.

$(\sqrt{6x+13})^2 = (2x+1)^2$ — Square both sides.

$6x + 13 = 4x^2 + 4x + 1$ — $(A+B)^2 = A^2 + 2AB + B^2$

$0 = 4x^2 - 2x - 12$ — Subtract $6x + 13$ from both sides.

$0 = 2(2x^2 - x - 6)$ — Factor out the GCF.

$0 = 2(2x+3)(x-2)$ — Factor completely.

$2x + 3 = 0$ or $x - 2 = 0$ — Set variable factors equal to zero.

$2x = -3$ $\quad x = 2$ — Solve for x.

$x = -\frac{3}{2}$

Check both proposed solutions. 2 checks, but $-\frac{3}{2}$ is extraneous. The solution is 2 and the solution set is {2}.

Section 10.7 Complex Numbers

The imaginary unit i is defined as

$$i = \sqrt{-1}, \quad \text{where} \quad i^2 = -1.$$

The set of numbers in the form $a + bi$ is called the set of complex numbers; a is the real part and b is the imaginary part. If $b = 0$, the complex number is a real number. If $b \neq 0$, the complex number is an imaginary number.

- $\sqrt{-81} = \sqrt{81(-1)} = \sqrt{81}\sqrt{-1} = 9i$
- $\sqrt{-75} = \sqrt{75(-1)} = \sqrt{25 \cdot 3}\sqrt{-1} = 5i\sqrt{3}$

To add or subtract complex numbers, add or subtract their real parts and add or subtract their imaginary parts.

$(2 - 4i) - (7 - 10i)$
$= 2 - 4i - 7 + 10i$
$= (2 - 7) + (-4 + 10)i = -5 + 6i$

Definitions and Concepts	Examples

Section 10.7 Complex Numbers (continued)

To multiply complex numbers, multiply as if they were polynomials. After completing the multiplication, replace i^2 with -1. When performing operations with square roots of negative numbers, begin by expressing all square roots in terms of i. Then multiply.

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- $(2-3i)(4+5i) = 8 + 10i - 12i - 15i^2$
  $= 8 + 10i - 12i - 15(-1)$
  $= 23 - 2i$
- $\sqrt{-36}\cdot\sqrt{-100} = \sqrt{36(-1)}\cdot\sqrt{100(-1)}$
  $= 6i \cdot 10i = 60i^2 = 60(-1) = -60$

The complex numbers $a + bi$ and $a - bi$ are conjugates. Conjugates can be multiplied using the formula

$$(A+B)(A-B) = A^2 - B^2.$$

The multiplication of conjugates results in a real number.

$$(3+5i)(3-5i) = 3^2 - (5i)^2 = 9 - 25i^2 = 9 - 25(-1) = 34$$

To divide complex numbers, multiply the numerator and the denominator by the conjugate of the denominator in order to obtain a real number in the denominator. This real number becomes the denominator of a and b in the quotient $a + bi$.

$$\frac{5+2i}{4-i} = \frac{5+2i}{4-i}\cdot\frac{4+i}{4+i} = \frac{20+5i+8i+2i^2}{16-i^2}$$
$$= \frac{20+13i+2(-1)}{16-(-1)}$$
$$= \frac{20+13i-2}{16+1}$$
$$= \frac{18+13i}{17} = \frac{18}{17} + \frac{13}{17}i$$

To simplify powers of i, rewrite the expression in terms of i^2. Then replace i^2 with -1 and simplify.

Simplify: i^{27}.

$$i^{27} = i^{26}\cdot i = (i^2)^{13} i$$
$$= (-1)^{13} i = (-1)i = -i$$

CHAPTER 10 REVIEW EXERCISES

10.1 *In Exercises 1–5, find the indicated root, or state that the expression is not a real number.*

1. $\sqrt{81}$
2. $-\sqrt{\frac{1}{100}}$
3. $\sqrt[3]{-27}$
4. $\sqrt[4]{-16}$
5. $\sqrt[5]{-32}$

In Exercises 6–7, find the indicated function values for each function. If necessary, round to two decimal places. If the function value is not a real number and does not exist, so state.

6. $f(x) = \sqrt{2x-5};\quad f(15), f(4), f\left(\frac{5}{2}\right), f(1)$
7. $g(x) = \sqrt[3]{4x-8};\quad g(4), g(0), g(-14)$

In Exercises 8–9, find the domain of each square root function.

8. $f(x) = \sqrt{x-2}$
9. $g(x) = \sqrt{100-4x}$

In Exercises 10–15, simplify each expression. Assume that each variable can represent any real number, so include absolute value bars where necessary.

10. $\sqrt{25x^2}$
11. $\sqrt{(x+14)^2}$
12. $\sqrt{x^2-8x+16}$
13. $\sqrt[3]{64x^3}$
14. $\sqrt[4]{16x^4}$
15. $\sqrt[5]{-32(x+7)^5}$

10.2 *In Exercises 16–18, use radical notation to rewrite each expression. Simplify, if possible.*

16. $(5xy)^{\frac{1}{3}}$
17. $16^{\frac{3}{2}}$
18. $32^{\frac{4}{5}}$

In Exercises 19–20, rewrite each expression with rational exponents.

19. $\sqrt{7x}$
20. $\left(\sqrt[3]{19xy}\right)^5$